#### **Revealed Political Power**

#### Jinhui Bai and Roger Lagunoff

(Georgetown University) (Georgetown University)

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### Introduction

 Political equality/egalitarianism is an ingrained governing philosophy in most democracies.

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#### Introduction

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- The principle is used to judge a government's legitimacy.
- ... embodied in rhetoric.

"one-man-one-vote", "justice is blind", "equality in the eyes of the law" - Cleisthenes, 508BC, etc.,...

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#### ► Channels of Wealth Bias in Policy Making Process.

 Differential participation rates. Rosenstone and Hansen (1993), Bartels (2008).

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- Differential participation rates. Rosenstone and Hansen (1993), Bartels (2008).
- Political Knowledge and Contact. Bartels (2008).
- Campaign contributions. Austen-Smith (1987), Grossman and Helpman (1996), Prat (2002), Coate (2004), Campante (2008).

## Research Agenda

#### • Our Focus: Identify Wealth Bias from Policy Outcome:

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- When can wealth bias be inferred?
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Non-parametric approach from Revealed Preference Tradition

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#### Contrast with Existing Literature:

- Implication from Parametric Models: Benabou (2000), Campante (2008), Bai & Lagunoff (2010)
- Reduced Form Statistical Analysis: Bartels (2008)

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3. Contracted Set of Allowed Preference. Add Preference Restrictions: supermodularity and "weakly separable utility"

*Result: unbiased polity imposes monotonicity restriction on the data.* 

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**Assumptions**: One dimensional policies, states. Each state is distinct. **y** increasing in **i**, and its structure is known/observed by outside observer.

#### Preferences

 $U(i, \omega_t, a_t) = i$ 's payoff fnct. Outside observer does not know/observe U directly, but knows that U belongs to an "admissible" class defined by

(A1) (Single peakedness) U single peaked in a.

(A2) (Single Crossing) **U** satisfies single crossing in (a; i).

## A Static Example

$$\begin{split} \mathsf{u}(\mathsf{c}_{\mathsf{t}},\mathsf{G}_{\mathsf{t}}) \;&=\;\; \mathsf{c}_{\mathsf{t}} \;+\; \frac{\mathsf{G}_{\mathsf{t}}^{1-\rho}}{1-\rho} \;\;\; \mathsf{s.t.} \\ \mathsf{c}_{\mathsf{t}} \;&=\; (1-\tau_{\mathsf{t}}) \, \mathsf{y} \left(\mathsf{i},\omega_{\mathsf{t}}\right), \\ \mathsf{G}_{\mathsf{t}} \;&=\; \tau_{\mathsf{t}} \int_{0}^{1} \mathsf{y} \left(\mathsf{i},\omega_{\mathsf{t}}\right) \mathsf{d}\mathsf{i} \;=\; \tau_{\mathsf{t}} \overline{\mathsf{y}} \left(\omega_{\mathsf{t}}\right) \end{split}$$

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Define  $a_t = 1 - \tau_t$  so the problem becomes

$$\mathsf{U}\left(\mathsf{i},\omega_{\mathrm{t}},\mathsf{a}_{\mathrm{t}}\right) = \mathsf{a}_{\mathrm{t}}\mathsf{y}\left(\mathsf{i},\omega_{\mathrm{t}}\right) + \frac{\left[\left(1-\mathsf{a}_{\mathrm{t}}\right)\overline{\mathsf{y}}\left(\omega_{\mathrm{t}}\right)\right]^{1-\rho}}{1-\rho}.$$

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ight]^{1-
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Problem accommodates:

- (a) pure growth.  $\mathbf{y}(\mathbf{i}, \omega_t) = \mathbf{g}(\mathbf{i})\omega_t$ .
- (b) pure (mean-preserving) inequality change.  $\overline{\mathbf{y}}(\omega) = \overline{\mathbf{y}}$ .

Power is measured by a *wealth-weighted vote share*  $\lambda(\mathbf{y}, \alpha, \omega)$ where  $\alpha(\omega) = bias$  in each state.

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 $lpha(\omega) > 0$  an elitist bias  $lpha(\omega) < 0$  a populist bias  $lpha(\omega) = 0$  an unbiased polity.

## Political Lorenz Curve with Elitist Bias



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## Weighted Majority Winners

**Def'n** A policy **a** is an  $\alpha$ -Weighted Majority Winner (WMW) in state  $\omega$  under admissible profile **U** if,

$$\int_{\{\mathbf{i}: \ \mathsf{U}(\mathbf{i},\omega,\mathbf{a}) \geq \mathsf{U}(\mathbf{i},\omega,\hat{\mathbf{a}})\}} \lambda\left(\mathsf{y}(\mathbf{i},\omega),\alpha(\omega),\omega\right) \, \mathsf{d}\mathbf{i} \ \geq 1/2 \quad \forall \ \hat{\mathbf{a}}$$

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Rationalizing Policy Data

Policy rule  $\Psi(\omega) = a$ .



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**Def'n** A weighting function  $\alpha$  rationalizes the policy data  $\{a_t\}_{t=1}^T$  if  $\exists$  admissible profile U and policy rule  $\Psi$  consistent with the data such that for each  $\omega$ ,  $\Psi(\omega)$  is an  $\alpha$ -Weighted Majority Winner under U.

# A Benchmark

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Let  $\{a_t\}_{t=1}^T$  be any policy data and  $\alpha$  be any weighting function. Then  $\alpha$  rationalizes  $\{a_t\}_{t=1}^T$ .

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### "Anything Goes" Theorem

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Bottom line: any distribution of political power - including one implied by equal representation (an unbiased polity) - can rationalize the policy data.

 Prf is constructive. Apply Gans-Smart Med. Voter Thm. Then construct admissible preference profile (adapt Boldrin-Montruccio quadratic form).

$${\sf U}\left({\sf i},\omega,{\sf a};\Psi
ight)=-rac{1}{2}\left({\sf a}-\widetilde{\Psi}\left({\sf i},\omega
ight)
ight)^2,$$
  
where  $\widetilde{\Psi}(\mu(\omega,lpha),\omega)=\Psi(\omega).$ 

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## Polling Data

 $\blacktriangleright$  N polls taken at each t comparing  $a_t$  to policy alternatives

$$\mathsf{a}^1 < \mathsf{a}^2 < \cdots < \mathsf{a}^\mathsf{N}$$

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- Poll data: Fraction p<sup>n</sup><sub>t</sub> of population prefer a<sub>t</sub> to policy alternative a<sup>n</sup>.
- ▶ No measurement error (!!).

**Def'n** An  $\alpha$  rationalizes both policy data  $\{a_t\}_{t=1}^T$  and poll data  $\{p_t^n\}_{t=1,n=1}^{T,N}$  if  $\exists$  admissible U and  $\Psi$  consistent with the data such that

(i)  $\forall \, \omega, \, \Psi(\omega)$  is an  $\alpha$ -Weighted Majority Winner under U, and

(ii)  $\forall t \forall n$ , U satisfies

 $\mathsf{p}_{\mathsf{t}}^{\mathsf{n}} \;=\; |\{\mathsf{i}:\;\mathsf{U}(\mathsf{i},\omega_{\mathsf{t}},\mathsf{a}_{\mathsf{t}})\geq\mathsf{U}(\mathsf{i},\omega_{\mathsf{t}},\mathsf{a}^{\mathsf{n}})\}|$ 

Let 
$$n_t^*$$
 satisfy  $a^{n_t^*-1} < a_t < a^{n_t^*}$ 

 $a^{n_t^*-1} = \text{closest "left-wing" alternative.}$  $a^{n_t^*} = \text{closest "right-wing" alternative.}$ 

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### Theorem

Let  $\{a_t\}$  be any policy data and  $\{p_t^n\}_{t=1,n=1}^{\mathsf{T},\mathsf{N}}$  any arbitrary polling data. Then:

1. There exists an lpha that rationalizes the data iff orall t,

 $1 - p_t^1 < \ldots < \ 1 - p_t^{n_t^* - 2} \ < \ 1 - p_t^{n_t^* - 1} \ < \ p_t^{n_t^*} \ < \ p_t^{n_t^* + 1} \ < \ldots < p_t^N$ 

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2. Any given lpha rationalizes the data iff

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# Inverse Pivotal Function M



# **Bias Band**



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- For some applied questions, we may have additional information on the preference beyond (A1) and (A2)
- These narrower class of preference may help reveal political wealth bias

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- For some applied questions, we may have additional information on the preference beyond (A1) and (A2)
- These narrower class of preference may help reveal political wealth bias
- ▶ We illustrate this from two canonical examples:
  - $\omega$  introduces a change in mean income
  - $\omega$  introduces a mean-preserving change of income inequality

Recall our canonical example of public goods provision

$$\mathsf{U}(\mathsf{i},\omega_{\mathsf{t}},\mathsf{a}_{\mathsf{t}}) = \mathsf{a}_{\mathsf{t}}\mathsf{y}(\mathsf{i},\omega_{\mathsf{t}}) + \frac{\left[(1-\mathsf{a}_{\mathsf{t}})\,\overline{\mathsf{y}}\,(\omega_{\mathsf{t}})\right]^{1-\rho}}{1-\rho}$$

Two interesting cases:

(a) pure growth. 
$$\mathbf{y}(\mathbf{i}, \omega_t) = \mathbf{g}(\mathbf{i})\omega_t$$
.

The preference satisfies a single-crossing restriction in  $(a; \omega)$ 

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(b) pure (mean-preserving) inequality change.  $\overline{\mathbf{y}}(\boldsymbol{\omega}) = \overline{\mathbf{y}}$ .

The preference only depends on  $y(i, \omega)$  and a, i.e.,

$$\mathsf{U}(\mathsf{i},\omega,\mathsf{a})=\mathsf{u}(\mathsf{y}(\mathsf{i},\omega),\mathsf{a})$$

Additional Single-Crossing Preference Restrictions

(A3) **U** satisfies single crossing in the pair (a;  $\omega$ ) for each **i**.

 $\implies$  each citizen's preferred policy rule is incr. in the state.

#### Theorem

A weighting function  $\alpha$  rationalizes  $\{a_t\}$  with preference in (A1)-(A3) iff for any pair of observed states such that  $\omega_t > \omega_{\tau}$ ,

$$\mathsf{a}_\mathsf{t} < \mathsf{a}_ au \implies \mu(\omega_\mathsf{t}, lpha) < \mu(\omega_ au, lpha)$$

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#### Corollary

The unbiased weighting system rationalizes the policy data only if the data is wkly increasing in the state. Additional Separable Preference Restrictions

(A4) U(i,  $\omega$ , a) = u(y(i,  $\omega$ ), a).

 $\implies$  each citizen's preferred policy rule is incr. in the income.

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Additional Separable Preference Restrictions

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#### Theorem

A weighting function  $\alpha$  rationalizes  $\{a_t\}$  with preference in (A1), (A2) and (A4) iff for any pair of observations,

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### Corollary

The unbiased weighting system rationalizes the policy data iff a policy change is associated with a change of the median income in the same direction.

# Conclusions

- 1. Toward a formal theory of revealed political power.
- 2. Policy data alone with only weak preference requirement is not discerning.
- Policy + polling data jointly describe the boundaries of wealth bias.
- 4. Additional preference restrictions rule out some types of bias.
- 5. Biggest challenge: Multi-dimensional policy and state spaces. Some success with order restricted preferences, but inherent difficulties.

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Single Crossing in (i, a):

For all  $a > \hat{a}$ ,

$$U(i, \omega_t, a) - U(i, \omega_t, \hat{a}) > 0$$

implies

$$U(j, \omega_t, a) - U(j, \omega_t, \hat{a}) > 0 \quad \forall j > i.$$

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## Two Interpretations

### 1. A Classic Static RPT Interpretation

The observer sees  $\{a_t, \omega_t\}_{t=1}^T$ . No intertemporal connection. Data is a time series generated by myopic citizens.

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### 1. A Classic Static RPT Interpretation

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### 2. A Dynamic Economy Interpretation

The observer sees  $\{a_t, \omega_t\}_{t=1}^T$ . He infers intertemporal connections, backing out transition rule  $\omega_{t+1} = Q(\omega_t, a_t)$ . Data is generated by forward looking citizens (U is a long run payoff). Underlying time horizon is infinite.

# The Political Side

Power is measured by a *wealth-weighted vote share*  $\lambda(\mathbf{y}, \alpha, \omega)$ where  $\alpha(\omega) = bias$  in each state.

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- $\lambda$  is a density in income **y**.
- λ is increasing in income if α > 0, decreasing in income if α < 0; constant if α = 0.</li>

▶  $\lambda$  satisfies strict single crossing in ( $\alpha$ ; y) and  $\rightarrow$  0 as  $\alpha \rightarrow \pm \infty$ 

Observer knows  $\lambda(\cdot)$  but must infer  $\alpha(\cdot)$  from data.

# Political Lorenz Curve with Dampened Elitist Bias



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# Political Lorenz Curve with Populist Bias



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## Algorithm

Necessity is the easy part. Sufficiency is harder.

Step 1° Use a recursive algorithm to construct  $\tilde{\Psi}(\mathbf{i}, \omega)$  satisfying 1.  $\tilde{\Psi}(\mu(\omega_t, \alpha), \omega_t) = \mathbf{a}_t \quad \forall t = 1, \dots, T.$ 

2.  $\tilde{\Psi}(\mathbf{i}, \omega)$  increasing in  $\mathbf{i}$  and  $\omega$ .

Step 2° Use the constructed  $\tilde{\Psi}$  to define  $\Psi$  and U given by

$$\mathsf{U}\left(\mathsf{i},\omega,\mathsf{a};\Psi
ight)=-rac{1}{2}\left(\mathsf{a}-\widetilde{\Psi}\left(\mathsf{i},\omega
ight)
ight)^{2},$$