Credit Ratings and Structured Finance*

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Abstract

The poor performance of credit ratings on structured finance products has prompted investigation into the role of Credit Rating Agencies (CRAs) in designing and marketing these products. We analyze a two-period reputation model where a CRA both designs and rates securities that are sold to different clienteles: unconstrained investors and investors constrained by minimum quality requirements. Ratings inflation increases when quality requirements for constrained investors are higher and decreases when the quality of the asset pool is higher. Securities for both types of investors may have inflated ratings. The motivation for pooling assets derives from tailoring to clienteles and from reputational incentives.

Keywords: Credit rating agencies, reputation, structured finance

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1 Introduction

The recent financial crisis has prompted much investigation into the role of credit-rating agencies (CRAs). With the dramatic increase in the use of structured finance products, the agencies quickly expanded their business and earned outsize profits (Moody's, for example, tripled its profits between 2002 and 2006). Ratings quality seems to have suffered, as structured finance products were increasingly given top ratings shortly before the financial markets collapsed. In this paper, we ask how the design of such products is influenced by CRAs, and how their structure changes with market incentives.

The design of structured finance products is marked by close collaboration between issuers and rating agencies. Issuers depend on rating agencies to certify quality and to be able to sell to regulated investors. Beyond directly paying CRAs for ratings (the "issuer pays" system), Griffin and Tang (2012) write that "The CRA and underwriter may engage in discussion and iteration over assumptions made in the valuation process." Agencies also provide their models to issuers even before the negotiations take place (Benmelech and Dlugosz, 2009). These products are characterized by careful selection of the underlying asset pool and private information about asset quality.

We present a reputation-based two-period model of rating structured products. Each period an issuer has a set of good and bad assets that it can put into multiple pools and issue simple securities against. A monopoly CRA assists in the design of these securities and rates them. The prospect of earning future profits can give the CRA reputational incentives to provide accurate ratings. We model reputation by positing that the CRA is long-lived and can be one of two types: truthful or opportunistic. The type of the CRA is revealed between periods with a probability that is increasing in the amount of ratings inflation.

Securities are sold to rational investors who cannot observe the type of the CRA or the quality of the securities, but make inferences from the ratings and the amount of underlying assets. There are two types of investors, constrained and unconstrained. Constrained investors need the expected quality of securities to be above a certain level, while unconstrained investors can purchase any type of security. A principal motivation

for securitization is to appeal to investor groups with heterogeneous preferences. The obvious example of this was the increased demand for highly rated investments in the 2000s by regulated entities (e.g. banks, pension funds, insurance companies).

The level of ratings inflation depends on the opportunistic CRA's trade-off between passing off bad assets as good ones, which allows it to extract more rents from the issuer that retains the good assets, and having the issuer include more good assets, which makes it less likely that the CRA will be identified as opportunistic and increases its expected future profits.

We present several findings on the drivers of this ratings inflation. First, when quality requirements for constrained investors are higher, ratings inflation increases. The tighter requirements reduce the amount of securities that can be created for constrained investors, decreasing the benefits of maintaining reputation for the future. This implies that tighter regulation of constrained investors has negative equilibrium effects on the quality of assets sold. Second, when the quality of the future asset pool increases, either through a larger supply of good assets or higher valuations of assets, ratings inflation decreases as there is a larger reward for maintaining reputation. This provides a link between fundamental asset values and ratings inflation, suggesting that ratings quality will be countercyclical.[1] Third, if the quality requirements for constrained investors are moderate, only securities sold to constrained investors will have their ratings inflated. However, when quality requirements are sufficiently high, constrained investors will be sold fewer securities and inflation will spill over into securities for unconstrained investors.

We provide two new motivations for the pooling of assets. First, in our model structuring motives derive from the need to tailor products for constrained investors. Second, a CRA can balance the informational advantage over investors with the need to maintain its reputation by choosing the right mix of good and bad assets to include.

There is substantial evidence of asymmetric information and strategic asset pool selection for structured finance products. Downing, Jaffee, and Wallace (2009) compare the performance of pools of mortgages that are pass-through MBS with no tranching

[1] This is consistent with theoretical (Bar-Isaac and Shapiro, 2013) and empirical (Auh, 2013) studies of rating accuracy over the business cycle. We discuss this further in the text.

with securitized REMICs (Real Estate Mortgage Investment Conduits) with tranching. The extra layer of securitization and anonymity in sales allows for a selection of worse performing pools due to private information. This is shown to be true with ex-post performance data. Moreover, there is a "lemons spread" due to rational discounting of these securities. An, Deng, and Gabriel (2011) show that portfolio lenders use private information to pass off lower quality loans to commercial mortgage backed securities (CMBS). Conduit lenders, who originate loans for direct sale into securitization markets do not select loans and hence have higher quality loans conditioning on the observables. The analysis shows a lemons discount for portfolio loans. This lemons discount is lower for multifamily loans, which have lower levels of uncertainty and lender private information than retail, office, and industrial loans. Elul (2011) demonstrates that securitized mortgages perform worse than portfolio loans, with the largest differences in prime mortgages in private (non-GSE) securitizations, consistent with the presence of adverse selection. Ashcraft, Goldsmith-Pinkham, and Vickery (2011) find that the MBS deals that were most likely to underperform were the ones with more interest-only loans (because of limited performance history) and lower documentation, that is, loans that were more opaque or difficult to evaluate.

We find that ratings inflation is an important element of structured finance. In the data, Gorton and Metrick (2012) show that AAA-rated asset backed securities have significantly higher cumulative default rates compared to AAA-rated corporate bonds. This is also true for lower rating categories, but the differences lessen as ratings worsen. Cornaggia, Cornaggia, and Hund (2013), also find that structured products are overrated compared to corporate issues, while municipal and sovereign bonds are underrated, over the sample period 1980-2010. Ashcraft, Goldsmith-Pinkham, and Vickery (2011) find that as MBS issuance volume shot up between 2005 and mid-2007, ratings quality declined. Specifically, subordination levels[2] for subprime and Alt-A MBS deals decreased over this period when conditioning on the overall risk of the deal. Subsequent ratings downgrades

[2]The subordination level they use is the fraction of the deal that is junior to the AAA tranche. A smaller fraction means that the AAA tranche is less "protected" from defaults, and therefore less costly from the issuer's point of view.

for the 2005 to mid-2007 cohorts were dramatically larger than for previous cohorts. Vickery (2012) shows that ratings inflation occurred for subprime mortgage backed securities at all investment grade rating levels, not just AAA. Griffin and Tang (2012) show that CRA adjustments to their models' predictions of credit risk in the CDO market were positively related to future downgrades. These adjustments were overwhelmingly positive and the amount adjusted (the width of the AAA tranche) increased sharply from 2003 to 2007 (from 6% to 18.2%). He, Qian, and Strahan (2012) find that top rated MBS tranches sold by larger issuers[3] performed significantly worse (prices drop more) and have higher initial yields than those sold by small issuers during the boom period of 2004 to 2006. Stanton and Wallace (2012) demonstrate that the spread between CMBS and corporate bond yields for ratings AA and AAA fell significantly after 2002 (and did not fall for bonds with worse ratings), when risk-based capital requirements for top rated CMBS were lowered significantly. Also, CMBS rated below AA were upgraded to AA or AAA significantly more than the rate observed in a comparable sample of RMBS leading up to the crisis.

1.1 Theoretical Literature

The link between ratings quality and reputation is key for our results. Mathis, McAndrews, and Rochet (2009) examines how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation in which a monopolist CRA may mix between lying and truthtelling to build up/exploit its reputation. The authors focus on whether an equilibrium in which the CRA tells the truth in every period exists, and they demonstrate that truthtelling incentives are weaker when the CRA has more business from rating complex products. Strausz (2005) is similar in structure to Mathis et al. (2009), but examines information intermediaries in general. Bar-Isaac and Shapiro (2013) incorporate economic shocks and show that CRA accuracy may be countercyclical, which is also consistent with our results. Our model of reputation is similar to those above, but the ability of the CRA to strategically structure what type of securities are

[3] They define larger by market share in terms of deals. As a robustness check, they also look at market share in terms of dollars and find similar results.

sold while at the same time rating those securities is new and links our work directly to the phenomenon of structured finance.

Cohn, Rajan, and Strobl (2013) show that issuer manipulation of the signal the CRA receives about asset quality may cause CRAs to exert less effort in gathering information. Opp, Opp, and Harris (2013) examine how ratings-contingent regulation affects the informativeness of ratings.

Hartman-Glaser (2013) models an issuer who plays an infinitely repeated game with reputation concerns. The issuer signals through the amount retained, an explicitly costly signal. In our paper, we focus on the ability of the issuer to select assets, while pooling and issuing multiple securities can occur due to the clientele effect.

In addition to their empirical results, An, Deng, and Gabriel (2011) have a theoretical model where a portfolio lender can only pass off some loans because of the lemons problem and must sell at a discount. Their results suggest that the magnitude of the lemons discount associated with portfolio loan sales varies positively with the dispersion of loan quality in the pool and inversely with the seller's cost of holding the loans in its portfolio.

2 The Model without a Rating Agency

We begin with two types of agents: an issuer and investors. All agents are risk neutral. We will analyze the issuer's problem first without any rating agency, and then look at the effect of introducing a rating agency.

The issuer has assets of measure N, of which a mass μ are good and worth G to investors, and a mass $N - \mu$ are bad and worth B to investors. Good assets are worth g to an issuer, while bad assets are worth b to an issuer.

We assume the following ordering:

$$b < B < g < G$$

The issuer's valuations of the assets are lower than the investors' values for the assets. This can occur for several reasons: the issuer may have valuable alternative investment

opportunities, regulatory capital requirements for holding the assets, and/or the need to transfer risk off of its balance sheet. The inequality of $g > B$ indicates that issuers prefer to keep good assets rather than sell them off if investors perceive them as B.

There is a continuum of risk-neutral investors, each with a wealth of 1.[4] Investors can be one of two types: *constrained* (C) or *unconstrained (U).* A measure $I_C > 0$ of them are constrained and a measure $I_U > 0$ of them are unconstrained.

Constrained investors will only purchase securities that they believe are of high quality and have an expected value of at least $\bar{V}$. We assume that $g < \bar{V} < G$, which implies that $\bar{V} > B$, i.e. a constrained investor would not buy a security worth B. As will be explained later, $\bar{V} > g$ guarantees a positive profit margin if constrained investors can be served, reducing the number of cases to study.

Constrained investors may be constrained because of regulations (for example banks, pension funds, and insurance companies are often restricted in the types of assets they may hold), internal by-law restrictions, or because of their portfolio hedging requirements. In practice, regulations currently require these types of institutions to hold investment products that have specific ratings. We relax this requirement for two reasons. First, regulations are being changed to weaken the dependence on ratings, and are tending toward using institutional risk models.[5] Second, we do not want ratings to be driven by ratings-based regulation, which has been discussed amply in the literature (see Opp, Opp, and Harris (2013) and White (2010)).[6] Lastly, an important argument for securitization is the clientele effect, which is what we are directly modeling here.

The unconstrained investors are willing to purchase any security. They may be hedge

[4]This assumes that investors are credit constrained, which might arise from borrowing frictions (see, for example, Boot and Thakor (1993)).

[5]In the U.S., the Dodd-Frank bill mandates removing references to credit ratings and replacing them with alternatives. The alternatives suggested are using internal models in conjunction with market and rating information (see http://www.financialstabilityboard.org/wp-content/uploads/c_140429z.pdf?page_moved=1) The E.U., in the CRA III legislation, mandates eliminating the mechanistic reliance on ratings and finding alternatives. Alternatives have not been settled on, although the internal ratings based approach is referenced (see http://ec.europa.eu/internal_market/rating-agencies/docs/140512-fsb-eu-response_en.pdf).

[6]In a different version of this paper, we look at a model where constrained investors need certain ratings. This model is more complex, but has very similar qualitative properties (although the interpretation of those properties will vary given the interpretation of rating-based regulation versus quality constraints).

funds or other institutional investors. We assume both types of investor are rational in the sense that they update given available information and maximize their payoff.[7]

The issuer can put together portfolios of good and bad assets for unconstrained and constrained investors through *securitization.* We define securitization as selling securities based on the payoffs of the portfolio. We restrict the space of securities by defining the payoff of a security as the average payoff of the underlying pool of assets. Letting μ_i and ν_i denote the measures of good and bad assets backing a portfolio $i \in \{U, C\}$ of positive measure, the payoff for securities based on this portfolio i will be $(\mu_i G + \nu_i B) / (\mu_i + \nu_i)$, and the quantity of such securities $\mu_i + \nu_i$.

This, of course, is an extremely stylized model of how securitization works, in practice things are much more complex (see Coval, Jurek, and Stafford (2009) for a detailed description of the process). In fact, the securities designed here resemble pass-through securities, where investors get pro-rata shares of cash flows from the underlying mortgages. We do not model the seniority structure/waterfall of non-pass-through securities.

We will assume that the total value of all bad assets is greater than the aggregate wealth of all investors: $(N - \mu) B \geq I_C + I_U$. This means that an issuer will always be able to replace a good asset with a bad asset, setting the stage for a severe lemons problem. We will also assume that unconstrained investors have enough wealth to buy all of the good assets, i.e. $I_U > \mu G$. This is completely for ease of exposition and does not affect results.

We will assume that the demand of all constrained investors cannot be met, as there is a scarcity of good assets.

$$I_C > \bar{V}\mu \frac{G - B}{\bar{V} - B} \tag{A1}$$

This constraint says that constrained investors have more wealth than the value of securities that could be generated for them. It follows that $I_C > \bar{V}\mu$. We also assume

[7] There has been much discussion about the naivete of investors in the RMBS market, e.g. see Bolton, Freixas, and Shapiro (2012). However, not all structured finance markets are necessarily characterized in such a way, as Stanton and Wallace (2012) point out: "All agents in the CMBS market can reasonably be viewed as sophisticated, informed investors."

that the issuer can't observe investor types. This will not matter, as the issuer can use simple incentive contracts (giving an epsilon more of surplus to unconstrained investors) to perfectly screen them. Finally, we will assume issuers make take it or leave it offers to investors. Investors' reservation utility is normalized to zero.

2.1 Full Information

Suppose that there is full information about the securities' profile. The issuer's payoff net of the initial value of the portfolio, $(N-\mu)\,b+\mu g$, is (we will use the convention of reporting net payoffs for the rest of the paper):

$$(\mu_U+\mu_C)\,(G-g)+(\nu_U+\nu_C)\,(B-b). \tag{1}$$

The full information profit-maximizing solution entails selling as many assets as possible to constrained investors, and securities worth B to all unconstrained investors. Note that this dominates selling only to constrained investors as unconstrained investors place a higher value on any remaining assets than the issuer does.

Lemma 1 *The profit-maximizing allocation has:*

1. *A constrained pool containing all of the good assets and a measure of bad assets* $\mu(G-\bar{V})/\left(\bar{V}-B\right)$ *such that the average value in the pool equals* $\bar{V}$*,*

2. *An unconstrained pool containing a measure* I_U/B *of bad assets,*

The issuer thus engages in securitization by selling different securities to different types of investors and retaining the remaining bad assets.

2.2 Asymmetric Information

When the quality of the issuer's securities is private information, the issuer faces the problem of persuading investors that the securities are of a certain quality. We will demonstrate that this directly leads to a lemons problem. This is similar to the adverse selection problem found in the empirical work of Downing, Jaffee, and Wallace (2009)

and An, Deng, and Gabriel (2011), who document a lemons spread and worse ex-post performance when issuers have more scope for selecting the loans that are securitized.

We assume the issuer will offer a range of securities to investors with labels of their quality. Investors will observe the total measure of assets issued against each pool (the quantity of securities), $\mu_i + \nu_i$, and the reported measures of good and bad assets in the pools, $\bar{\mu}_i$ and $\bar{\nu}_i$, where $i \in \{U, C\}$. We employ the equilibrium concept of Perfect Bayesian Equilibrium. In the following lemma, we describe the equilibrium allocation.

Lemma 2 *In equilibrium, the issuer will sell securities backed by a measure I_U/B of bad assets to unconstrained investors.*

This represents a breakdown of the market typical for adverse selection problems. The issuer can't include any good assets in equilibrium. If it did, and investors believed the good assets were included and raised their valuations, the issuer would then replace the assets with bad ones to capture the extra rents. This temptation leads to only bad assets being sold and consequently constrained investors being excluded.

3 The Model with a Rating Agency

In this section, we examine whether a rating agency can reduce or eliminate the asymmetric information problem. We also study how ratings interact with the structuring of the investments. We focus on a monopoly rating agency.

The CRA reduces the lemons problem through the reputation it acquires over time. We model two types of rating agency: truthful (T) and opportunistic (O).[8] The opportunistic CRA will announce the value for each security, but will choose its announcement and the structure on the basis of its incentives. The truthful CRA is behavioral in the sense that it is restricted to truthful announcements of security values, but is strategic in the way it designs the securities. This is a significant departure from the literature,

[8] This follows the approach of Fulghieri, Strobl, and Xia (2014) and Mathis, McAndrews, and Rochet (2009) (who in turn follow the classic approach of modeling reputation of Kreps and Wilson (1984) and Milgrom and Roberts (1984)).

which reduces the behavioral player to a nonstrategic player.[9] The literature generally uses the behavioral player as a device to create reputational incentives for the opportunistic player. In our model, this will limit the amount of ratings inflation (and mis-selling) the opportunistic CRA chooses in the first period.

Our model will have two periods. The CRA will be the same for both periods and each period there will be a different issuer. For ease of exposition, we will begin by describing a one-period version of this model. The probability of facing a truthful CRA at the beginning of the period is given by θ_t where $t \in \{1, 2\}$, which, together with the structure of the game and payoffs, is common knowledge. We also assume the issuer knows the type of the rating agency.[10]

The CRA observes perfectly the quality of the issuer's assets and makes a take-it-or-leave-it offer to the issuer. As part of its services, the CRA designs and rates the securities offered by the issuer for a fee $f \geq 0$. This fee is unobservable to investors. While in practice, the issuer will initially design the securities and get feedback from the rating agencies about modifications necessary to achieve certain ratings[11], we incorporate this back and forth into one step for simplicity. If the issuer does not use a rating agency it may issue securities nevertheless. Therefore the issuer can get at least its asymmetric information net payoff of $I_U (B - b) / B$ by not purchasing ratings. We will assume the CRA incurs a positive, but arbitrarily small cost of issuing a rating. Hence, in any equilibrium, the CRA is hired if and only if it can create additional surplus.

Denote a message that is sent by a CRA by $\bar{m} = (\bar{\mu}_C, \bar{\mu}_U, \bar{\nu}_C, \bar{\nu}_U)$ and the set of such messages by M, where $\bar{\mu}_i$ ($\bar{\nu}_i$) is the reported measure of good (bad) assets in a pool with securities intended for an investor of type $i \in \{C, U\}$. Denote the true measures

[9] The only exception we are aware of is Hartman-Glaser (2013) where the truthful issuer can decide how much to retain of a security.

[10] As the issuer knows the quality of its securities, this is the most natural assumption; otherwise, both types of rating agency would be involved in a two-sided signaling game as in Bouvard and Levy (2013), Frenkel (2012), and Bar-Isaac and Deb (2014). Other papers on CRAs do not need to make an assumption about this as the issuer has no choice variable.

[11] See details in Griffin and Tang (2012). Rating agencies also provide their basic model to issuers to communicate further. For example, Benmelech and Dlugosz (2009) write, "The CDO Evaluator software [from S&P, publicly available] enabled issuers to structure their CDOs to achieve the highest possible credit rating at the lowest possible cost... the model provided a sensitivity analysis feature that made it easy for issuers to target the highest possible credit rating at the lowest cost."

of assets by $m = (\mu_C, \mu_U, \nu_C, \nu_U)$. This message is equivalent to the CRA reporting a quality ("rating") of $(\bar{\mu}_i G + \bar{\nu}_i B) / (\bar{\mu}_i + \bar{\nu}_i)$ for securities of type $i \in \{C, U\}$, since we assume the quantity of assets in each pool is observable.[12]

A strategy for a CRA of type d is a triplet $s^d = (\bar{m}^d, m^d, f^d) \in S^d$, where S^d is the strategy space of type d. Since we assume the true quantities are observable to investors, any message $\bar{m}$ must fulfill $\bar{\mu}_C + \bar{\nu}_C = \mu_C + \nu_C$ and $\bar{\mu}_U + \bar{\nu}_U = \mu_U + \nu_U$. If the CRA is truthful, then the strategy space is further restricted such that $(\bar{\mu}_C, \bar{\mu}_U, \bar{\nu}_C, \bar{\nu}_U) \equiv (\mu_C, \mu_U, \nu_C, \nu_U)$.

Let $\beta : M \to \Delta$ be the belief function of the investors, assigning a probability distribution over the set of CRA types upon observing $\bar{m}$, so that $\beta(d|\bar{m})$ is the conditional belief that a CRA is of type $d \in \{T, O\}$ given a message $\bar{m}$. Let $V_i^{\beta}(\bar{m})$ be the investors' expected valuation of security i conditional on message $\bar{m}$ under the beliefs β. Unconstrained investors are willing to pay a total of $p_U(\bar{m}) = V_U^{\beta}(\bar{m})$ for the unconstrained securities, and constrained investors a total of $p_C(\bar{m}) = V_C^{\beta}(\bar{m})$ for the constrained securities if $V_C^{\beta}(\bar{m}) \geq \bar{V}\,(\bar{\mu}_C + \bar{\nu}_C)$ and $p_C(\bar{m}) = 0$ otherwise.

Note that our assumption that $(N - \mu)\, B \geq I_U + I_C$ guarantees that the opportunistic CRA has sufficiently many bad assets to create pools of size equal to the truthful CRA's that contain only bad assets.

To summarize, the timing of the game with one issuer is as follows:

1. Investors believe that the CRA is truthful with probability θ_t. In period 1, the probability is a prior given by nature, and in period 2, the probability is a posterior.

2. The CRA offers the issuer a contract for a fee f^d. The contract specifies the measures of good and bad assets to be included in each pool, and that ratings will be produced.

3. If the issuer accepts, then the securities are constructed. The CRA decides on the measure of good and bad assets to report to investors (ratings).

[12] This is equivalent in the model to assuming that the quantity of securities issued against each pool is observable.

Otherwise, the issuer selects the measures of good and bad assets to be included in each pool and sells the securities itself, without any rating.

4. Investors observe the total quantity of assets and the reported measures of good and bad assets in each pool (if the CRA was hired) and buy securities at their conditional expected value.

We suppose that the steps are repeated in a second period, and that the issuer is different in each period.

If the different types of CRAs separate in the first period, then second-period investors update their priors about the type of the CRA accordingly. If the different types of CRAs pool in the first period, investors are still able to update their priors. The reason is that in this case, we will assume that investors discover the type of the opportunistic CRA between periods with a positive probability. This probability depends on the amount of ratings inflation the opportunistic CRA chooses. We will define this probability and the dynamics explicitly in Section 4. Now, we focus on the second-period choices.

3.1 The Second Period

In this section, we will analyze the second period, when the type of the CRA has not been revealed in the first period and the posterior that the CRA is truthful is θ_2. Since this is the last period, the opportunistic CRA has no reputation concerns. An alternative interpretation of this section is that it analyzes a one-period version of the model.

Our first result concerns the securities offered by the issuer at the opportunistic CRA.

Lemma 3 *In any equilibrium of the second period, any security rated by the opportunistic CRA will have a value of B.*

Without reputation concerns, the opportunistic CRA has no incentive to include good assets in the pool of assets to sell since the actual composition is not observable to investors.

We say that an equilibrium is *pooling* if it has the property that both types of CRAs report the same values of all securities and the quantity of securities issued are the same

(we will also include any equilibrium where both types of CRA are not hired in this category). We call any equilibrium which is not pooling and where at least one type of CRA is hired, a *separating* equilibrium.

Lemma 4 *In the second period, there is no separating equilibrium.*

This is an important result in the characterization of the equilibria. If there were a separating equilibrium, the opportunistic CRA would be recognized and the best it could do is sell bad assets to unconstrained investors at fair value. As the issuer could do this without the CRA, the opportunistic CRA would not be hired given the small fixed cost of operating.

Given this result, we examine pooling equilibria. The possible pooling equilibria where CRAs are active could have securities sold only to unconstrained investors, securities sold only to constrained investors, or two types of securities sold, one meant for each type of investor. All of these possible pooling equilibria exist. However, after we refine the set of equilibria, there will no longer be one where securities are sold only to constrained investors.

Given the numerous equilibria that can be supported by a variety of off-the-equilibrium path beliefs, we use the refinement concept of *Undefeated Equilibrium*, introduced by Mailath, Okuno-Fujiwara, and Postlewaite (1993). Placing restrictions on off-the-equilibrium path beliefs using a concept such as the Intuitive Criterion (Cho and Kreps, 1987) has little bite in this environment, whereas the Undefeated Equilibrium concept selects a unique equilibrium outcome for a given set of parameters. We give a brief intuitive discussion of the concept here, and define it formally in the Appendix.

The undefeated equilibrium concept is used to select among different Pure-Strategy Perfect Bayesian Equilibria (PBEs). In our setting, these are equilibria such that (1) each type of CRA is using a pure strategy and maximizing profits given the investors' bids and the other CRA's strategy, (2) each investor bids his expected value conditional upon observed amount of securities issued and reported values, and (3) beliefs are calculated using Bayes' rule for amount of securities issued and reported values used with positive probability.

The undefeated concept essentially works by checking that no types in one equilibrium are better off in another equilibrium where they choose a different action/message.[13] In the appendix, we demonstrate that for our model, this selects equilibria that are payoff maximizing - i.e. equilibria that give *each* type of CRA weakly higher payoffs than any other equilibria.

We now write two conditions which will help define the parameter space for the unique undefeated equilibrium outcome.

$$\theta_2(G-B)b/B > g-b \tag{C1}$$

$$\theta_2 G + (1-\theta_2)B \geq \bar{V} \tag{C2}$$

The first condition says that if the posterior that the CRA is truthful is sufficiently high in the second period, the truthful CRA strictly prefers to add one more good asset rather than a bad asset to the asset pool being sold. The second condition states that if the same posterior is sufficiently high, it is possible to serve constrained investors, in spite of the fact that the opportunistic CRA includes only bad assets. The assumption that $\bar{V} > g$ guarantees a positive profit margin under this condition, and reduces the number of cases to study.

We now proceed to find the undefeated equilibria.

Proposition 1 *If and only if C2 holds, the unique outcome of any undefeated equilibrium, E_{**}, has two pools with the following features:*

1. *For constrained investors, the opportunistic CRA includes only bad assets, and the truthful CRA includes all good assets and a measure*

$$\mu\left(\theta_2 G + (1-\theta_2)B - \bar{V}\right) / \left(\bar{V} - B\right)$$

[13] While this works by comparing equilibrium payoffs, Mailath, Okuno-Fujiwara, and Postlewaite (1993) suggest this places more realistic restrictions on off-the-equilibrium path beliefs than other concepts by using beliefs from an actual equilibrium. In the examples they examine, this selects the most reasonable equilibria. This concept is also used in several other papers, including Taylor (1999), Gomes (2000), and Fishman and Hagerty (2003).

of bad assets such that, given the opportunistic CRA's choice, the expected value of a security backed by the pool equals $\bar{V}$.

2. *For unconstrained investors, both CRA types includes a measure I_U/B of bad assets.*

In the proposition, the unique undefeated equilibrium outcome has two pools: one for constrained investors and one for unconstrained investors. In the constrained pool, the issuer at the opportunistic CRA puts in only bad assets, while the issuer at the truthful CRA puts all of its good assets and enough bad assets to weakly satisfy the constraint of the constrained investors (given the constrained investors expect a truthful CRA with probability θ_2). Both put in only bad assets for the unconstrained pool. Both pools are priced according to the rational expectations of investors, meaning the prices are dependent on the investors' perception that the CRA is truthful. The opportunistic CRA makes strictly larger profits than the truthful CRA as it receives the same price and sells off more bad assets (and retains more good assets). The issuer with an opportunistic CRA offloads more bad assets than if there were asymmetric information with no CRA.

The above equilibrium is undefeated since it is the one that maximizes profits for both types of CRAs. To get an intuition for this, note that in this equilibrium, the truthful includes all of its good assets and as many bad assets as possible given that the opportunistic CRA only includes bad assets *and* the constraint of the constrained investors binds. Since both CRAs sell pools of the same size, this means that it is also the equilibrium where the opportunistic CRA can include as many bad assets as possible.

Investors who interact with an opportunistic CRA see ratings above the actual value of the securities offered (ratings inflation) and pay a price larger than the actual value for those securities. In the next section, we will detail a mechanism whereby these investors in the first period will realize with some probability that there is a difference between the rating and the value. They will thus learn the CRA they are observing is opportunistic. When they learn a CRA is opportunistic, they will ignore all of its future ratings, creating a reputational punishment that will limit the amount of ratings inflation in the first period.

For our next set of parameters, we find a unique one-pool undefeated equilibrium outcome.

Proposition 2 *If and only if $C1$ holds but $C2$ does not, the unique outcome of any undefeated equilibrium, E_*, has one pool for the unconstrained investors with the following features: the opportunistic CRA includes only bad assets, and the truthful CRA includes a measure μ of good assets and a measure $\left(I_U - \mu\left(\theta_2 G + (1-\theta_2) B\right)\right)/B$ of bad assets.*

In this proposition, the unique undefeated equilibrium outcome has one pool with securities sold to all of the unconstrained investors. The truthful CRA places all of its good assets in the pool, and as many bad assets as it can to satisfy the demand of the unconstrained investors. The price of the securities reflects the value and the perceived probability that the CRA is truthful. Once again, the opportunistic CRA makes higher profits than the truthful CRA. The intuition for why this is an undefeated equilibrium is that both CRA types are selling as many assets as possible (and given C1, the truthful CRA finds it profitable to include good assets) given that θ_2 is not high enough to serve constrained investors.

For the last set of parameters, no CRA is hired:

Corollary 1 *If $C1$ and $C2$ do not hold, any equilibrium, $E_\varnothing$, has neither of the CRAs being hired.*

This follows from the proofs of Proposition 1 and Proposition 2. In these equilibria the CRA can't generate value for the issuer, so the issuer does not hire the CRA but issues securities of value B, which are purchased by unconstrained investors.

From the above, it follows immediately that any undefeated equilibrium where the CRAs are hired has ratings inflation. For the equilibrium with two types of securities, the constrained securities' rating is equal to the value of what the truthful CRA is offering, but this is above the expected value by investors since the opportunistic CRA sells only bad assets. For the equilibrium with one type of security, there is a similar type of inflation. Despite the potential for a large amount of ratings inflation, it is clear that

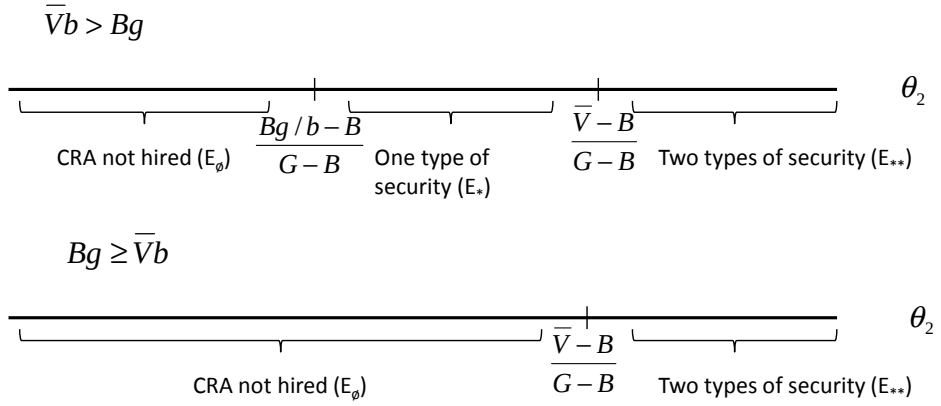

Figure 1: Equilibrium Configuration in Period 2.

securitization improves welfare in the second period compared to the benchmark of no CRA, as otherwise the issuers would not hire the CRA.

Given Proposition 1, Proposition 2, and Corollary 1, we can now look at the equilibrium configuration, i.e. the parameter space for which each equilibrium exists.

Corollary 2 *The equilibrium configuration has the following features:*

1. *If $\bar{V}b > Bg$: for $\theta_2 \geq \frac{\bar{V}-B}{G-B}$, the equilibrium is of type E_{**}, for $\frac{\bar{V}-B}{G-B} > \theta_2 > \frac{Bg/b-B}{G-B}$, the equilibrium is of type E_*, and for $\frac{Bg/b-B}{G-B} \geq \theta_2$, the equilibrium is of type $E_\varnothing$.*

2. *If $Bg \geq \bar{V}b$: for $\theta_2 \geq \frac{\bar{V}-B}{G-B}$, the equilibrium is of type E_{**}, for $\frac{\bar{V}-B}{G-B} > \theta_2$, the equilibrium is of type $E_\varnothing$.*

We do not prove the corollary, as it follows directly from the above propositions and the assumption that $\bar{V} > g$. We illustrate the equilibrium configuration in Figure 1.

The corollary provides several insights. First, a one-security equilibrium only exists if $\bar{V}b > Bg$. This reflects the fact that the quality requirement of constrained investors is high relative to the benefit of retaining G assets and securities dedicated to constrained investors will not always be sustainable. It also means that the benefit of pushing B assets onto investors is not that large, which makes it desirable to sell off G assets to the unconstrained investors. Second, the two-security equilibrium exists when θ_2 is large. This means that it takes a sufficient reputation for honesty to be able to sell to constrained

investors. Third, the larger the quality requirement of constrained investors, the less likely it is that there will be a two-security equilibrium.[14]

In the next section, we proceed to the first period and examine how the payoffs of the second period create reputation effects for the opportunistic CRA and whether they can eliminate conflicts of interest.

4 The First Period

In this section, we will analyze equilibrium behavior in the first period. We begin by defining a reputation mechanism to link periods 1 and 2. We then extend the undefeated equilibrium concept to a two-period game. Using these building blocks, we thereafter find the unique undefeated equilibrium outcome for a given set of parameter conditions.

4.1 Ratings Inflation and the Reputation Mechanism

We will introduce reputation concerns in the model by assuming that the type of the opportunistic CRA is discovered with a positive probability between periods. We start by defining *ratings inflation* - the variable z will be our measure of how inflated (or inaccurate) ratings are. We assume a functional form for z:

$$\begin{aligned} z &= (\bar{\mu}_C^O G + \bar{v}_C^O B) - (\mu_C^O G + v_C^O B) \\ &\quad +(\bar{\mu}_U^O G + \bar{v}_U^O B) - (\mu_U^O G + v_U^O B). \end{aligned} \tag{2}$$

This represents the aggregate difference between reported and actual values for all securities issued. This depends on both the magnitude of the divergence between the ratings and the actual quality *and* on the quantity of securities that had inflated ratings. It is important to include both dimensions in the reputation mechanism. CRAs are more likely to be punished when they have poorer ratings quality and when that quality has affected more investors (as it is more likely to be observed and acted on).

[14] This can be found directly from the corollary by shifting $\bar{V}$.

Using the fact that $\bar{\mu}_C^O + \bar{v}_C^O = \mu_C^O + v_C^O$ and $\bar{\mu}_U^O + \bar{v}_U^O = \mu_U^O + v_U^O$ we can simplify this to:

$$z = \left(\bar{\mu}_C^O + \bar{\mu}_U^O - \mu_C^O - \mu_U^O\right)(G - B). \tag{3}$$

The maximum level of ratings inflation occurs when the opportunistic CRA reports that it has included all of its good assets (and possibly some bad assets), while it actually has included only bad assets. In this case, $z = \mu(G - B)$.

Define p as the probability that the type of the opportunistic CRA is discovered after period 1 ends and before period 2 begins. Each CRA wants to maximize its expected discounted profits. Since the opportunistic CRA will not be hired in the second period if its type is known, its expected discounted profits are given by:

$$\Pi^O = \pi_1^O + \delta(1 - p)\pi_2^O.$$

Here, π_1^O represents first-period profits, π_2^O represents second-period profits, and δ is the discount factor.

We posit that the type of the opportunistic CRA will be more likely to be discovered the more inaccurate its ratings are.[15] More precisely, we assume $p = 1$ if the CRAs separate in the first period, and otherwise $p = h(z)$. The function h is assumed to be increasing, strictly convex, and continuously differentiable on $[0, \mu(G - B)]$, such that $h(0) = 0$, $h'(0) = 0$, $h(\mu(G - B)) \leq 1$, and $h'(\mu(G - B)) > \frac{g-b}{\delta\mu(G-B)\left(\bar{V}-b\right)}$. As will be demonstrated in the Appendix (see the proof of Lemma 8), this functional form rules out corner solutions where the opportunistic CRA only includes bad assets ($\mu_U^O = \mu_C^O = 0$) whenever the truthful CRA includes all of the issuer's good assets, for the class of equilibria we study.

If there is no ratings inflation at all, the opportunistic CRA is secure and will earn its full second-period profits. If there is ratings inflation and the opportunistic CRA is

[15] Note that in the CRA literature (e.g. Fulghieri, Strobl, and Xia (2012), Mathis, McAndrews, and Rochet (2009), and Bar-Isaac and Shapiro (2013)) the reputation mechanism is much simpler, as those papers have an investment that is binary, and only defaults in the bad state. Therefore something rated good that defaults leads directly to learning. Because of the generality of our setup, we define this mechanism as ex-post learning from the divergence between the rating and the realized performance.

discovered, it is not hired in period 2. If the CRA's type is not revealed in period 1, then the equilibrium posterior in the beginning of period 2 that it is truthful is:

$$\theta_2 = \theta_1/(\theta_1 + (1-p)(1-\theta_1)),$$

where θ_1 denotes the prior at the beginning of period 1. It follows immediately from this formula that $\theta_1 \leq \theta_2$, i.e. given that an opportunistic CRA was not found in the first period, it is more likely that the CRA is truthful.

We have already shown that there are no separating equilibria in the second period. The following lemma extends this result to the first period.

Lemma 5 *There is no equilibrium where the CRAs separate in the first period.*

If the CRAs separated in the first period, the opportunistic CRA wouldn't have any business in any period and it would therefore have a profitable deviation by mimicking the truthful CRA. We can thus restrict ourselves to looking only at pooling equilibria.

In any pooling equilibrium where the CRA is hired, the opportunistic CRA's choice of how many good assets to include in the pools, (μ_U^O, μ_C^O) must be optimal given the first-period message of the truthful CRA, $(\bar{\mu}_U^*, \bar{\nu}_U^*, \bar{\mu}_C^*, \bar{\nu}_U^*)$. Furthermore, the beliefs of investors are held fixed when the opportunistic CRA chooses the amount of good assets to include, meaning that the choice does not affect the price received. More specifically, (μ_U^{O*}, μ_C^{O*}) must be a solution to the following maximization problem:

$$\begin{aligned}
\max_{\mu_U^O, \mu_C^O \geq 0} \{(1-\theta_1) &\begin{pmatrix} \left(\mu_U^{O*} + \mu_C^{O*}\right) G + \\ \left(\bar{\mu}_U^* + \bar{\nu}_U^* + \bar{\mu}_C^* + \bar{\nu}_C^* - \mu_U^{O*} - \mu_C^{O*}\right) B \end{pmatrix} + \\
&\theta_1 \left(\left(\bar{\mu}_U^* + \bar{\mu}_C^*\right) G + \left(\bar{\nu}_U^* + \bar{\nu}_C^*\right) B\right) - I_U \left(B - b\right)/B - \\
&\left(\mu_U^O + \mu_C^O\right) g - \left(\bar{\mu}_U^* + \bar{\nu}_U^* + \bar{\mu}_C^* + \bar{\nu}_U^* - \mu_U^O - \mu_C^O\right) b + \\
&(1 - h(\left(\bar{\mu}_C^* + \bar{\mu}_U^* - \mu_C^O - \mu_U^O\right)(G - B)))\delta\pi_2^O\}
\end{aligned}$$

The first two lines represent the opportunistic CRA's net revenues in the first period.

As the price depends on the equilibrium beliefs of investors and the quantity is observable and identical for both types of CRAs, net revenues are held fixed in the choice problem for the opportunistic CRA. The third line represents the opportunity cost of not holding the assets. The fourth line represents the expected second-period profits. This consists of the probability the opportunistic CRA will operate in the second period times the discounted equilibrium profits in the second period. Note that the probability depends on the opportunistic CRA's choice, as more distortion away from the reported value will lower its likelihood of survival, but the equilibrium second-period profits do not, as the beliefs of investors over the updated type of the CRA are held fixed.

In any pooling equilibrium, the first order conditions with respect to the amount of good assets included in the constrained and unconstrained pools in period 1 are given by:

$$b - g + (G - B)h'(z)\delta\pi_2^O \leq 0, \tag{4}$$

where the inequality can be replaced by an equality when $\mu_U^O > 0$ or $\mu_C^O > 0$.

4.2 Equilibrium Definition and Assumptions

We will now characterize the equilibria of the two-period game. This game has multiple equilibria and in order to select among them we would ideally like to apply something similar to the undefeated equilibrium concept that was employed to the second-period game in the previous section. However, the undefeated equilibrium concept is formally defined for one-stage signaling games and therefore has to be amended to fit our framework.[16]

Let the *second-period game given prior* θ_2 be the one-period game described in Section 3 where the prior is given by θ_2 and CRA payoffs are defined by corresponding one-period profits. Let the *first-period game* be the one-period game described in Section 3 where the prior is given by θ_1 and CRA payoffs are defined by the first-period profits plus the discounted expected second-period profits in an undefeated equilibrium of the second-

[16] Mailath et al (1993) briefly discuss the possibility of extending their concept to general games with more stages and multiple players.

period game given prior θ_2, where θ_2 is the posterior conditional upon the first-period actions and whether the CRA's type was revealed between periods.

Definition 1 *We say that $\mathcal{E}$ is an **undefeated equilibrium of the full game** if:*

1. *For every prior θ_2, the restriction of $\mathcal{E}$ to the second period is an undefeated equilibrium of the second-period game given prior θ_2.*

2. *The restriction of $\mathcal{E}$ to the first period is an undefeated equilibrium of the first-period game.*

In order to simplify the analysis, we will limit the parameter space to guarantee a unique undefeated equilibrium outcome in the second period. This allows us to avoid multiple discontinuities in the choice of first-period ratings inflation, which depends on second-period profits. The simplest way to do this is to first assume

$$Bg \geq \bar{V}b. \tag{A2}$$

It follows from Corollary 2 (which is graphically depicted in Figure 1), that this limits the second-period undefeated equilibria to two possibilities: two types of securities are sold or the CRA is not hired.

The undefeated equilibrium where the CRA is not hired in the second period is of little interest, as there are no reputational concerns and it reduces the first period to the static game that we had previously solved. Moreover, since $\theta_1 < \theta_2$ if the type of the CRA is not revealed between periods, it implies the CRA would not be hired in the first period either. We will therefore make a second assumption to focus on the two security equilibrium in period 2.

We will use the following notation to denote the posterior in period 2 if the opportunistic CRA is not discovered, $\theta_2(z) := \theta_1/(\theta_1 + (1 - h(z))(1 - \theta_1))$. The profits for the opportunistic CRA (as given in Proposition 1) are $\left(\bar{V} - b\right)\mu\theta_2(z)(G - B)/\left(\bar{V} - B\right)$. Plugging these second-period profits into the the incentive constraint (4), and replacing

the inequality by an equality, we define z^* implicitly by

$$h'(z^*)\theta_2(z^*) = (g-b)\left(\bar{V}-B\right)/\delta\mu\left(G-B\right)^2\left(\bar{V}-b\right). \tag{5}$$

Our assumptions about $h()$ guarantee that that this equation has a unique and interior solution for θ_1 sufficiently large.[17] Our second assumption is that θ_1 is so large that condition C2 holds for $\theta_2(z^*)$:

$$\theta_2(z^*)G + (1-\theta_2(z^*))B \geq \bar{V}. \tag{A3}$$

With this assumption, the undefeated equilibrium outcome in the second period is unique and given by Proposition 1.

4.3 Undefeated Equilibria of the Full Game

The following two conditions will determine which type of undefeated equilibrium of the full game will be observed.

$$\left(\mu(G-B) - z^*\left(1-\theta_1\right)\right)\frac{b}{B} - \mu(g-b) + \frac{\bar{V}\left(1-\theta_1\right)\left(\mu(G-B)-z^*\right)}{\bar{V}-\theta_1 G-\left(1-\theta_1\right)B}\frac{B-b}{B} > 0 \quad \text{(C1')}$$

$$\mu\left(G-\bar{V}\right) \geq \left(1-\theta_1\right)z^* \tag{C2'}$$

Condition (C1') implies that the truthful CRA makes positive profits from selling good assets when it can't sell all of them to constrained investors. Condition (C2') implies that if the truthful CRA places all of the good assets in the constrained pool it can satisfy constrained investors.

Notice that the second-period game in section 3 can be described as a first-period game without reputation concerns, i.e. where the opportunistic CRA includes only bad

[17] This follows since $h'(z)\theta_2(z)$ is a continuous and strictly increasing function of z on $[0, \mu(G-B)]$ such that $h'(0)\theta_2(0) = 0$ and $h'(\mu(G-B))\theta_2(\mu(G-B)) > (g-b)\left(\bar{V}-B\right)/\delta\mu\left(G-B\right)^2\left(\bar{V}-b\right)$ if θ_1 is so large that A3 holds.

assets. We can see this directly from the above conditions - when we set ratings inflation to its maximum $\mu(G-B)$, C1' collapses to C1 and C2' collapses to C2.

We can now characterize the unique undefeated equilibrium outcome for a given set of parameters, which we will do in Propositions 3 and 4.

Proposition 3 *If and only if C2′ holds, the unique outcome of any undefeated equilibrium of the full game, $\mathcal{E}_{**}$, has two types of securities with the following features in the first period:*

1. *The amount of ratings inflation by the opportunistic CRA is z^* as defined in equation (5).*

2. *For constrained investors, the truthful CRA includes a measure μ of good assets and*

$$\nu_C = \left(\mu(G-\bar{V}) - (1-\theta_1)z^*\right) / \left(\bar{V}-B\right)$$

of bad assets, and the opportunistic CRA includes a measure $\mu - z^*/(G-B)$ of good assets and $\nu_C + z^*/(G-B)$ of bad assets, such that the expected value of a security backed by the pool equals $\bar{V}$.

3. *For unconstrained investors, both CRAs types include a measure I_U/B of bad assets.*

In $\mathcal{E}_{**}$, a two-security equilibrium similar to E_{**} is played in the first period, although in $\mathcal{E}_{**}$ the opportunistic CRA will now include some good assets. The motivation for the opportunistic CRA to include some good assets is reputational; it is trading off extracting more rents from the issuer in the first period by inflating ratings and allowing the issuer to retain more good assets versus increasing the likelihood that the opportunistic CRA survives to enjoy its second period profits. More specifically, in the first period all unconstrained investors will purchase securities of value B. The constrained investors are sold as many securities as possible with an expected value of $\bar{V}$. The truthful CRA will place all of its good assets and some bad assets in this pool, whereas the opportunistic CRA will place a fraction of the good assets, and fill the rest with bad assets. In $\mathcal{E}_{**}$, the second-period equilibrium outcome is E_{**}.

In the Appendix, we prove Proposition 3 by showing that given $C2'$, the above equilibrium outcome maximizes *both* the truthful and the opportunistic CRAs' payoffs over the set of potential pooling equilibria.[18] Hence, the equilibrium outcome is not just Pareto efficient in the sense that no type could be made better off without another being made worse off. It goes beyond this to say that these are the equilibria both types of CRA would select.

There is also a second type of undefeated equilibrium.

Proposition 4 *If and only if $C1'$ holds but $C2'$ does not, the unique outcome of any undefeated equilibrium of the full game, $\mathcal{E}_*$, has two types of securities with the following features in the first period:*

1. *The amount of ratings inflation by the opportunistic CRA is z^* as defined in equation (5).*

2. *For constrained investors, the truthful CRA includes a measure*

$$\mu_C = \frac{(1-\theta_1)(\mu(G-B)-z^*)}{\bar{V}-\theta_1 G-(1-\theta_1)B}$$

   of good assets and no bad assets, and the opportunistic CRA includes a measure $\mu - z^*/(G-B)$ of good assets and $\mu_C - (\mu - z^*/(G-B))$ of bad assets, such that the expected value of a security backed by the pool equals $\bar{V}$.

3. *For unconstrained investors, the truthful CRA includes a measure $\mu - \mu_C$ of good assets and $(I_U - (\mu-\mu_C)(\theta_1 G + (1-\theta_1)B))/B$ of bad assets, and the opportunistic CRA includes only bad assets.*

In $\mathcal{E}_*$, two securities are issued and the opportunistic CRA allocates a mix of good and bad assets to the constrained pool, but only bad assets to the unconstrained pool. The truthful CRA, on the other hand, allocates some, but not all of its good assets to

[18] Interestingly, if one replaced the 3 assumptions A2, A3, and $C2'$ with the condition $\theta_1 G + (1-\theta_1)B \geq \bar{V}$, one would find that any undefeated equilibrium of the full game is of type $\mathcal{E}_{**}$.

the constrained pool, and a mix of good and bad assets to the unconstrained pool.[19]

This equilibrium shares many features of $\mathcal{E}_{**}$. The amount of ratings inflation and thus the opportunistic CRA's allocation of good assets to the constrained pool are the same. The expected value of securities sold to constrained investors is still $\bar{V}$ and the second-period equilibrium outcome is still E_{**}. The difference is that in $\mathcal{E}_{*}$, it is more difficult to satisfy the quality requirements of the constrained investors – the truthful CRA cannot include any bad assets in the constrained pool and it cannot include all of its good assets (given the equilibrium ratings inflation choice of the opportunistic CRA). The truthful CRA sells the rest of its good assets to unconstrained investors. Notice that as the opportunistic CRA allocates good assets only to constrained investors, there is equilibrium ratings inflation for *both* types securities[20]. In $\mathcal{E}_{**}$, there was only ratings inflation in the securities meant for constrained investors. Vickery (2012) shows evidence of substantial ratings inflation at all investment grade rating levels for subprime RMBS.

Thus the difficulty in serving constrained investors, either because of their high quality requirements or the lower quality of good assets, 'pushes' ratings inflation to the securities meant for the unconstrained investors. Conversely, when it is easier to serve constrained investors, the ratings inflation gets concentrated in their securities. This is consistent with the experiment of loosened capital requirements described in Stanton and Wallace (2012).

Our last result in the characterization is:

Corollary 3 *If and only if C1' and C2' do not hold, no CRA is hired in any period.*

The corollary is straightforward – if C2' and C1' do not hold, then an equilibrium where the truthful CRA makes positive profits in the first period is not possible.

[19] One might wonder if the equilibria $\mathcal{E}_{**}$ and $\mathcal{E}_{*}$ exist given the assumed conditions. For $\mathcal{E}_{**}$, footnote 20 demonstrates a simple condition that is easy to satisfy for which it exists. For $\mathcal{E}_{*}$, we provide the following example: Let $h(z) = z^2, b = 1/5, B = 1/3, g = 2/3, G = 1, \bar{V} = 5/6, \mu = 3/2, \delta = 1/2$, and $\theta_1 = 6/10$. It is easy to see that A2 holds. Solving numerically gives: $z^* \approx 0.726$, implying $\theta_2(z^*)G + (1 - \theta_2(z^*))B - \bar{V} \approx 0.00707$ (A3 holds), $\mu(G - \bar{V}) - (1 - \theta_1)\, z^* \approx -0.0406$ (C2' does not hold), and first-period profits for the truthful CRA of approximately 0.0902 (C1' holds).

[20] This is because the quality of the securities rated by the truthful CRA are strictly better than the quality of the securities rated by the opportunistic CRA for *both* types of investor.

4.4 Ratings Inflation

Using the above results, it is also straightforward to compute how ratings inflation changes with the parameters in the equilibria $\mathcal{E}_{**}$ and $\mathcal{E}_{*}$. In order to clearly understand the dynamics, we will now mark each variable with a subscript t, $t \in \{1, 2\}$, to denote which period it is from. For example, the value of good assets for investors in period 2 is given by G_2.

Proposition 5 *In $\mathcal{E}_{**}$ and $\mathcal{E}_{*}$, the ratings inflation by the opportunistic CRA in period 1 is:*

1. *Increasing in* g_1, B_1, and $\bar{V}_2$
2. *Decreasing in* δ, μ_2, G_1, G_2, B_2, b_1 and θ_1.

Ratings inflation increases if second-period constrained investors demand higher quality assets (higher $\bar{V}_2$) . This occurs because second-period profits are decreasing in the quality requirement of constrained investors, as it is more difficult to push securities onto them. As second-period profits decline, the cost of inflating ratings dissipates. Intriguingly, this suggests that tighter constraints on investors decrease the quality of ratings through an equilibrium effect. Therefore stricter regulation surrounding the quality of assets that financial institutions, pension funds, or insurance companies may hold can backfire as CRAs lower their standards in response.

Consider some of the second-period variables. ratings inflation decreases if the premium for good or bad assets (G_2 or B_2, respectively) is larger, as second-period profits will be larger and the benefit to the opportunistic CRA of maintaining its reputation is thus larger. Similarly, inflation decreases with the fraction of good assets (μ_2). This set of results is quite interesting; if the quality of the future asset pool improves, then there will be less ratings inflation. This can be given a business cycle interpretation; in recessions, there will be less ratings inflation than in booms. This is consistent with theoretical results found in Bar-Isaac and Shapiro (2013), Bolton, Freixas, and Shapiro (2012), and Fulghieri, Strobl, and Xia (2014) and empirical results in Auh (2013).

Ratings inflation goes down if the prior that the CRA is truthful in period 1 is larger. The insight on the prior comes from the fact that the more likely the period 1 CRA is truthful, the more there is to gain for the opportunistic CRA in period 2, implying it will choose less ratings inflation in period 1 to increase the chance of survival. Interestingly, one might posit that there should be a trade-off, as if the prior is larger, the opportunistic CRA has higher gains from inflating ratings in period 1. However, this is incorrect since the gain from unilaterally deviating by reducing the quantity of good assets in the first period is independent of the price obtained then.

There is a subtle effect with respect to the values first-period investors place on good and bad assets (G_1 and B_1, respectively). The reputation mechanism depends on the amount ratings are inflated. This amount, given in equation 3, represents the difference between perceived value and the actual value of the securities. The larger the term $G_1 - B_1$ is, the more likely the opportunistic CRA will get caught (and punished by withdrawn business) for inflating its ratings, as the substitution of a bad asset for a good asset is more likely to be noticed. Therefore, a larger $G_1 - B_1$ leads to less ratings inflation.

Lastly, ratings inflation decreases if reputation is more important, which is proxied for by the discount factor δ.

4.5 Welfare

It is natural to ask about the welfare effects of ratings inflation. While we view the role of the truthful CRA as providing reputational incentives for the opportunistic CRA, we must incorporate both types of CRA into a welfare calculation. Given that all agents are rational, ratings inflation will be detrimental to those investors who face the opportunistic CRA, but benefit those who face the truthful CRA. Therefore, it is not obvious ex-ante that ratings inflation has a negative impact on welfare.

We provide here a welfare analysis for limited parameters, where the results are analytically tractable. Specifically, we look at the welfare properties of $\mathcal{E}_{**}$. Welfare is given by the the weighted sum of CRA payoffs for the two-period game plus the surplus of the issuer. Note that the welfare of investors is implicitly included as their rents are

extracted. Welfare is thus given by:

$$\begin{aligned} W_{**} &= \theta_1 \pi_1^T + (1-\theta_1)\pi_1^O + \delta\{\theta_1 \pi_2^T + (1-\theta_1)(1-p)\pi_2^O\} \\ &\quad +(1+\delta) I_U (B-b)/B. \end{aligned} \tag{6}$$

Using this, the following comparative statics are straightforward to compute.

Proposition 6 *The ex-ante welfare in any undefeated equilibrium of the full game $\mathcal{E}_{**}$ is increasing in θ_1 and μ, and decreasing in $\bar{V}$ and z^*.*[21]

Welfare increases in the probability of a CRA being truthful, θ_1, and in the measure of good assets, μ. Both of these allow the total amount of assets sold to increase.

An increase in the probability of detecting ratings inflation will increase welfare. Transparency and provision of historical data is beneficial in this environment.

First period ratings inflation enters negatively into the expression of welfare. ratings inflation has a negative effect because it decreases the amount of assets sold. The more inflation there is, the harder it is to satisfy constrained investors and the amount of assets sold to them must be restricted.

Welfare decreases with the minimum quality requirement of constrained investors, $\bar{V}$, as that reduces the possibility of selling assets. This suggests that any benefits of regulation that constrains investors, such as a reduced risk of financial contagion, must be traded off with the reduced efficiency of capital allocation. Further examination of this trade-off is beyond the scope of the current model.

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Appendix

Proof of Lemma 1

The issuer's problem translates into the following optimization program:

$$\max_{\nu_U,\nu_C,\mu_U,\mu_C\geq 0} \left\{(\mu_U+\mu_C)(G-g)+(\nu_U+\nu_C)(B-b)\right\},$$

subject to the constraints:

$$I_U-\mu_U G-\nu_U B\geq 0, \tag{A}$$

$$\mu-\mu_U-\mu_C\geq 0, \tag{C}$$

$$\nu_C B+\mu_C G-(\nu_C+\mu_C)\bar{V}\geq 0. \tag{D}$$

Note that a restriction on the size of the constrained pool is redundant by Assumption A1. We assign multipliers A, C, D to the above constraints and form the Langrangian function L. Constraint (A) states that the unconstrained pool cannot have a value greater than the wealth of the unconstrained investors I_U. Constraint (C) states that the amount of good assets that can be included is μ. Constraint (D) states that constrained investors require a quality level of at least $\bar{V}$.

The Kuhn-Tucker first-order conditions are as follows, where each inequality can be replaced by an equality if the corresponding measure is positive:

$$\frac{\partial L}{\partial \nu_U}=B-b-AB\leq 0 \tag{7}$$

$$\frac{\partial L}{\partial \mu_U} = G - g - AG - C \leq 0 \tag{8}$$

$$\frac{\partial L}{\partial \nu_C} = B - b + D\left(B - \bar{V}\right) \leq 0 \tag{9}$$

$$\frac{\partial L}{\partial \mu_C} = G - g - C + D\left(G - \bar{V}\right) \leq 0. \tag{10}$$

1. From (7), it follows that $A \geq (B-b)/B > 0$, and hence that constraint (A) binds. In fact, it must be the case that $A = (B-b)/B$, as $A > 0$ means all unconstrained investors will be served and since $\mu G < I_U$ this can only be the case if $\nu_U > 0$.

2. From (9), it follows that $D \geq \frac{B-b}{\bar{V}-B} > 0$, and hence constraint (D) binds. This implies either that the constrained pool is empty or that each constrained security has a value of $\bar{V}$.

3. From (10), it follows that $C \geq G - g + D\left(G - \bar{V}\right)$ and constraint (C) binds.

Substituting the binding constraints into the objective function yields $\mu(G-g) + \left(\frac{I_U - (\mu - \mu_C)G}{B} + \mu_C(\frac{G-\bar{V}}{\bar{V}-B})\right)(B-b)$. As this is increasing in μ_C, the solution has $\mu_C = \mu$ and $\mu_U = 0$. This implies that $\nu_U = I_U/B$ and $\nu_C = \mu\frac{G-\bar{V}}{\bar{V}-B}$.

Undefeated Equilibria: Definition and Application

In this subsection, we define the concept of Undefeated Equilibria, as put forth by Mailath, Okuno-Fujiwara, and Postlewaite (1993). We begin with the definition of a Pure Strategy Perfect Bayesian Equilibrium (PBE).

In addition to the notation in section 3.1, we add the following. Denote an arbitrary CRA type by d and the set of such types by $D = \{T, O\}$. Let $p = (p_U, p_C)$ be the vector of aggregate bids for the two types of securities. The profits to the CRA of type d are denoted by $\pi(s, p, d)$. Let $1_{\bar{m}(d)=\bar{m}}$ be an indicator function that takes the value 1 if type d sends message $\bar{m}$. Finally, define the probability function $\Theta(d)$ such that $\Theta(T) = \theta$ and $\Theta(O) = 1 - \theta$.

Definition 2 *$E^* = (s^*, p^*, \beta^*)$ is a **Pure Strategy Perfect Bayesian Equilibrium (PBE)** if and only if:*

1. $\forall d \in D : s^*(d) \in \arg\max_{s \in S^d} \pi(s, p, d)$,
2. $\forall \bar{m} \in M : p_U(\bar{m}) = V_U^{\beta}(\bar{m})$, *and* $p_C(\bar{m}) = V_C^{\beta}(\bar{m})$ *if* $V_C^{\beta}(\bar{m}) \geq \bar{V}\,(\bar{\mu}_C + \bar{\nu}_C)$ *and* $p_C(\bar{m}) = 0$ *otherwise,*
3. $\forall d \in D$ *and* $\forall \bar{m} \in M : \beta^*(d|\bar{m}) = \Theta(d) 1_{\bar{m}(d)=\bar{m}} / \sum_{d' \in D} \Theta(d') 1_{\bar{m}(d')=\bar{m}}$ *if the denominator is positive.*

In words, a strategy profile and a belief function constitute a Pure Strategy Perfect Bayesian Equilibrium if: 1. each type of CRA is using a pure strategy maximizing profits given the investors' bids and the other CRA's strategy, 2. investors bid their expected value conditional upon observed amount of securities and reported values, 3. beliefs are calculated using Bayes' rule for measures of securities and reported values used with positive probability.

Definition 3 *A PBE, $E = (s, p, \beta)$, **defeats** another PBE, $E' = (s', p', \beta')$, if and only if:*

1. $\forall d \in D : \bar{m}'(d) \neq \bar{m}$ *and* $K = \{d \in D : \bar{m}(d) = \bar{m}\} \neq \emptyset$,
2. $\forall d \in K : u(s, p, d) \geq u(s', p', d)$ *and* $\exists d \in K : \pi(s, p, d) > \pi(s', p', d)$,
3. $\exists d \in K : \beta'(d|\bar{m}) \neq \Theta(d)\eta(d) / \sum_{d' \in D} \Theta(d')\eta(d')$ *for some* $\eta : D \to [0, 1]$ *satisfying:*
   $d' \in K$ *and* $\pi(s', p', d') < \pi(s, p, d') \Rightarrow \eta(d') = 1$, *and*
   $d' \notin K \Rightarrow \eta(d') = 0$.

In words, an equilibrium E defeats another equilibrium E' if: 1. there is a message $\bar{m}$ sent only in E, 2. the set of types K who send this message are all better off in E than in E' and at least one of them is strictly better off, and 3. under E', the (off-the-equilibrium path) beliefs about some such a type are not a posterior assuming only types in K send $\bar{m}$ and that they do so with probability one if they are strictly worse off than under E.

A PBE is said to be *undefeated* if the game has no other PBE that defeats it. In order to apply the undefeated concept, we define a *payoff-maximizing equilibrium* as a PBE that, for a given set of parameters, gives each type of CRA weakly higher payoffs than any other PBE.

We use the following lemmas to relate a payoff-maximizing equilibrium to an undefeated equilibrium. Lemma 6 proves that any payoff-maximizing equilibrium is undefeated. Lemma 7 is then used to show that there are no other undefeated equilibria besides those which are payoff-maximizing equilibria. Therefore the two concepts are equivalent in our setting.

Lemma 6 *A payoff-maximizing equilibrium is undefeated.*

Since no type can be strictly better off in another PBE, it follows immediately from the definition of an undefeated equilibrium that a payoff-maximizing equilibrium, E, must be undefeated.

Lemma 7 *A PBE is defeated by another if the latter is weakly more profitable for both CRAs and strictly so for the truthful CRA.*

Suppose there are two PBEs E and E' such that E is weakly more profitable for both CRAs and strictly so for the truthful CRA. First note that by Lemma 4, both equilibria must be pooling (although the CRAs may not be hired in one of the equilibria). Second, since (1) the truthful CRA is restricted to honest reports and (2) the truthful CRA must use different strategies in E and E', the messages sent in the two equilibria must be different, $\bar{m} \neq \bar{m}'$ (if the CRAs are not hired in one of the equilibria, the corresponding message is empty). Third, beliefs in E' given the message $\bar{m}$ cannot be a posterior assuming the truthful CRA sends this message with probability one, or it would have a profitable unilateral deviation. Therefore, E defeats E'.

Therefore, it suffices to find a payoff-maximizing equilibrium.

Proof of Proposition 1 and 2

Using Lemmas 6 and 7, we can restrict ourselves to look for payoff-maximizing equilibria. We thus begin by finding the equilibria that maximize the profits of the truthful CRA. We will then show that these also maximize the profits of the opportunistic CRA.

By Lemma 3 and Lemma 4, this implies solving:

$$\max_{\mu_U,\mu_C,\nu_U,\nu_C\geq 0} \left\{ \begin{array}{c} (\mu_U+\mu_C)\left(\theta_2 G+(1-\theta_2)B-g\right) \\ +(\nu_U+\nu_C)(B-b)-I_U\left(B-b\right)/B \end{array} \right\}$$

The first line represents the gain the truthful CRA makes by including good assets. As the opportunistic CRA only includes bad assets, the price that the truthful CRA receives reflects this. The second line has two terms. The first is the gain the truthful CRA makes by including bad assets, which will be priced at B. The second is the surplus the truthful CRA must give up to the issuer in order to be hired. Note that these profits could be rewritten as coming from two different securities, but for simplicity we have written everything in terms of aggregates.

This maximization is subject to the restrictions:

$$I_U-\mu_U\left(\theta_2 G+(1-\theta_2)B\right)-\nu_U B\geq 0, \qquad (A_2)$$

$$\mu-\mu_U-\mu_C\geq 0, \qquad (C)$$

$$\theta_2\left(\mu_C G+\nu_C B\right)+(1-\theta_2)\left(\mu_C+\nu_C\right)B-\bar{V}\left(\mu_C+\nu_C\right)\geq 0. \qquad (D_2)$$

Note that a restriction on the size of the constrained pool is redundant by Assumption A1. We assign multipliers A_2, C, D_2 to the above constraints and form the Langrangian function L. The subscripts signify that solution is for the second period. Constraint (A_2) states that the value of the unconstrained pool, given that the opportunistic CRA includes only bad assets, cannot be greater than the wealth of the unconstrained investors I_U. Constraint (C) states that the amount of good assets that can be included is μ. Constraint (D_2) states that constrained investors require an expected quality level of at

least $\bar{V}$.

The Kuhn-Tucker first-order conditions are as follows, where each inequality can be replaced by an equality if the corresponding measure is positive:

$$\begin{aligned}
\frac{\partial L}{\partial \nu_U} &= B - b - A_2 B \leq 0 && (11)\\
\frac{\partial L}{\partial \mu_U} &= \theta_2 G + (1-\theta_2)B - g - A_2\left(\theta_2 G + (1-\theta_2)B\right) - C \leq 0 && (12)\\
\frac{\partial L}{\partial \nu_C} &= B - b + D_2\left(B - \bar{V}\right) \leq 0 && (13)\\
\frac{\partial L}{\partial \mu_C} &= \left(\begin{array}{l} \theta_2 G + (1-\theta_2)B - g - C + \\ D_2\left(\theta_2 G + (1-\theta_2)\,B - \bar{V}\right) \end{array}\right) \leq 0. && (14)
\end{aligned}$$

1. From (11), it follows that $A_2 \geq (B-b)/B > 0$, and hence that constraint (A_2) binds. In fact, it must be the case that $A_2 = (B-b)/B$, as $A_2 > 0$ means all unconstrained investors will be served and since $\mu G < I_U$ this can only be the case if $\nu_U > 0$.

2. From (13), it follows that $D_2 \geq \frac{B-b}{\bar{V}-B} > 0$, and hence constraint (D_2) binds. This implies either that the constrained pool is empty or that each constrained security has a value of $\bar{V}$.

3. From (12), it follows that $C \geq \theta_2(G-B)b/B + b - g$. Hence, if $\theta_2(G-B)b/B + b - g > 0$, then constraint (C) binds.

4. From (14), it follows that

$$\begin{aligned}
C &\geq \theta_2 G + (1-\theta_2)B - g + \frac{B-b}{\bar{V}-B}\left(\theta_2 G + (1-\theta_2)\,B - \bar{V}\right)\\
&= \theta_2\,(G-B) - g + \frac{B-b}{\bar{V}-B}\theta_2\,(G-B) + b\\
&= \theta_2\,(G-B)\left(\frac{\bar{V}-b}{\bar{V}-B}\right) + b - g
\end{aligned}$$

5. From (12) and (14), it follows that $\frac{\partial L}{\partial \mu_C} > \frac{\partial L}{\partial \mu_U}$ if $\theta_2 G + (1-\theta_2)\,B \geq \bar{V}$, i.e. if

constrained investors can be served then there will be no good assets in the unconstrained pool. Moreover, from the assumption that $\bar{V} > g$ follows that in this case $\theta_2 G + (1-\theta_2)B - g > 0$, and hence by (14) constraint (C) binds. Of course, if constraint (C) binds, then it can't be the case that $0 > \frac{\partial L}{\partial \mu_C} > \frac{\partial L}{\partial \mu_U}$ (which would imply $\mu_C = \mu_U = 0$).

The above implies that if $\theta_2 G + (1-\theta_2)\, B \geq \bar{V}$, then the solution has $\mu_C = \mu$, $\mu_U = 0$, $\nu_C = \mu\left(\theta_2 G + (1-\theta_2)\, B - \bar{V}\right) / \left(\bar{V} - B\right)$, and $\nu_U = I_U/B$, giving strictly positive profits of $\left(\bar{V} - b\right) \mu\theta_2 (G-B) / \left(\bar{V} - B\right) - \mu(g-b)$.

If $\theta_2 G + (1-\theta_2)\, B < \bar{V}$, then there are no securities for constrained investors. The possibilities are either that (a) Condition C1 holds: $\mu_U = \mu$, $\nu_U = \frac{I_U - \mu(\theta_2 G + (1-\theta_2)B)}{B}$, and $\mu_C = \nu_C = 0$, or (b) Condition C1 does not hold: $\mu_C = \mu_U = \nu_C = 0$, and $\nu_U = I_U$. The first gives profits of $\mu\theta_2\,(G-B)\, b/B - \mu(g-b)$ and the second implies zero profits.

Hence, if and only if C2 holds, then the profit-maximizing solution has $\mu_C = \mu$, $\nu_C = \mu\left(\theta_2 G + (1-\theta_2)\, B - \bar{V}\right) / \left(\bar{V} - B\right)$, $\nu_U = I_U/B$, and $\mu_U = 0$. If and only if C2 does not hold but C1 does, then the solution has $\mu_U = \mu$, $\nu_U = \frac{I_U - \mu(\theta_2 G + (1-\theta_2)B)}{B}$, and $\mu_C = \nu_C = 0$. Finally, if and only if neither C1 nor C2 holds, then the solution has no CRA being hired.

It is easy to see that these solutions can be implemented as equilibria. For example, if beliefs are equal to the prior for any out-of-equilibrium message, they can be sustained.

The above equilibria also maximize the profits of the opportunistic CRA. If we denote the truthful CRA's profits by π_2^T, then the profits for the opportunistic CRA (when hired) can be written $\pi_2^T + (\mu_U + \mu_C)(g-b)$. Since the partial of profits with respect to μ_C is even higher than for the truthful CRA, if the equilibrium is payoff maximizing for the truthful CRA, then it is as well for the opportunistic CRA. Therefore the above equilibria are payoff maximizing equilibria, which, by Lemma 6, are also undefeated. It follows by Lemma 7 that there are no undefeated equilibria with different strategy-profiles since any other equilibrium is less profitable for the truthful CRA.

Proof of Lemma 5

In a separating equilibrium, the type of each CRA would be revealed perfectly. Hence, by Lemma 3 the opportunistic CRA would only be able to issue securities worth B in period 2, and it would thus not be hired then. This implies that it has no reputation concerns and would never issue a security worth more than B in period 1 either, and as it is separating in the first period, it would not be hired in the first period either.

The truthful CRA could not issue securities in period 1 resulting in a positive surplus on its own, or the opportunistic CRA would have a profitable deviation by mimicking the sizes and ratings of its issues (with actual values equal to or lower than the reported). Furthermore, if it were issuing securities worth B, it would not be hired.

Proof of Propositions 3 and 4

We will prove the propositions by showing that the equilibrium outcomes under $\mathcal{E}_{**}$ and $\mathcal{E}_{*}$ yield each type of CRA a higher payoff than any other equilibrium outcome of the first-period game, thus demonstrating that they are payoff-maximizing and that the truthful earns strictly less in any other equilibrium, and thereafter invoke Lemma 6 and 7 to show that they are the only undefeated equilibria. Throughout we assume A2 and A3, which, by Corollary 2 implies that the undefeated equilibrium in the second period is of type E_{**}.

We start with a number of useful Lemmas.

Lemma 8 *If the incentive constraint (4) does not bind in an equilibrium where the CRAs are hired, there is another equilibrium where both types are strictly better off.*

Suppose there is a payoff-maximizing equilibrium where the CRAs are hired in the first period and where the incentive constraint does not bind. It follows from the opportunistic CRA's first-order condition (4) that this can occur only when the opportunistic CRA issues only securities of type B. Furthermore, the truthful CRA keeps some, but not all of its good assets. If it kept all of the good assets, the truthful CRA wouldn't be hired. If it included all of the good assets, the incentive constraint of the opportunistic CRA

would be violated.[22] To see this, note that (4) in this case could be written

$$h'(\mu\,(G-B)) \leq \frac{(g-b)\left(\bar{V}-B\right)}{\theta_2\left(\mu\,(G-B)\right)\delta\mu\left(\bar{V}-b\right)(G-B)^2},$$

which is inconsistent with assumption A3 and the assumption that

$$h'(\mu(G-B)) > \frac{g-b}{\delta\mu\,(G-B)\left(\bar{V}-b\right)}.$$

Define $\pi\,(z) := \mu\left(\left(\bar{V}-b\right)\theta_2\,(z)\,(G-B)/\left(\bar{V}-B\right)\right)$. An equilibrium candidate maximizing the truthful CRA's payoff must be a solution to the following program, where the second-period profits are given by Proposition 1:

$$\max_{\mu_U,\mu_C,\nu_U,\nu_C\geq 0}\left\{\begin{array}{c}(\mu_U+\mu_C)\,(\theta_1 G+(1-\theta_1)B-g)\\ +(\nu_U+\nu_C-I_U/B)(B-b)\\ +\delta\pi\,(z)-\delta\mu\,(g-b))\end{array}\right\}$$

s.t.

$$I_U-\mu_U\,(\theta_1 G+(1-\theta_1)B)-\nu_U B\geq 0, \qquad (A_1)$$

$$\theta_1\,(\mu_C G+\nu_C B)+(1-\theta_1)\,(\mu_C+\nu_C)B-\bar{V}\,(\mu_C+\nu_C)\geq 0, \qquad (D_1)$$

$$(g-b)\,/(G-B)-h'(z)\delta\pi\,(z)\geq 0, \qquad (E)$$

Constraint (A_1) states that the value of the assets in the unconstrained pool cannot be greater than I_U. Constraint (D_1) states that constrained investors require an expected quality level of at least $\bar{V}$. Constraint (E) is the opportunistic CRA's incentive constraint (4).

We form the Lagrangian function L, with multipliers A_1, D_1, and E. Recalling $z = (\mu_U+\mu_C)\,(G-B)$, we obtain the following Kuhn-Tucker first-order conditions, where

[22] This implies that constraint (C), which says that the measure of good assets included is at most μ, used in Propositions 1 and 2, is redundant here and therefore left out.

each holds with equality if the corresponding variable is positive:

$$\frac{\partial L}{\partial \nu_U} = B - b - A_1 B \leq 0 \tag{15}$$

$$\frac{\partial L}{\partial \nu_C} = B - b + D_1 \left(B - \bar{V}\right) \leq 0 \tag{16}$$

$$\frac{\partial L}{\partial \mu_U} = \theta_1 G + (1-\theta_1)B - g + \delta\pi'(z)(G-B) \tag{17}$$

$$-A_1\left(\theta_1 G + (1-\theta_1)B\right) - E(G-B)\left(h''(z)\delta\pi(z) + h'(z)\delta\pi'(z)\right) \leq 0$$

$$\frac{\partial L}{\partial \mu_C} = \theta_1 G + (1-\theta_1)B - g + \delta\pi'(z)(G-B) \tag{18}$$

$$+D_1\left(\theta_1 G + (1-\theta_1)B - \bar{V}\right)$$

$$-E(G-B)\left(h''(z)\delta\pi(z) + h'(z)\delta\pi'(z)\right) \leq 0.$$

1. From condition (15), it follows that $A_1 \geq (B-b)/B > 0$. Given the assumption that $I_U > G\mu$, $\nu_U > 0$ and $A_1 = (B-b)/B$.

2. Condition (16) gives us $D_1 \geq (B-b)/\left(\bar{V} - B\right) > 0$.

3. If $\theta_1 G + (1-\theta_1)B - \bar{V} \geq 0$, then by the assumption in the text that $\bar{V} > g$, $\theta_1 G + (1-\theta_1)B - g \geq 0$. However, given that $\pi'(z) > 0$, if this holds E must be positive, since otherwise $\frac{\partial L}{\partial \mu_C} > 0$. We also note that $\frac{\partial L}{\partial \mu_C} - \frac{\partial L}{\partial \mu_U} = D_1\left(\theta_1 G + (1-\theta_1)B - \bar{V}\right) + A_1\left(\theta_1 G + (1-\theta_1)B\right) > 0$, implying that $\mu_U = 0$.

4. If $\theta_1 G + (1-\theta_1)B - \bar{V} < 0$, then given $D_1 > 0$, it must be that $\mu_C = \nu_C = 0$, i.e. there is no constrained pool. If additionally $\theta_1(G-B)b/B - g + b \geq 0$, then E must be positive since otherwise $\frac{\partial L}{\partial \mu_U} > 0$. If, on the other hand, $\theta_1(G-B)b/B - g + b < 0$, then first-period profits for the truthful CRA are negative. As fees are assumed to be non-negative, this can't be the case.

Hence, if the CRAs are hired the solution must have all three constraints bind, and either $\mu_U = 0$, if $\theta_1 G + (1-\theta_1)B - \bar{V} \geq 0$, or $\mu_C = 0$, if $\theta_1 G + (1-\theta_1)B - \bar{V} < 0$. This uniquely determines all four variables. More importantly, it shows that the truthful CRA is better off if the incentive constraint binds.

Analogously to the above, an equilibrium candidate maximizing the opportunistic CRA's payoff under the assumption that the incentive constraint does not bind must be a solution to the following program:

$$\max_{\mu_U,\mu_C,\nu_U,\nu_C\geq 0}\left\{\begin{array}{c}(\mu_U+\mu_C)\left(\theta_1 G+(1-\theta_1)B-b\right)\\ +(\nu_U+\nu_C-I_U/B)(B-b)\\ +\left(1-h\left(\left(\mu_U+\mu_C\right)(G-B)\right)\right)\delta\pi\left(z\right)\end{array}\right\}$$

s.t.

$$I_U-\mu_U\left(\theta_1 G+(1-\theta_1)B\right)-\nu_U B\geq 0, \tag{A_1}$$

$$\theta_1\left(\mu_C G+\nu_C B\right)+(1-\theta_1)\left(\mu_C+\nu_C\right)B-\bar{V}\left(\mu_C+\nu_C\right)\geq 0, \tag{D_1}$$

$$\left(g-b\right)/(G-B)-h'(z)\delta\pi\left(z\right)\geq 0. \tag{E}$$

We obtain the Kuhn-Tucker first-order conditions:

$$\frac{\partial L}{\partial \nu_U}=B-b-A_1 B\leq 0 \tag{19}$$

$$\frac{\partial L}{\partial \nu_C}=B-b+D_1\left(B-\bar{V}\right)\leq 0 \tag{20}$$

$$\frac{\partial L}{\partial \mu_U}=\theta_1 G+(1-\theta_1)B-b+\left(1-h\left(z\right)\right)\delta\pi'\left(z\right)(G-B)-h'\left(z\right)\delta\pi\left(z\right)(G-B) \tag{21}$$

$$-A_1\left(\theta_1 G+(1-\theta_1)B\right)-E(G-B)\left(h''\left(z\right)\delta\pi\left(z\right)+h'(z)\delta\pi'\left(z\right)\right)\leq 0$$

$$\frac{\partial L}{\partial \mu_C}=\theta_1 G+(1-\theta_1)B-b+\left(1-h\left(z\right)\right)\delta\pi'\left(z\right)(G-B)-h'\left(z\right)\delta\pi\left(z\right)(G-B) \tag{22}$$

$$+D_1\left(\theta_1 G+(1-\theta_1)B-\bar{V}\right)-E(G-B)\left(h''\left(z\right)\delta\pi\left(z\right)+h'(z)\delta\pi'\left(z\right)\right)\leq 0$$

As in the previous case, constraints (A_1) and (D_1) must bind. Moreover, by constraint (E),

$$-h'\left(z\right)\delta\pi\left(z\right)(G-B)\geq -g+b.$$

This implies that

$$\frac{\partial L}{\partial \mu_U} \geq \theta_1 G + (1-\theta_1)B - g + (1-h(z))\,\delta\pi'(z)\,(G-B)$$
$$-A_1\left(\theta_1 G + (1-\theta_1)B\right) - E(G-B)\left(h''(z)\,\delta\pi(z) + h'(z)\delta\pi'(z)\right)$$

and

$$\frac{\partial L}{\partial \mu_C} \geq \theta_1 G + (1-\theta_1)B - g + (1-h(z))\,\delta\pi'(z)\,(G-B)$$
$$+D_1\left(\theta_1 G + (1-\theta_1)B - \bar{V}\right) - E(G-B)\left(h''(z)\,\delta\pi(z) + h'(z)\delta\pi'(z)\right).$$

Hence, by the same arguments as above, $E > 0$ and we obtain the same (one- and two-security type) solutions as above. It remains to show that these solutions can be sustained as equilibria. However, this follows trivially by assuming out-of-equilibrium path beliefs that assigns probability one to the opportunistic CRA.

It follows that both types of CRA earn strictly higher payoffs in the above equilibria, where the incentive constraint binds, than in any equilibrium where this is not the case.

Lemma 9 *If and only if $C2'$ holds, $\mathcal{E}_{**}$ is a payoff-maximizing equilibrium of the first-period game. If and only if $C1'$ holds but $C2'$ does not, the outcome of $\mathcal{E}_{*}$ is a payoff-maximizing equilibrium of the first-period game.*

We know from Lemma 5 that any equilibrium of the first-period game where the CRAs are hired must be pooling. Consider the objective function of the truthful CRA:

$$\theta_1(\mu_U G + \nu_U B) + (1-\theta_1)\left(\mu_U^O G + (\mu_U + \nu_U - \mu_U^O)B\right) - \mu_U g - \nu_U b +$$
$$\theta_1(\mu_C G + \nu_C B) + (1-\theta_1)\left(\mu_C^O G + (\mu_C + \nu_C - \mu_C^O)B\right) - \mu_C g - \nu_C b$$
$$-I_U(B-b)/B + \delta\pi_2^T(\theta_2),$$

where μ_U^O and μ_C^O are the measures of good assets sold by the opportunistic CRA for the unconstrained and constrained pools respectively, and $\pi_2^T(\theta_2)$ is the second-period profits of the truthful CRA in the unique undefeated equilibrium outcome of the corresponding

second-period game. Note that we have proven (i) that the incentive constraint binds in any payoff maximizing equilibrium in Lemma 8 and (ii) there is a unique interior level of ratings inflation z^*. This implies that $\pi_2^T(\theta_2)$ does not change with respect to the choice variables (as we are comparing equilibria).

The first line of the objective function is the net revenue from the unconstrained securities, i.e. price (which depends on θ_1) times quantity minus opportunity cost of holding the assets. The second line is the net revenue from the constrained securities. The third line has the surplus that the CRA must leave to the issuer and the expected profits from the second period.

We are looking for the payoff-maximizing equilibrium, which implies that this expression should be maximized with respect to all of the choice variables $\mu_U, \mu_C, \nu_U, \nu_C, \mu_U^O$, and μ_C^O given non-negativity constraints and the restrictions:

$$I_U - \theta_1 \left(\mu_U G + \nu_U B\right) - (1-\theta_1)\left(\mu_U^O G + \left(\mu_U + \nu_U - \mu_U^O\right) B\right) \geq 0, \qquad (\bar{A}_1)$$

$$\mu - \mu_U - \mu_C \geq 0, \qquad (C)$$

$$\begin{array}{c} \theta_1 \left(\mu_C G + \nu_C B\right) + (1-\theta_1)\left(\mu_C^O G + (\mu_C + \nu_C - \mu_C^O) B\right) \\ -\bar{V}\left(\mu_C + \nu_C\right) \end{array} \geq 0, \qquad (\bar{D}_1)$$

$$\left(\mu_U - \mu_U^O\right)(G-B) + \left(\mu_C - \mu_C^O\right)(G-B) - z^* = 0, \qquad (\bar{E})$$

$$\mu_C + \nu_C - \mu_C^O \geq 0, \qquad (F)$$

$$\mu_U + \nu_U - \mu_U^O \geq 0, \qquad (H)$$

where z^* is the inflation when the opportunistic CRA's incentive compatibility constraint binds (equation 4). We know that this is the case from Lemma (8). Constraint $\left(\bar{A}_1\right)$ states that the value of the assets in the unconstrained pool cannot be greater than I_U. Note that due to assumption A1, a corresponding constraint for the the constrained

pool is redundant. Constraint (C) states that the amount of good assets that can be included is μ. Constraint $(\bar{D}_1)$ states that constrained investors require a quality level of at least $\bar{V}$. Constraint $(\bar{E})$ is the binding incentive constraint. Constraints (F) and (H) state that in each pool, the opportunistic CRA cannot include more good assets than the total measure of assets (good and bad) included by the truthful CRA.

We set up the Lagrangian L with multipliers named after each constraint ($\bar{A}_1, C, \bar{D}_1, \bar{E}$, F, and H) and obtain the following (simplified) first-order conditions. Each holds with equality if the relevant variable is greater than zero.

$$\frac{\partial L}{\partial \mu_U^O}\frac{1}{G-B} = 1-\theta_1-\bar{A}_1(1-\theta_1)-\bar{E}-H/(G-B)\leq 0 \tag{23}$$

$$\frac{\partial L}{\partial \mu_C^O}\frac{1}{G-B} = 1-\theta_1+\bar{D}_1(1-\theta_1)-\bar{E}-F/(G-B)\leq 0 \tag{24}$$

$$\frac{\partial L}{\partial \nu_U} = B-b-\bar{A}_1 B+H\leq 0 \tag{25}$$

$$\frac{\partial L}{\partial \mu_U} = \theta_1 G+(1-\theta_1)B-g-\bar{A}_1(\theta_1 G+(1-\theta_1)B) \tag{26}$$

$$-C+\bar{E}(G-B)+H\leq 0$$

$$\frac{\partial L}{\partial \nu_C} = B-b+\bar{D}_1\left(B-\bar{V}\right)+F\leq 0 \tag{27}$$

$$\frac{\partial L}{\partial \mu_C} = \theta_1 G+(1-\theta_1)B-g-C+F+ \tag{28}$$

$$\bar{D}_1\left(\theta_1 G+(1-\theta_1)B-\bar{V}\right)+\bar{E}(G-B)\leq 0.$$

We can find the unique solution in 5 steps.

1. Condition (25) implies that $\bar{A}_1 > 0$, and hence, by the assumption that $I_U > G\mu$; $\nu_U > 0$, $\bar{A}_1 = \frac{B-b+H}{B}$.

2. Condition (27) implies $\bar{D}_1 \geq \frac{B-b+F}{\bar{V}-B} > 0$. Hence, either there are no constrained securities or the constrained securities have a value of $\bar{V}$.

3. Solving for $\bar{E}$ from (24) and plugging into (28) gives: $C \geq \bar{D}_1\left(G-\bar{V}\right)+G-g > 0$.

Hence, the truthful CRA includes all of its good assets and the opportunistic a measure $\mu - z^*/(G-B)$ of such assets.

4. We have that

$$\frac{\partial L}{\partial \mu_C} - \frac{\partial L}{\partial \mu_U} = F - H + (\theta_1 G + (1-\theta_1)B)(B - b + H)/B + \bar{D}_1\left(\theta_1 G + (1-\theta_1)B - \bar{V}\right). \tag{29}$$

   (a) If (29) is positive, then $\mu_C = \mu$ and $\mu_U = 0$, implying $F = 0$ and $\bar{D}_1 = \frac{B-b}{\bar{V}-B}$. From $F = 0$ follows that $\frac{\partial L}{\partial \mu_C^O} > \frac{\partial L}{\partial \mu_U^O}$, giving $\mu_C^O = \mu - z^*/(G-B)$ and $\mu_U^O = 0$. Hence, $H = 0$. Using the binding constraints $(\bar{A}_1)$ and $(\bar{D}_1)$ we can calculate $\nu_U = I_U/B$, and $\nu_C = \left(\mu\left(G - \bar{V}\right) - (1-\theta_1)z^*\right)/\left(\bar{V} - B\right)$. This solution exists if and only if $\mu\left(G - \bar{V}\right) \geq (1-\theta_1)z^*$, in which case $\nu_C \geq 0$ and first-period profits, given by the following expression, are positive:

$$(\mu(G-B) - z^*(1-\theta_1))\frac{\bar{V}-b}{\bar{V}-B} - \mu(g-b).$$

   (b) If (29) is negative, which requires $\nu_C = 0$ (otherwise, the expression is equal to $\left(\frac{B-b+H}{B} + \frac{B-b+F}{\bar{V}-B}\right)\theta_1(G-B) > 0$) and $\theta_1 G + (1-\theta_1)B < \bar{V}$, then $\mu_U = \mu$ and $\mu_C = 0$. By constraints $(\bar{E})$ and (F) follow that in this case $\mu_C^O = 0$ and $\mu_U^O = \mu - z^*/(G-B)$. By constraint $(\bar{A}_1)$, $\nu_U = \frac{I_U - \mu G + z^*(1-\theta_1)}{B}$. First-period profits in this case are given by

$$(\mu(G-B) - z^*(1-\theta_1))b/B - \mu(g-b).$$

   (c) If (29) is zero, which like the previous case requires $\nu_C = 0$ and $\theta_1 G + (1-\theta_1)B < \bar{V}$, we can also have a solution where the truthful CRA places good assets in both pools. It must entail $\mu_C^O > 0$, or constrained investors could not be served due to $\theta_1 G + (1-\theta_1)B < \bar{V}$. Moreover, $F = 0$ since otherwise $\mu_C^O = \mu_C$, which is not consistent with $\bar{D}_1 > 0$. Hence, $\frac{\partial L}{\partial \mu_C^O} > \frac{\partial L}{\partial \mu_U^O}$, implying $\mu_U^O = 0$. Solving for μ_C from the binding constraint $(\bar{D}_1)$, using $\nu_C = 0$ and

$\mu_C^O = \mu - z^*/(G-B)$, gives:

$$\mu_C = \frac{(1-\theta_1)\left(\mu(G-B) - z^*\right)}{\bar{V} - \theta_1 G - (1-\theta_1) B}$$

and

$$\mu_U = \frac{\mu\left(\bar{V} - G\right) + (1-\theta_1) z^*}{\bar{V} - \theta_1 G - (1-\theta_1) B}.$$

The first expression (μ_C) is positive by the assumption regarding $h'(\mu(G - B))$. The second expression (μ_U) is positive if $\mu\left(G - \bar{V}\right) < (1-\theta_1) z^*$. This inequality also guarantees $\theta_1 G + (1-\theta_1) B < \bar{V}$, but not that first-period profits, given by the subsequent expression, are positive:

$$\left(\mu(G-B) - z^*(1-\theta_1)\right) b/B - \mu(g-b) + \frac{\bar{V}(1-\theta_1)\left(\mu(G-B) - z^*\right)}{\bar{V} - \theta_1 G - (1-\theta_1) B} \frac{B-b}{B}.$$

5. By comparing the profits of the three candidates follows that (a) is a solution if and only if $\mu\left(G - \bar{V}\right) \geq (1-\theta_1) z^*$, and that (c) is a solution if and only if $\mu\left(G - \bar{V}\right) < (1-\theta_1) z^*$ and first-period profits are positive.

The profit-maximizing equilibria for the truthful CRA are profit maximizing also for the opportunistic CRA. The reason is that given Lemma 8, inflation in any profit-maximizing equilibrium where the CRA is hired is given by z^*, implying that second-period profits are fixed and that the only difference in first-period profits between the opportunistic and the truthful CRA is given by a constant, $z^* \frac{g-b}{G-B}$. Hence, the maximization problem for the truthful CRA also maximizes the profits of the opportunistic CRA.

We are now in a position to complete the proof of Propositions 3 and 4. Propositions 1, 2, and Corollary 1 characterize the unique undefeated equilibrium outcome of the second-period game for any prior θ_2. Lemmas 8 and 9 demonstrate that the restriction of $\mathcal{E}_{**}$ and $\mathcal{E}_*$ to the first period are the only payoff-maximizing equilibria of the first-period game (although they can be supported by different beliefs) and therefore, by Lemmas 6 and 7, also the only undefeated equilibria.