

The Lindahl Approach to Household Behavior

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Introduction : Objectives

To extend the non-unitary approach to household behavior by filling the gap between the two models studied in the literature:

- The 'collective model', which entails Pareto-efficiency of household decisions (e.g. Chiappori, 1988, 1992, Browning & Chiappori, 1998, Cherchye-De Rock-Vermeulen, 2007).
- The 'fully non-cooperative model', resulting in a Nash equilibrium with voluntary contributions to public goods (e.g. Ulph, 1988, Chen & Wooley, 2001, Lechene & Preston, 2005, 2007, Browning, Chiappori & Lechene, 2006).

This extension was started in two previous papers (d'Aspremont and Dos Santos Ferreira, 2009, Cherchye, Demuynck and De Rock, 2009). The present paper is "work in progress"

Motivation: household money management systems

The International Social Survey Program (1994 and 2002). Two types of systems

- Systems in which couples operate more or less as *single economic units*:
 - money management by one of the two spouses (more than 50%), may be with some spending money left to the other
 - money pooled in a common bank account and managed jointly by the two spouses
- Individualized or privatized systems in which couples operate largely as two separate, *autonomous economic units* independent management system:
 - each spouse keeps his/her own income separate and is responsible for different items of household expenditure
 - the *partial pool* : couples pool some of their income to pay for collective expenditure and keep the rest separate to spend as they choose (13% 1994, 17% 2002)

To test the robustness of some household features:

- *Separate spheres*: there is at most one public good to which both spouses contribute (Lundberg and Pollak, 1993)
- *Local income pooling*: in the case they both contribute, income redistributions have locally no effect.

This is important from a policy point of view
(e.g. Targeting benefits to one household member)

The Lindahl Approach

The Lindahl equilibrium is used both conceptually and for testability

- It implies to introduce *contributive shares* (*Lindahl personalised prices*) by the spouses to public goods within the household in the collective and semi-cooperative models.
- To derive nonparametric tests based on a revealed preference characterization.

The (centralised) collective model

- Two-adult household ($J = A, B$)

$$\begin{aligned} & \max_{(q^A, q^B, Q) \in \mathbb{R}_+^{2n+m}} \mu U^A(q^A, Q) + (1 - \mu) U^B(q^B, Q) \\ & \text{s.t. } p(q^A + q^B) + PQ \leq Y, \end{aligned}$$

for some Pareto weight $\mu \in [0, 1]$, with

q^J : basket of purchased private goods

Q : basket of purchased public goods

Y : household income

(p, P) : market prices

- Non-unitary if Pareto weight μ depends on (Y, p, P) . Otherwise, we get a standard individual programme by first maximising the weighted sum of the two utilities under the constraint $q^A + q^B = q$.

The non-cooperative game with voluntary contributions

- Each spouse J chooses a strategy $(q^J, g^J) \in \mathbb{R}_+^{n+m}$ (q^J denoting J 's private consumptions and g^J his/her contributions to public goods) in order to solve the programme:

$$\begin{aligned} \max_{(q^J, g^J) \in \mathbb{R}_+^{n+m}} & U^J(q^J, g^J + g^{-J}) \\ \text{s.t.} & pq^J + Pg^J \leq Y^J. \end{aligned}$$

- A Nash equilibrium of this game can be characterized by the first order conditions (for $J = A, B$):

$$\begin{aligned} \frac{1}{\partial_{q_1} U^J(q^J, g^J + g^{-J})} \partial_q U^J(q^J, g^A + g^B) & \leq p \\ \tau^J(q^J, g^A + g^B) & \leq P \\ pq^J + Pg^J & = Y^J, \end{aligned}$$

with an equality for any private good i s.t. $q_i^J > 0$ or any public good k s.t. $g_k^J > 0$.

The Lindahl equilibrium

- A pair of (Lindahl) prices $(P^A, P^B) \in \mathbb{R}_+^{2m}$ are posted ($P^A + P^B = P$).
- Each J , anticipating contribution $g_k^{-J} \in \mathbb{R}_+$, chooses a contribution $g_k^J \in \mathbb{R}_+$ and pays $P_k^J (g_k^A + g_k^B)$.
- For private goods, J chooses the quantity vector $q^J \in \mathbb{R}_+^n$ to be bought in the market at prices $p \in \mathbb{R}_+^n$.

Definition

A vector $(q^A, g^A, q^B, g^B, P^A, P^B) \in \mathbb{R}_+^{2n+4m}$, with $P^A + P^B = P$, is a *Lindahl household equilibrium* if it satisfies the "budget consistency" condition $P_k^J (g_k^A + g_k^B) = P_k g_k^J$ (for $J = A, B$ and any public good k), and if the pair (q^J, g^J) solves the program for $J = A, B$,

$$\max_{(q^J, g^J) \in \mathbb{R}_+^{n+m}} U^J (q^J, g^J + g^{-J}) \quad \text{s.t.} \quad pq^J + P^J (g^J + g^{-J}) \leq Y^J,$$

The "budget consistency" condition

- The budget consistency condition $P_k^J (g_k^A + g_k^B) = P_k g_k^J$ can be interpreted as a kind of "participation constraint".
- It also ensures the equivalence of this household equilibrium to the standard definition of a Lindahl equilibrium where individualized contributions g_k^J are not introduced:
By the condition, $g_k^J = 0$ and $g_k^{-J} > 0$ imply $P_k^J = 0$, and hence a contradiction since, with $P_k^J = 0$, $g_k^J = 0$ could not be optimal ($U^J(q^J, Q)$ is increasing in Q_k). Hence, at a Lindahl household equilibrium, g_k^A and g_k^B are either both positive or both nil for any public good k .

The Lindahl equilibrium: First order conditions

The first order conditions for a Lindahl household equilibrium (for $J = A, B$):

$$\begin{aligned}\frac{1}{\partial_{q_1} U^J(q^J, g^J + g^{-J})} \partial_q U^J(q^J, g^J + g^{-J}) &\leq p \\ \tau^J(q^J, g^J + g^{-J}) &\leq P^J \\ pq^J + P^J(g^J + g^{-J}) &= Y^J,\end{aligned}$$

with an equality for any private good i s.t. $q_i^J > 0$ or any public good k s.t. $g_k^J > 0$. They entail the Bowen-Lindahl-Samuelson conditions (efficiency).

Partial cooperation: introducing intermediate degrees of autonomy

We introduce a more comprehensive model where there are spouses' arrangements which are variants of Lindahl's

- Each spouse J , for reasons that may be of many kinds, may want (or, sometimes, may be obliged) to keep some *degree of autonomy* $\theta^J \in [0, 1]$ in spending for the public goods
- The public good expenses Pg^J of spouse J is divided in two portions: one portion, $\theta^J Pg^J$, is autonomously spent by J and the other portion $\bar{\theta}^J Pg^J$, with $\bar{\theta}^J = (1 - \theta^J)$, is spent through Lindahl taxation.
- Again a pair of *contributive shares* P_k^A and P_k^B is posted for each public good k , such that $P_k^A + P_k^B = P_k$
- Then, each J , anticipating a contribution $g_k^{-J} \in \mathbb{R}_+$, chooses $g_k^J \in \mathbb{R}_+$ and pays $P_k^J (\bar{\theta}^A g_k^A + \bar{\theta}^B g_k^B)$
- Finally, J buys the basket $\theta^J g^J$ of public goods directly in the market at prices P .

A household equilibrium with some degree of autonomy

Definition

A vector $(q^A, g^A, q^B, g^B, P^A, P^B) \in \mathbb{R}_+^{2n+4m}$, with $P^A + P^B = P$, is a *household θ -equilibrium* with degrees of autonomy $(\theta^A, \theta^B) \in [0, 1]^2$ if it satisfies the "budget consistency" condition

$$P_k^J \left(\bar{\theta}^A g_k^A + \bar{\theta}^B g_k^B \right) = P_k \bar{\theta}^J g_k^J, \text{ for } J = A, B \text{ and any public good } k,$$

and if the pair (q^J, g^J) solves the following program for $J = A, B$:

$$\begin{aligned} & \max_{(q^J, g^J) \in \mathbb{R}_+^{n+m}} U^J \left(q^J, g^J + g^{-J} \right) \\ & \text{s.t. } p q^J + P \theta^J g^J + P^J \left(\bar{\theta}^J g^J + \bar{\theta}^{-J} g^{-J} \right) \leq Y^J. \end{aligned}$$

For $\theta^A = \theta^B = 0$, we get Lindahl ; and for $\theta^A = \theta^B = 1$, we get Nash

Budget consistency and voluntariness

- The budget consistency may be reformulated for, say, the wife A and public good k , as $P_k g_k^A = P_k \theta^A g_k^A + P_k^A (\bar{\theta}^A g_k^A + \bar{\theta}^B g_k^B)$ meaning that the market value $P_k g_k^A$ of the wife's voluntary contribution to public good k exactly decomposes into the market value of the autonomous portion $P_k \theta^A g_k^A$ and the remaining portion subject to Lindahl taxation.
- Moreover, we get $P_k^A (\bar{\theta}^B g_k^B) = P_k^B (\bar{\theta}^A g_k^A)$, so that $P_k^J = 0$ whenever $\bar{\theta}^J g_k^J = 0$ while $\bar{\theta}^{-J} g_k^{-J} > 0$. Knowing that her husband is not fully non-cooperative (i.e. $\theta^B < 1$) and that he is willing to contribute to public good k (i.e. $g_k^B > 0$), the wife A should not be taxed for public good k , either if she is fully non-cooperative (i.e. $\theta^J = 1$) or if she would rather like to decrease the household consumption of good k .

Voluntariness: consequences

For $0 < \theta^J < 1$, $J = A, B$, if there is a *separate spheres* equilibrium (an equilibrium where $g_k^A g_k^B = 0$ for all k), it coincides with an equilibrium of the game with voluntary contributions to public goods.

- To see this fact from the wife's viewpoint, denoting g_A^A the vector of public goods to which she contributes and P_A their corresponding market prices, her constraint becomes $p q^A + P_A g_A^A \leq Y^A$ (since the contributive share P_k^A is zero if she does not contribute to public good k).
- This constraint is then equivalent to the constraint in the fully non-cooperative game. So the two programs coincide for the private goods and the public goods to which she contributes.
- For a public good k she is not contributing to, if she deviated (by choosing $\tilde{g}_k^A > g_k^A = 0$), she would have to pay even more in the non-cooperative case (that is $P_k \tilde{g}_k^A$) than in the semi-cooperative one ($P_k \theta^A \tilde{g}_k^A$).

First order conditions

To contrast the semi-cooperative household decisions with the efficient case, let us consider the first order conditions relative to the public good k for both spouses' programs:

$$\tau_k^J(q^J, g^J + g^{-J}) \equiv \frac{\partial_{Q_k} U^J(q^J, g^J + g^{-J})}{\partial_{q_1} U^J(q^J, g^J + g^{-J})} \leq \theta^J P_k + \bar{\theta}^J P_k^J, \quad J = A, B, \quad (1)$$

with equality if $g_k^J > 0$.

- For efficiency, the Bowen-Lindahl-Samuelson condition requires that the sum of the two marginal willingnesses to pay $\tau_k^A + \tau_k^B$ be equal, for all k , to the market price $P_k = P_k^A + P_k^B$.
- If both spouses contribute to public good k , the sum of the two marginal willingnesses to pay is equal, to $P_k + \theta^A P_k^B + \theta^B P_k^A$, larger than P_k outside the case $\theta^A = \theta^B = 0$.
- Also, if spouse J contributes alone to public good k , $\tau_k^J = P_k$, so that $P_k < \tau_k^A + \tau_k^B$, leading to a similar conclusion.

Existence

For every $(\theta^A, \theta^B) \in [0, 1]^2$, there exists a household θ -equilibrium.

PROOF: Consider the household θ -equilibrium as an equilibrium of a "generalized game". For $(\theta^A, \theta^B) \neq (1, 1)$, we introduce a fictitious player with strategy space $S^0 = \{(P^A, P^B) \in \mathbb{R}_+^{2m} : P^A + P^B = P\}$ and payoff function $-\sum_{k=1}^m \left| P_k^A (\bar{\theta}^A g_k^A + \bar{\theta}^B g_k^B) - P_k (\bar{\theta}^A g_k^A) \right|$. The strategy spaces S^A and S^B of the two spouses can be compactified:

$$S^J = \left\{ (q^J, g^J) \in \mathbb{R}_+^{n+m} : q_i^J \leq Y^J / p_i, g_k^J \leq Y^J / P_k, \text{ all } i, \text{ all } k, \text{ and } p q^J + P \theta^J g^J + P^J (\bar{\theta}^J g^J + \bar{\theta}^{-J} g^{-J}) \leq Y^J \right\}.$$

Since all relations are linear in the relevant strategy variables and the payoff functions are continuous and quasi-concave, the best reply correspondences are upper hemicontinuous and convex-valued. Hence, there exists a "social equilibrium" by Debreu (1952) theorem. Clearly, at this equilibrium, both spouses' programs (conditionally on P^A and P^B) are solved, and $P_k^J (\bar{\theta}^A g_k^A + \bar{\theta}^B g_k^B) = P_k (\bar{\theta}^J g_k^J)$ for any J and any k ,

Some local properties

- We can examine how the properties of household θ -equilibria are affected by changes in the degrees of autonomy of the two spouses.
- We know from Browning, Chiappori and Lechene (2010) that, in the case of full autonomy ($\theta^A = \theta^B = 1$), there are generically only two possible regimes: pure *separate spheres* and *separate spheres up to one public good* to which both spouses contribute, the latter regime being characterized by *local income pooling*.
- Consider a household θ -equilibrium $(q^A, g^A, q^B, g^B, P^A, P^B) \in \mathbb{R}_+^{2n+4m}$ with degrees of autonomy $(\theta^A, \theta^B) \neq (1, 1)$, environment (p, P, Y) and income distribution (Y^A, Y^B) .
- Further, consider a partition $\{M^A, M^B, M^{AB}, M^0\}$ of the set M of public goods, where M^A and M^B are the subsets of goods to which A and B , respectively, contribute exclusively at this equilibrium, M^{AB} is the subset of goods to which both spouses contribute and M^0 is the subset of goods that are not at all consumed by the household.

Local properties for the cooperative or semi-cooperative cases

- $m^A + m^B + 2m^0$ are immediately determined, namely $g_k^J = 0$ for $k \in M^{-J} \cup M^0$, $J = A, B$.
- Besides $2m^0$ Lindahl prices corresponding to the non-consumed public goods can be ignored.
- To determine the remaining $2n + 4m - (m^A + m^B) - 4m^0$ unknowns, we have $2(n - 1)$ equations for the FOCs for private goods, 2 budget equations, $m^A + m^B + 2m^{AB}$ equations for the FOCs for public goods, $m - m^0$ equations $P_k^A + P_k^B = P_k$ and the $m - m^0$ consistency conditions.
- Hence, we have $2(n - 1) + 2 + m^A + m^B + 2m^{AB} + 2(m - m^0)$ equations in $2n + 4m - (m^A + m^B) - 4m^0$ unknowns, implying an excess $2(m - (m^A + m^B + m^{AB} + m^0)) = 0$ of the number of unknowns over the number of equations.
- Therefore, a Lindahl household equilibrium is (generically) locally determinate.

Full non-cooperation

- If $(\theta^A, \theta^B) = (1, 1)$, we eliminate the $2(m - m^0)$ unknowns P_k^A and P_k^B for $k \in M^A \cup M^B \cup M^{AB}$ and the corresponding $m - m^0$ equations $P_k^A + P_k^B = P_k$ together with the $m - m^0$ budget consistency conditions.
- Consider the subsystem with $2(n - 1) + m^A + m^B + 2m^{AB}$ FOCs in $2n + m^A + m^B + m^{AB}$ unknowns, namely q_i^J (for $J = A, B$ and $i = 1, \dots, n$), g_k^J (for $J = A, B$ and $k \in M^J$) and $g_k^A + g_k^B$ (for $k \in M^{AB}$); so there is $m^{AB} - 2$ more equations than unknowns: we have generically *overdeterminacy* if $m^{AB} \geq 2$.
- If $m^{AB} = 0$ (*separate spheres*), the two individual budget equations make the whole system determinate.
- If $m^{AB} = 1$ (*separate spheres up to one public good*), to obtain determinacy of the whole system we replace the two budget constraints by the single household budget equation. Then splitting of Y into Y^A and Y^B has no influence on the equilibrium: we have *local income pooling*.

To test for household behavior, two approaches have been used in the literature.

- One is to assume sufficient differentiability of the demand system (a parameterized system for empirical applications) and to derive testable local properties, such as properties of the Slutsky matrix. This is the approach introduced by Browning and Chiappori (1998) to discriminate the collective model from the (less general) unitary model. I will not discuss that.
- The second approach is the revealed preference approach consisting in rationalizing given data sets with a particular model. Such rationalization is based on global conditions and is non-parametric. This is the approach introduced by Cherchye, De Rock and Vermeulen (2007) for the collective model and by Cherchye, Demuynck and De Rock (2009) for their semi-cooperative model. We follow their route.

Comparison with Cherchye, Demuyne and De Rock (2009)

This last model is based on general exogenous donation vectors $\delta^J = (1 - \theta^{-J}) P^J$, $J = A, B$, but concentrate on the case

$$\delta^J = \zeta^J \tau^J (q^J, g^J + g^{-J}), \text{ for } 0 \leq \zeta^J \leq 1.$$

Since in our present model, $\tau^J (q^J, g^J + g^{-J}) = \theta^J P + (1 - \theta^J) P^J$ at equilibrium, to make the two models coincide we simply have to require

$$(1 - \theta^{-J}) P^J = \zeta^J (\theta^J P + (1 - \theta^J) P^J),$$

hence

$$P^J = \frac{1}{1 + \frac{1 - (\theta^{-J} + \zeta^J)}{\zeta^J \theta^J}} P$$

This condition, which requires that the personalized prices be co-linear with the market prices for public goods, is not generally compatible with our budget consistency requirement.

However *rationalizability* can also be shown for the present model.

Definition

A data set $(p_t, P_t, q_t, Q_t)_{t \in T}$ is θ -rationalizable for some pair of degrees of autonomy $(\theta^A, \theta^B) \in [0, 1]^2$, if there exist pairs of continuous, concave, monotonic utility functions (U^A, U^B) , of individual incomes $(Y_t^A, Y_t^B)_{t \in T} \in \mathbb{R}_+^{2|T|}$, of personalized prices $(P_t^A, P_t^B)_{t \in T} \in \mathbb{R}_+^{2m|T|}$, of individual private consumptions $(q_t^A, q_t^B)_{t \in T} \in \mathbb{R}_+^{2n|T|}$ and of voluntary contributions to public goods $(g_t^A, g_t^B)_{t \in T} \in \mathbb{R}_+^{2m|T|}$, such that, for any $t \in T$,

$$Y_t^A + Y_t^B = p_t q_t + P_t Q_t, P_t^A + P_t^B = P_t, q_t^A + q_t^B = q_t, g_t^A + g_t^B = Q_t$$

and such that $(q_t^A, g_t^A, q_t^B, g_t^B, P_t^A, P_t^B)$ is a household θ -equilibrium with degrees of autonomy (θ^A, θ^B) .

A data set $(p_t, P_t, q_t, Q_t)_{t \in T}$ satisfies the *Generalized Axiom of Revealed Preferences* (GARP) if, for any $s, t \in T$, $p_s q_s + P_s Q_s \leq p_s q_t + P_s Q_t$ whenever (q_t, Q_t) is revealed preferred to (q_s, Q_s) .

Theorem

The data set $(p_t, P_t, q_t, Q_t)_{t \in T}$ is θ -rationalizable if and only if, for any $t \in T$, there exist pairs of individual private consumptions $(q_t^A, q_t^B) \in \mathbb{R}_+^{2n}$, of voluntary contributions to public goods $(g_t^A, g_t^B) \in \mathbb{R}_+^{2m}$, and of personalized prices of public goods $(P_t^A, P_t^B) \in \mathbb{R}_+^{2m}$ such that

$$q_t^A + q_t^B = q_t, g_t^A + g_t^B = Q_t, P_t^A + P_t^B = P_t,$$

$$\text{and } P_k^A \left(\bar{\theta}^B g_k^B \right) = P_k^B \left(\bar{\theta}^A g_k^A \right), \text{ for all } k,$$

and, for $J = A, B$, the data set $(p_t, P_t^J, q_t^J, g_t^J)_{t \in T}$ with $P_t^J \equiv \theta^J P_t + (1 - \theta^J) P_t^J$ satisfies GARP.

Proof: necessity

By concavity of the utility function U^J ($J = A, B$), for any $(s, t) \in T^2$.

$$\begin{aligned} & U^J(q_s^J, Q_s) - U^J(q_t^J, Q_t) \\ & \leq \partial_q U^J(q_t^J, Q_t) \cdot (q_s^J - q_t^J) + \partial_Q U^J(q_t^J, Q_t) \cdot (Q_s - Q_t) \end{aligned}$$

By the FOC of spouse J 's program in the θ -household game, namely

$$\begin{aligned} \partial_q U^J(q_t^J, Q_t) & \leq \lambda_t^J p_t \text{ and } \partial_q U^J(q_t^J, Q_t) \cdot q_t^J = \lambda_t^J p_t \cdot q_t^J \\ \partial_Q U^J(q_t^J, Q_t) & \leq \lambda_t^J (\theta^J P_t + (1 - \theta^J) P_t^J) \equiv \lambda_t^J \mathcal{P}_t^J, \end{aligned}$$

the above inequality can be rewritten as, leading to GARP by Afriat's theorem

$$U^J(q_s^J, Q_s) \leq U^J(q_t^J, Q_t) + \lambda_t^J (p_t, \mathcal{P}_t^J) \cdot (q_s^J - q_t^J, Q_s - Q_t).$$

Proof: sufficiency

Again by GARP and Afriat's theorem, there exist numbers $U_t^J \in \mathbb{R}$ and $\lambda_t^J \in \mathbb{R}_{++}$ ($J = A, B$, $t \in T$) such that,

$$U_s^J \leq U_t^J + \lambda_t^J (p_t, \mathcal{P}_t^J) (q_s^J - q_t^J, Q_s - Q_t), \text{ for each } J, \text{ and any } (s, t)$$

We may accordingly define J 's utility function

$$U^J(q^J, Q) \equiv \min_{t \in T} \left\{ U_t^J + \lambda_t^J (p_t, \mathcal{P}_t^J) (q^J - q_t^J, Q - Q_t) \right\}.$$

This function is continuous, concave and increasing, as required.

Proof: sufficiency

Let us prove that $U^J(q_t^J, g_t^J + g_t^{-J})$ is no smaller than $U^J(q^J, g^J + g_t^{-J})$ for any consumption bundle (q^J, g^J) satisfying J 's budget constraint at t in the θ -household game:

$$p_t (q^J - q_t^J) + [\theta^J P_t + (1 - \theta^J) P_t^J] (g^J - g_t^J) \leq 0.$$

Since $\mathcal{P}_t^J g_t^J \equiv [\theta^J P_t + (1 - \theta^J) P_t^J] g_t^J$, the preceding inequality implies

$$(p_t, \mathcal{P}_t^J) (q^J - q_t^J, Q - Q_t) \leq 0.$$

Hence, deviating from (q_t^J, g_t^J) can only decrease $U^J(q^J, g^J + g_t^{-J})$. Since the equalities $P_k^A(\bar{\theta}^B g_k^B) = P_k^B(\bar{\theta}^A g_k^A)$, for all k , are imposed by assumption, we may conclude that $(q_t^A, g_t^A, q_t^B, g_t^B, P_t^A, P_t^B)$ is a household θ -equilibrium.