Rationality and the Bayesian Paradigm

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- Objectivity as second-order subjectivity

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- Clearly, $\succsim^* \subset \succsim^{\wedge}$

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- **P5** $\exists\ f \succ g$
- **P6** $f \succ g$ $\exists$ a partition of S, $\{A_1, ..., A_n\}$ $f^h_{A_i} \succ g$ and $f \succ g^h_{A_i}$

Savage's Theorem

- Assume that X is finite. Then $\succsim$ satisfies P1-P6 if and only if there exist a non-atomic finitely additive probability measure μ on S $(=(S, 2^S))$ and a non-constant function $u : X \to \mathbb{R}$ such that, for every $f, g \in F$

$$f \succsim g \quad \text{iff} \quad \int_S u(f(s))d\mu(s) \geq \int_S u(g(s))d\mu(s)$$

  Furthermore, in this case μ is unique, and u is unique up to positive linear transformations.

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- A problem for a behavioral derivation
- Where would the probability come from?

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- The Bayesian approach is good at representing knowledge, poor at representing ignorance

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- Rule-based approaches: logit
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- But none extends to the cases of wars, stock market crashes...

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- Maxmin EU: there exists a set of probabilities C such that

$$V(f) = \min_{P \in C} \int_S u(f(s))\, dP(s)$$

Other Multiple-Priors Models

- Nau, Klibanoff-Marinacci-Mukerji: “smooth preferences”

$$\varphi : \mathbb{R} \rightarrow \mathbb{R}$$

$$\int_{\Delta(S)} \varphi\left(\int u(f)\, dp\right) d\mu$$

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- Maccheroni-Marinacci-Rustichini: "variational preferences"

$$V(f) = \min_{P \in \Delta(S)} \left\{ \int_S u(f(s))\, dP(s) + c(P) \right\}$$

Incomplete Preferences

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- Can be combined with the maxmin criterion as "subjective rationality"