The Lindahl approach to household behavior*

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Abstract

In a model of household behavior allowing for various degrees of autonomy from full autonomy to full cooperation, the paper introduces the concept of household L-equilibrium, based on a generalized notion of Lindahl prices, namely contributive shares that satisfy both a consistency and a voluntariness condition. In this model, except for the full autonomy case, all regimes of equilibria are generically possible: separate spheres, separate spheres up to one public good and the two spouses contributing to more than one public good. It is further shown that a revealed preference approach could be used to construct nonparametric tests of the model.

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1 Introduction

An important objective in studying household behavior is to explore how household spending decisions, and in particular decisions concerning public consumption within the household, can be "personalised", in the sense of being attributed to a specific member of the household. This is important from a public policy point of view where some benefits may have to be "targeted" to some particular member of the household. An important case, which has been documented in several empirical investigations, is the positive effect on children welfare of targeting benefits to the wife. Targeting might also be relevant from the firm point of view. If the consumption of some (public) good by the household is in the decision sphere of one member (according to age or gender) a firm may want to adjust product design or marketing accordingly.

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It is clear that, for such an objective, the household behavior model cannot be "unitary", that is, we cannot assume that household income is pooled and that there is a single household utility function. In the literature, a first non-unitary approach to address the targeting issue (Blundell, Chiappori and Meghir, 2005) has been to use the so-called "collective model" of Browning and Chiappori (1998) in which it is assumed that the household still has a pooled income but that it maximizes a collective utility function (implying Pareto efficiency) where the weight given to each spouse utility may vary according to his/her decision power and other variables such as prices and income. A second approach is the pure non-cooperative model in which each spouse contributes voluntarily to the provision of public goods within the household. In this approach household spending may be organised in "separate spheres", whereby each spouse specializes in the provision of different goods. Separate spheres is either assumed, as in Lundberg and Pollak (1993), or derived at equilibrium, as in the work of Browning, Chiappori and Lechene (2006) who show that, generically at the non-cooperative equilibrium, there are "pure separate spheres" or only one public good to which both spouses contribute ("separate spheres up to one public good") and that, in the case there is such a single joint contribution, income redistributions have locally no effect ("local income pooling"). For the targeting issue this is an important result. Indeed, in case of local income pooling, increasing the income of only one spouse (say, the mother) in order to increase household expenditure on some targeted public good (say, healthcare for the children) does not have the desirable effect.

However, both the collective model and the non-cooperative model may appear as two extreme models of household behavior: "neither the assumption of fully efficient cooperation nor of complete absence of collaboration is likely to be an entirely accurate description of typical household spending behaviour and analysis of such extreme cases can be seen as a first step towards understanding of a more adequate model" (Lechene and Preston, 2005, p. 19). In trying to cope with this issue, we have introduced in a previous paper (d'Aspremont and Dos Santos Ferreira, 2009) a very general strategic model of household behavior allowing for a parameterized set of equilibria that includes not only the fully cooperative solutions of the collective model and the fully non-cooperative equilibria of the pure non-cooperative model (corresponding to full autonomy), but also a continuum of intermediate equilibria corresponding to semi-cooperative models based on various degrees of spouses' autonomy. In this general strategic model the personalised Lindahl prices play a crucial role in order to determine the contribution of each spouse to the household public goods. This is clearly true for efficient equilibria which coincide with Lindahl equilibria. But Lindahl prices also play a role as long as all household members do not have full autonomy.

An important conclusion derived in this general model, as far as generic properties are concerned, is that the income pooling property never holds for equilibria outside full autonomy, but that the separate spheres property holds at all equilibria except the efficient ones. In the present paper we want to investigate further the robustness of these generic properties. This will be done in a variant of our previous model where the degree of autonomy of each spouse is exogenous, but may vary from nil to full autonomy, and where we introduce a generalized version of Lindahl prices, seen as "contributive shares" that have to satisfy some conditions (consistency and voluntariness). Again, when the degree of autonomy is nil for both spouses an equilibrium coincides with a Lindahl equilibrium (and the contributive shares coincide with the Lindahl prices), and when there is full autonomy an equilibrium is a fully non-cooperative equilibrium.

As well illustrated by Cherchye, De Rock and Vermeulen (2007) for the collective model, the Lindahl approach can also be useful in empirical investigations. Although Lindahl prices are not observable, they may be used to characterize the rationalizability of the collective model and to construct non-parametric tests to check the consistency of a data set that only includes observed prices and quantities. In Cherchye, Demuynck and De Rock (2009), a similar revealed preference method is used to derive nonparametric tests for the noncooperative model and for a semi-cooperative model based on general exogenous donation vectors[1]. We shall show that this revealed preference approach is also applicable to the household model presented here.

This model will be presented for a two-adult household but, as it will become clear, the concept of household L-equilibrium that we propose, based on a generalized notion of Lindahl prices (the contributive shares), is applicable for any larger group and indeed for an economy with both private goods and public goods where the costs of producing the public goods are linear.

In section 2, we present the model and the concept of household L-equilibrium. In section 3, we shall examine the robustness of the separate spheres and local income poolig property and illustrate our conclusion by a simple example. Section 4 shows the rationalizability of the model and the last section draws some conclusions.

2 From cooperative to non-cooperative household behavior: an encompassing concept of equilibrium

For simplicity, let us consider a two-adult household, consuming goods that are either private or public (within the household). Denote the two household members by A (the wife) and B (the husband), and let $\left(q^A, q^B\right) \in \mathbb{R}_+^{2n}$ be the pair of vectors of consumption by the two spouses of n private goods and $Q \in \mathbb{R}_+^m$ the consumption vector of m public goods. The preferences of each individual J ($J = A, B$) are represented by a utility function $U^J\left(q^J, Q\right)$, which is defined on $\mathbb{R}_+^{n+m}$, increasing and strongly quasi-concave. Each member J of the household is supposed to receive an initial income $Y^J \in \mathbb{R}_+$, possibly after a preliminary redistribution of the household income $Y = Y^A + Y^B$. The spouses decide on their total consumption given the vectors of private good prices $p \in \mathbb{R}_{++}^n$ and

[1] In this paper they also compare their semi-cooperative model to the model in d'Aspremont and Dos Santos Ferreira (2009).

of public good prices $P \in \mathbb{R}^m_{++}$. The first private good, assumed to be desired in any household environment, is taken as numéraire ($p_1 = 1$).

A first approach to the spouses' decision is to assume that they always reach through cooperation an efficient outcome. This is the so called "collective model" of household behaviour (Browning and Chiappori, 1998). This model can be formulated using the Lindahl approach. Let us suppose that, in order to determine the contribution of each spouse to the household public goods, there is a pair of personalized (Lindahl) prices $\left(P^A, P^B\right) \in \mathbb{R}^{2m}_+$, satisfying $P^A + P^B = P$, which are posted within the household. Each spouse J suggests a quantity vector $g^J \in \mathbb{R}^m_+$ of public goods. The total quantity vector bought by the household in the market at prices $P \in \mathbb{R}^m_+$ is then simply $g^A + g^B$. The corresponding expense $P\left(g^A + g^B\right)$ will be covered by the *budgetary contributions* of both spouses using the personalized prices, the budgetary contribution of spouse J being $P^J\left(g^A + g^B\right)$. For private goods, each spouse J decides individually the quantity vector $q^J \in \mathbb{R}^n_+$ to be bought in the market at prices $p \in \mathbb{R}^n_+$. In this framework, the standard Lindahl equilibrium definition can be formulated as follows:

Definition 1 *A Lindahl household equilibrium is a vector* $\left(q^A, g^A, q^B, g^B, P^A, P^B\right) \in \mathbb{R}^{2n+4m}_+$ *such that* $P^A + P^B = P$, *such that, for* $J = A, B$, *the pair* $\left(q^J, g^J\right)$ *solves the program*

$$\max_{(q^J, g^J) \in \mathbb{R}^{n+m}_+} U^J\left(q^J, g^J + g^{-J}\right) \tag{1}$$
$$s.t.\ pq^J + P^J\left(g^J + g^{-J}\right) \leq Y^J,$$

and such that g^A_k *and* g^B_k *are either both positive or both nil for any public good* k.[2]

As well known, a Lindahl equilibrium outcome is Pareto efficient. However, it is far from clear that households always reach collectively efficient outcomes. In order to introduce a more comprehensive approach, we assume that there is some arrangement within the household according to which each spouse, say the wife A, divides into two portions her *intended contribution* g^A to the basket of public goods. One portion, $\theta^A g^A$ (with $\theta^A \in [0,1]$) is autonomously spent by her in the market at prices P. It will be called the *autonomous contribution of spouse* A. The parameter θ^A can thus be interpreted as A's *degree of autonomy* left by the household arrangement. A particular case is to assume that $\theta^A = \theta^B = \theta$, and then to see θ as the *degree of non-cooperation* of the household. Now, letting $\overline{\theta}^A = 1 - \theta^A$, the other portion $\overline{\theta}^A g^A$ – called the *concerted contribution of spouse* A – is meant to be collectively acquired by the

[2] Notice that the consumption of public good k in both the argument of the utility function and the budget constraint is the aggregate household consumption $g^A_k + g^B_k$. By imposing the same sign to both g^A_k and g^B_k, we eliminate the case where some spouse J would like to diminish the consumption of good k but cannot since $g^J_k = 0$ (the non-negativity condition is binding).

household, together with the corresponding portion $\overline{\theta}^B g^B = \left(1-\theta^B\right) g^B$ of the husband's intended contribution. Hence, $\overline{\theta}^A g^A + \overline{\theta}^B g^B$ is the total quantity of public goods that will be collectively acquired by the household according to the household arrangement. Also, part of this arrangement is the fixing, *à la* Lindahl, of a pair of *contributive shares* P_k^A and P_k^B of the market price P_k of each public good k, such that $P_k^A + P_k^B = P_k$. For a given pair of vectors of concerted contributions $\left(\overline{\theta}^A g^A, \overline{\theta}^B g^B\right) \in \mathbb{R}_+^{2m}$ of public goods, these shares determine the budgetary contribution $P^J\left(\overline{\theta}^A g^A + \overline{\theta}^B g^B\right)$ that each spouse J has to pay. We impose the following condition:

Condition 1 (Consistency) *The vector pairs of concerted contributions* $\left(\overline{\theta}^A g^A, \overline{\theta}^B g^B\right) \in \mathbb{R}_+^{2m}$ *of public goods and of contributive shares* $\left(P^A, P^B\right) \in \mathbb{R}_+^{2m}$ *satisfy* $P\overline{\theta}^J g^J = P^J\left(\overline{\theta}^A g^A + \overline{\theta}^B g^B\right)$ *for* $J = A, B$.

This condition, that we may reformulate for, say, the wife A as $Pg^A = P\theta^A g^A + P^A\left(\overline{\theta}^A g^A + \overline{\theta}^B g^B\right)$ means that the market value Pg^A of the wife's intended contribution to public consumption exactly decomposes into the market value of her autonomous contribution plus her budgetary contribution. Clearly, if it holds for A it holds for B, and it is equivalent to $P^A\left(\overline{\theta}^B g^B\right) = P^B\left(\overline{\theta}^A g^A\right)$.

A further natural condition is the *voluntariness condition*, that a spouse should not be asked to pay for a public good the consumption of which he/she would rather like to decrease.

Condition 2 (Voluntariness) *The vector pairs of concerted contributions* $\left(\overline{\theta}^A g^A, \overline{\theta}^B g^B\right) \in \mathbb{R}_+^{2m}$ *of public goods and of contributive shares* $\left(P^A, P^B\right) \in \mathbb{R}_+^{2m}$ *are such that, for any* $J \in \{A, B\}$ *and any* $k \in \{1, ..., m\}$, $P_k^J = 0$ *whenever* $\overline{\theta}^J g_k^J = 0$ *while* $\overline{\theta}^{-J} g_k^{-J} > 0$.

On the basis of these two conditions, we may formulate the following equilibrium concept:

Definition 2 *A* household L-equilibrium *with degrees of autonomy* $\left(\theta^A, \theta^B\right) \in [0,1]^2$ *is a vector* $\left(q^A, g^A, q^B, g^B, P^A, P^B\right) \in \mathbb{R}_+^{2n+4m}$ *satisfying the consistency and the voluntariness conditions and such that, for* $J = A, B$, *the pair* $\left(q^J, g^J\right)$ *solves the following program:*

$$\max_{(q^J, g^J) \in \mathbb{R}_+^{n+m}} U^J\left(q^J, g^J + g^{-J}\right) \tag{2}$$

$$s.t.\ pq^J + P\theta^J g^J + P^J\left(\overline{\theta}^J g^J + \overline{\theta}^{-J} g^{-J}\right) \leq Y^J. \tag{3}$$

Notice that, for the extreme case $\theta^A = \theta^B = 0$, spouse J is exclusively confronted to the personalized price vector P^J for public goods, so that we obtain the conditions for a Lindahl household equilibrium, hence for a Pareto efficient outcome. Indeed, the two programs coincide in that case, and $g_k^J = 0$ while $g_k^{-J} > 0$ for some J and some k is excluded at equilibrium: by the voluntariness condition, this would imply $P_k^J = 0$, so that J would like to increase (for free) the consumption of public good k, given preference monotonicity. In the other extreme case $\theta^A = \theta^B = 1$, the consistency and voluntariness conditions are trivially satisfied, and the personalized prices cease to play a role, so that J's program reduces to the corresponding program in the fully non-cooperative *game with voluntary contributions to public goods* (Browning, Chiappori and Lechene, 2006). Observe also that in all cases, extreme or intermediate, the strategic choices of the spouses $\left(q^A, g^A, q^B, g^B\right)$ are always collectively feasible at market prices in the sense that

$$p\left(q^A + q^B\right) + P\left(g^A + g^B\right) \leq Y^A + Y^B. \tag{4}$$

This can be easily verified by adding the two individual budget constraints. Observe finally that the individual budget constraints (3) can by consistency be written *at equilibrium* as

$$pq^J + Pg^J \leq Y^J. \tag{5}$$

An alternative approach to the analysis of cases intermediate to full non-cooperation and full cooperation would consist in fixing exogenously the contributive shares $\left(P^A, P^B\right) \in \mathbb{R}_+^{2m}$ while letting different degrees of autonomy $\left(\theta^A, \theta^B\right)$ be endogenously associated with different household equilibria. In d'Aspremont and Dos Santos Ferreira (2009) this is done by fixing the contributive shares to be equal to the Lindahl prices.

Let us now consider, for any $\left(\theta^A, \theta^B\right) \in [0,1]^2$, the first order conditions relative to the public good k for both spouses' programs (**??**):

$$\tau_k^J\left(q^J, g^J + g^{-J}\right) \equiv \frac{\partial_{Q_k} U^J\left(q^J, g^J + g^{-J}\right)}{\partial_{q_1} U^J\left(q^J, g^J + g^{-J}\right)} \leq \theta^J P_k + \overline{\theta}^J P_k^J,\ J = A, B, \tag{6}$$

with equality if $g_k^J > 0$. For efficiency, the Bowen-Lindahl-Samuelson condition requires that the sum of the two marginal willingnesses to pay $\tau_k^A + \tau_k^B$ be equal, for all k, to the market price $P_k = P_k^A + P_k^B$ (implying $\theta^A = \theta^B = 0$). More generally, for less than full cooperation, the Bowen-Lindahl-Samuelson condition is violated for all public goods. Indeed, the sum of the two marginal willingnesses to pay will then be equal, if both spouses contribute to public good k, to $P_k + \theta^A P_k^B + \theta^B P_k^A$, larger than P_k and the more so the higher the degrees of autonomy of the two spouses. Also, if spouse J contributes alone to public good k, $\tau_k^J = P_k$, so that $P_k < \tau_k^A + \tau_k^B < P_k + \theta^{-J} P_k$, leading to a similar conclusion.

An important result is the following.

Proposition 1 *For every* $\left(\theta^A, \theta^B\right) \in [0,1]^2$*, there is a household L-equilibrium.*

Proof. We can use a standard argument by considering the household L-equilibrium as an equilibrium of a game (where strategy spaces are non constant correspondences). For $\left(\theta^A, \theta^B\right) \neq (1,1)$, in addition to spouses A and B we introduce a fictitious player with strategy space $S^0 = \left\{\left(P^A, P^B\right) \in \mathbb{R}_+^{2m} : P^A + P^B = P\right\}$ and payoff function $-\sum_{k=1}^m \left|P_k^A \left(\overline{\theta}^A g_k^A + \overline{\theta}^B g_k^B\right) - P_k \left(\overline{\theta}^A g_k^A\right)\right|$. The strategy spaces S^A and S^B of the two spouses can be compactified by defining:

$$S^J = \left\{ \begin{array}{c} \left(q^J, g^J\right) \in \mathbb{R}_+^{n+m} : q_i^J \leq Y^J/p_i, g_k^J \leq Y/P_k, \text{ all } i, \text{ all } k, \text{ and} \\ pq^J + P\theta^J g^J + P^J \left(\overline{\theta}^J g^J + \overline{\theta}^{-J} g^{-J}\right) \leq Y^J \end{array} \right\}.$$

Since all relations are linear in the relevant strategy variables and the payoff functions are continuous and quasi-concave, the best reply correspondences of the two spouses as well as the one of the fictitious player are upper hemicontinuous and convex-valued. Hence, there exists a social equilibrium by Debreu (1952) theorem. Clearly, $P_k^J \left(\overline{\theta}^A g_k^A + \overline{\theta}^B g_k^B\right) = P_k \left(\overline{\theta}^J g_k^J\right)$ for any J and any k at equilibrium, so that $\overline{\theta}^J g_k^J = 0$ and $\overline{\theta}^{-J} g_k^{-J} > 0$ for some J and some k will imply $P_k^J = 0$ (the voluntariness condition). Also, $P^J \left(\overline{\theta}^A g^A + \overline{\theta}^B g^B\right) = P\left(\overline{\theta}^J g^J\right)$, so that the consistency condition is also satisfied. ■

3 The regimes of household public good expenditure

The next step consists in examining the properties of household L-equilibria, and in particular how they respond to changes in the degrees of autonomy of the two spouses. We know from Browning, Chiappori and Lechene (2006) that, in the case of full autonomy ($\theta^A = \theta^B = 1$), there are generically only two possible regimes: pure *separate spheres* and *separate spheres up to one public good* to which both spouses contribute, this second regime being characterized by *local income pooling*. In our previous model (d'Aspremont and Dos Santos Ferreira, 2009), with varying degrees of autonomy and fixed Lindahl prices, we have shown that the two-regime property extends to all intermediate equilibria, although the local income pooling property breaks down. We shall now examine what happens in the present model.

3.1 Local determinacy and L-equilibrium regimes

Consider a household L-equilibrium $\left(q^A, g^A, q^B, g^B, P^A, P^B\right) \in \mathbb{R}_+^{2n+4m}$ with degrees of autonomy $\left(\theta^A, \theta^B\right) \in [0,1]^2$, environment (p, P, Y) and income distribution $\left(Y^A, Y^B\right)$. Since, at equilibrium, the spouses' budget constraints are binding and of the form $pq^J + Pg^J = Y^J$, each vector q^J must maximize J's utility $U^J(\cdot, Q)$, with $Q = g^A + g^B$, under the constraint $pq^J = y^J$ (with

$y^J = Y^J - Pg^J$). Hence, q^J is uniquely determined by $\left(Q, y^J\right)$ in addition to the prices p of private goods. Further, consider a partition $\left\{M^A, M^B, M^{AB}, M^0\right\}$ of the set M of public goods, where M^A and M^B are the subsets of goods to which A and B contribute exclusively at this equilibrium, M^{AB} is the subset of goods to which both spouses contribute and M^0 is the subset of goods that are not consumed by the household. Denote by m^A, m^B, m^{AB} and m^0 the cardinals of the corresponding subsets in the partition. The $m^A + m^B + m^{AB}$ positive coordinates of Q and the $2m$ contributive shares P_k^A and P_k^B are then determined, given y^A and y^B, by the $m^A + m^B + 2m^{AB}$ first order conditions

$$\begin{aligned} \frac{\partial_{Q_k} U^A\left(q^A\left(Q, y^A\right), Q\right)}{\partial_{q_1} U^A\left(q^A\left(Q, y^A\right), Q\right)} &= \theta^A P_k + \overline{\theta}^A P_k^A, \quad k \in M^A \cup M^{AB}, \qquad (7) \\ \frac{\partial_{Q_k} U^B\left(q^B\left(Q, y^B\right), Q\right)}{\partial_{q_1} U^B\left(q^B\left(Q, y^B\right), Q\right)} &= \theta^B P_k + \overline{\theta}^B P_k^B, \quad k \in M^B \cup M^{AB}, \end{aligned}$$

and by the m equations $P_k^A + P_k^B = P_k$ ($k \in M$). Hence, we have $m^A + m^B + 2m^{AB} + m$ equations in $m^A + m^B + m^{AB} + 2m$ unknowns, implying an excess $m - m^{AB}$ of the number of unknowns over the number of equations.

Thus, except when both spouses contribute to all the public goods (the typical situation in Lindahl equilibrium), underdetermination is the rule, as long as we do not take the voluntariness condition into account. If $m^A = m^B = 0$, this underdetermination is of order m^0, concerning the sole contributive shares applying to the public goods which are not consumed by the household, so that equilibrium is in fact determined in this case. We shall consequently simply assume that $m^0 = 0$.

In the fully non-cooperative model with $\theta^A = \theta^B = 1$, we eliminate $2m$ unknowns P_k^A and P_k^B and m equations $P_k^A + P_k^B = P_k$, so that the excess of the number of unknowns over the number of equations becomes $-m^{AB}$. As a consequence, there is generically *over*determination unless $m^{AB} = 0$ (*separate spheres*). Another case in which this overdetermination is avoided is the case $m^{AB} = 1$ (*separate spheres up to one public good*), where we may add 2 unknowns, by taking y^A and y^B as adjustment variables, and just 1 equation, by replacing the pair of budget equations $y^A = Y^A - Pg^A$ and $y^B = Y^B - Pg^B$ by the single aggregate equation $y^A + y^B = Y - PQ$. In this case, the splitting of Y into Y^A and Y^B does not influence the equilibrium outcome: *local income pooling*.[3]

In the general model, as already stated, local underdeterminacy of the household L-equilibrium would be generically of order $m - m^{AB} = m^A + m^B$ without the voluntariness condition. This condition introduces precisely the $m^A + m^B$ additional equations required to obtain generic local determinacy: $P_k^A = P_k$ for $k \in M^A$ and $P_k^A = 0$ for $k \in M^B$. As to the quantities of public goods g_k^A and g_k^B contributed by the two spouses, they are determined by the equations

[3] In d'Aspremont and Dos Santos Ferreira (2009), where the $2m$ contributive shares P_k^A and P_k^B are given, taken equal to the Lindahl prices, we obtain precisely the same situation of generic separate spheres (at least up to one public good). Local income pooling does however not apply, because the Lindahl prices depend themselves upon the income distribution.

$g_k^A = Q_k$ if $k \in M^A$ and $g_k^B = Q_k$ if $k \in M^B$. However, if $k \in M^{AB}$, we have $g_k^A + g_k^B = Q_k$: one additional equation for each additional pair of unknowns. By taking into account the consistency equation $P^A \left(\overline{\theta}^A g^A + \overline{\theta}^B g^B \right) = P \left(\overline{\theta}^A g^A \right)$, we thus end up with an excess of the number of unknowns over the number of equations equal to $m^{AB} - 1$ (the two unknowns y^A and y^B being determined by the two budget equations). Hence, the household L-equilibrium is locally determined in the case of separate spheres, at least up to one public good, underdetermined if $m^{AB} > 1$. This underdeterminacy is inocuous (in the sense that it does not affect the equilibrium outcome) when the two spouses have the same degree of autonomy θ (the degree of non-cooperation in the household) since, by the consistency condition, for $J = A, B$, $Pg^J = P^J \left(g^A + g^B \right)$ in this particular case, so that y^A and y^B only depend on the sum $g^A + g^B$, which is also the sole argument of U^J as concerns public consumption.

The preceding analysis shows that the separate spheres and the local income pooling properties that generically hold in the pure non-cooperative case do not generalize to the semi-cooperative cases. To illustrate this conclusion, we will now use the example introduced by Browning, Chiappori & Lechene (2006).

3.2 An example

Assume Cobb-Douglas preferences over one private good and two public goods. We denote by x and z the private consumptions of spouses A and B, respectively, and by X and Z the quantities of the two public goods. The utility functions are given by:

$$U^A (x, X, Z) = x X^a Z^\alpha \text{ and } U^B (z, X, Z) = z X^b Z^\beta, \tag{8}$$

with positive parameter values a, α, b and β. The wife A is supposed to care more about the first public good, and the husband B about the second, so that

$$\frac{\alpha/a}{\beta/b} < 1, \tag{9}$$

where the term on the LHS can be taken as the *degree of symmetry* of the spouses' preferences for the two public goods. We use the normalization

$$p_x = p_y = P_X = P_Z = Y = 1,$$

with an income distribution given by $Y^A = \rho$ and $Y^B = 1 - \rho$.

Browning, Chiappori & Lechene (2006) use this example to study the fully non-cooperative game with voluntary contributions to public goods. They show the existence of three kinds of regimes. For extremely unequal income distributions, the spouse with the higher income contributes alone to both public goods. For relatively equal income distributions, the prevailing regime is the one of pure separate spheres, each spouse contributing to her/his preferred public good. Finally, in the intermediate cases, one finds separate spheres up to one public good: the spouse with the higher income contributes to both public goods, while the other spouse contributes solely to her/his preferred public good. In the context of varying degrees of autonomy, with contributive shares equal to the Lindahl prices, d'Aspremont and Dos Santos Ferreira (2009) use the same example and find the same configuration of regimes. This is also true in the present model (for $\left(\theta^A, \theta^B\right) \in (0,1)^2$), but with a crucial difference concerning the regime prevailing for relatively equal income distributions. We shall accordingly limit our analysis to this case.

First consider the regime of pure *separate spheres*: A contributes to her preferred public good (X) and B to his (Z). By the voluntariness condition, $P_Z^A = P_X^B = 0$, and by the first order conditions for public goods,

$$\begin{aligned} ax/X &= \theta^A + \overline{\theta}^A P_X^A = 1 \text{ and } \alpha x/Z < \theta^A + \overline{\theta}^A P_Z^A = \theta^A, \\ \beta z/Z &= \theta^B + \overline{\theta}^B P_Z^B = 1 \text{ and } bz/X < \theta^B + \overline{\theta}^B P_X^B = \theta^B. \end{aligned}$$

Further using the equilibrium budget equations

$$x + X = \rho \text{ and } z + Z = 1 - \rho,$$

we easily obtain the solution

$$x = \frac{\rho}{1+a},\ X = \frac{a\rho}{1+a},\ z = \frac{1-\rho}{1+\beta},\ Z = \frac{\beta(1-\rho)}{1+\beta}.$$

This solution is constrained by the two first order conditions expressed as inequalities:

$$\frac{\alpha}{\beta}\frac{1+\beta}{1+a}\frac{\rho}{1-\rho} < \theta^A \text{ and } \frac{b}{a}\frac{1+a}{1+\beta}\frac{1-\rho}{\rho} < \theta^B.$$

Clearly, one of these two conditions will be violated for small enough or large enough values of ρ, so that separate spheres (with both spouses contributing to public consumption) can indeed prevail only if the income distribution between the two spouses is not too unequal. Also, by multiplying both sides of the first inequality by the corresponding sides of the second, we obtain

$$0 < \frac{\alpha/a}{\beta/b} < \theta^A\theta^B < 1, \tag{10}$$

so that existence of the regime of separate spheres requires a relatively high average degree of autonomy of the two spouses, the higher the larger the degree of symmetry of their preferences for the public goods. The fully non-cooperative case, where $\theta^A\theta^B = 1$, always satisfies this condition, provided there is no full symmetry in the spouses' preferences.

Now consider the regime where *both spouses contribute to both public goods*, which is generically excluded under full autonomy of the two spouses. By first order conditions,

$$\begin{aligned} ax/X &= \theta^A + \overline{\theta}^A P_X^A \text{ and } \alpha x/Z = \theta^A + \overline{\theta}^A P_Z^A, \\ \beta z/Z &= \theta^B + \overline{\theta}^B P_Z^B \text{ and } bz/X = \theta^B + \overline{\theta}^B P_X^B. \end{aligned}$$

Division of both sides of the second and third equations by the corresponding sides of the first and fourth, respectively, leads to

$$\frac{a}{\alpha}\theta^A \le \frac{a}{\alpha}\frac{\theta^A + \overline{\theta}^A P_Z^A}{\theta^A + \overline{\theta}^A P_X^A} = \frac{X}{Z} = \frac{b}{\beta}\frac{\theta^B + \overline{\theta}^B P_Z^B}{\theta^B + \overline{\theta}^B P_X^B} \le \frac{b}{\beta}\frac{1}{\theta^B},$$

the two inequalities being easily checked to be true. We thus obtain

$$0 < \theta^A \theta^B \le \frac{\alpha/a}{\beta/b} < 1, \tag{11}$$

an existence condition opposite to the one we found for the regime of separate spheres. For both spouses to contribute to both public goods their average degree of autonomy must be small enough, the smaller the more asymmetric their preferences for the public goods.

Thus, separate spheres appear as a characteristic of high individual autonomy in household decisions. As the spouses become less and less autonomous, the regime prevailing when their incomes are not too different is rather the one where both contribute to both public goods, which is the rule under full cooperation.

4 Testable characterization of household rationality

To test for household behavior, two approaches have been used in the literature.

One is to assume sufficient differentiability of the demand system (a parameterized system for empirical applications) and to derive testable local properties, such as properties of the (pseudo-)Slutsky matrix. This is the approach introduced by Browning and Chiappori (1998) to discriminate the collective model from the (less general) unitary model. This is also the approach we have used in our previous model to discriminate the different types of household behavior: full cooperation, full autonomy and partial autonomy of the two spouses. We shall not use this approach here.

The second approach is the revealed preference approach consisting in rationalizing given data sets with a particular model. Such rationalization is based on global conditions and is non-parametric. This is the approach introduced by Cherchye, De Rock and Vermeulen (2007) for the collective model and by Cherchye, Demuynck and De Rock (2009) for their semi-cooperative model. This model is based on general exogenous donation vectors δ^J from spouse J to spouse $-J$, $J = A, B$, per unit of each public good. Transposed to our model, the vector δ^J corresponds precisely to our $\overline{\theta}^{-J} P^J$. The authors concentrate on the case where δ^J is co-linear with τ^J, the vector of J's marginal willingness to pay for the public goods. By contrast, the vector P^J of spouse J's contributive shares cannot be taken as co-linear with $\tau^J > 0$, since this would in general imply a violation of the voluntariness condition. However, *rationalizability* can also be shown to prevail in our model.

Let us start with some definitions, considering some given price-quantity data set $(p_t, P_t, q_t, Q_t)_{t\in T} \in \mathbb{R}_+^{2(n+m)|T|}$.

Definition 3 *Consider a data set* $S = (p_t, P_t, q_t, Q_t)_{t\in T} \in \mathbb{R}_+^{2(n+m)|T|}$. *We call* $\left(q_t^A, g_t^A, q_t^B, g_t^B, P_t^A, P_t^B\right)_{t\in T} \in \mathbb{R}_+^{2(n+2m)|T|}$ *a disaggregation of* S *if, for any* $t \in T$, $q_t^A + q_t^B = q_t$, $g_t^A + g_t^B = Q_t$ *and* $P_t^A + P_t^B = P_t$.

Definition 4 *A data set* $S = (p_t, P_t, q_t, Q_t)_{t\in T}$ *is* θ*-rationalizable for some pair of degrees of autonomy* $\left(\theta^A, \theta^B\right) \in [0,1]^2$, *if there exist a pair of continuous, concave, monotonic utility functions* $\left(U^A, U^B\right)$ *and a disaggregation of* S *such that, for any* $t \in T$, $\left(q_t^A, g_t^A, q_t^B, g_t^B, P_t^A, P_t^B\right)$ *is a household L-equilibrium with degrees of autonomy* $\left(\theta^A, \theta^B\right)$ *for an income distribution* $\left(Y_t^A, Y_t^B\right)$ *satisfying* $Y_t^A + Y_t^B = p_t q_t + P_t Q_t$.

Definition 5 *A data set* $S = (p_t, P_t, q_t, Q_t)_{t\in T}$ *satisfies the Generalized Axiom of Revealed Preferences (GARP) if, for any* $s, t \in T$, $p_s q_s + P_s Q_s \leq p_s q_t + P_s Q_t$ *whenever* (q_t, Q_t) *is revealed preferred*[4] *to* (q_s, Q_s).

We can now state the following result.

Proposition 2 *The data set* $S = (p_t, P_t, q_t, Q_t)_{t\in T}$ *is* θ*-rationalizable for the pair of degrees of autonomy* $\left(\theta^A, \theta^B\right) \in [0,1]$ *if and only if, for some disaggregation of* S *such that, for any* $t \in T$, $P_t^A\left(1-\theta^B\right)g_t^B = P_t^B\left(1-\theta^A\right)g_t^A$, *the data set* $\left(p_t, \tau_t^J, q_t^J, g_t^J\right)_{t\in T}$, *where* $\tau_t^J \leq \theta^J P_t + \left(1-\theta^J\right)P_t^J$ *(for any* $t \in T$ *and with equality for any coordinate* k *such that* $g_{tk}^J > 0$*), satisfies GARP for* $J = A, B$.

Proof. Let us first prove necessity ("only if"). By concavity of the utility function U^J $(J = A, B)$,

$$\begin{aligned} & U^J\left(q_s^J, Q_s\right) - U^J\left(q_t^J, Q_t\right) \\ \leq\ & \partial_q U^J\left(q_t^J, Q_t\right)\left(q_s^J - q_t^J\right) + \partial_Q U^J\left(q_t^J, Q_t\right)\left(Q_s - Q_t\right) \end{aligned}$$

for any $(s,t) \in T^2$. By the FOC of spouse J's program (2), namely

$$\begin{aligned} \partial_q U^J\left(q_t^J, Q_t\right) &\leq \lambda_t^J p_t \text{ and } \partial_q U^J\left(q_t^J, Q_t\right) q_t^J = \lambda_t^J p_t q_t^J \\ \partial_Q U^J\left(q_t^J, Q_t\right) &\equiv \lambda_t^J \tau_t^J \leq \lambda_t^J\left(\theta^J P_t + \left(1-\theta^J\right)P_t^J\right), \end{aligned}$$

[4] Recall that (q_t, Q_t) is directly revealed preferred to (q_s, Q_s), or $(q_t, Q_t)\,R\,(q_s, Q_s)$, if $p_t q_t + P_t Q_t \geq p_t q_s + P_t Q_s$. Then we say (q_t, Q_t) is revealed preferred to (q_s, Q_s), if, for some sequence $t, r, v, ..., u, s$ in T,

$$(q_t, Q_t)\,R\,(q_r, Q_r)\,R\,(q_v, Q_v)\,...\,(q_u, Q_u)\,R\,(q_s, Q_s).$$

the above inequality can be rewritten as

$$U^J\left(q_s^J, Q_s\right) \leq U^J\left(q_t^J, Q_t\right) + \lambda_t^J\left(p_t, \tau_t^J\right)\left(q_s^J - q_t^J, Q_s - Q_t\right).$$

Using Afriat's theorem (Varian, 1982), we thus obtain equivalence with GARP. Let us second prove sufficiency ("if"). Again by GARP and Afriat's theorem, there exist numbers $U_t^J \in \mathbb{R}$ and $\lambda_t^J \in \mathbb{R}_{++}$ $(J = A, B,\ t \in T)$ such that, for each J and each $(s,t) \in T^2$,

$$U_s^J \leq U_t^J + \lambda_t^J\left(p_t, \mathcal{P}_t^J\right)\left(q_s^J - q_t^J, Q_s - Q_t\right).$$

We may accordingly define J's utility function

$$U^J\left(q^J, Q\right) \equiv \min_{t\in T}\left\{U_t^J + \lambda_t^J\left(p_t, \mathcal{P}_t^J\right)\left(q^J - q_t^J, Q - Q_t\right)\right\}.$$

This function is continuous, concave and increasing, as required. Let us prove that $U^J\left(q_t^J, g_t^J + g_t^{-J}\right)$ is no smaller than $U^J\left(q^J, g^J + g_t^{-J}\right)$ for any consumption bundle $\left(q^J, g^J\right)$ satisfying J's budget constraint at t:

$$p_t\left(q^J - q_t^J\right) + \left[\theta^J P_t + \overline{\theta}^J P_t^J\right]\left(g^J - g_t^J\right) \leq 0.$$

Since $\tau_{tk}^J = \theta^J P_{tk} + \overline{\theta}^J P_{tk}^J$ if $g_{tk}^J > 0$, $\tau_t^J g_t^J = \left(\theta^J P_t + \overline{\theta}^J P_t^J\right) g_t^J$, so that the preceding inequality implies

$$\left(p_t, \tau_t^J\right)\left(q^J - q_t^J, Q - Q_t\right) \leq 0.$$

Hence, deviating from $\left(q_t^J, g_t^J\right)$ can only decrease $U^J\left(q^J, g^J + g_t^{-J}\right)$. Since the equality $P_t^A\left(\overline{\theta}^B g_t^B\right) = P_t^B\left(\overline{\theta}^A g_t^A\right)$ is imposed by assumption, we may conclude that $\left(q_t^A, g_t^A, q_t^B, g_t^B, P_t^A, P_t^B\right)$ is a household L-equilibrium and the proof is complete. ■

5 Conclusion

Reconsidering a model of household behavior which allows for various (exogenous) degrees of autonomy – from full autonomy to full cooperation –, we have introduced the concept of household L-equilibrium, a concept based on a generalized notion of Lindahl prices, namely contributive shares that satisfy both a consistency and a voluntariness condition. These contributive shares coincide with the Lindahl prices only under full cooperation. In this model, if we except the full autonomy case, all regimes of equilibria are generically possible: separate spheres, separate spheres up to one public good and the two spouses contributing to more than one public good. The last regime is locally underdetermined except when the degrees of autonomy of the two spouses coincide. As illustrated in an example, for both spouses to contribute to all public goods their average degree of autonomy should be small enough, and the smaller the more asymmetric their preferences for the public goods. By contrast, the regime of separate spheres requires a relatively high average degree of autonomy of the two spouses, the higher the larger the degree of symmetry of their preferences for the public goods. We also show that a revealed preference approach could be used to construct nonparametric tests of the model.

Clearly several extensions of this model can be considered. First, the concept of L-equilibrium can be easily extended to more household members and the results adapted accordingly. Second, we could introduce the possibility of public good production within the household. These two possibilities would make our model equivalent to a standard economy with several private and several public goods and any number of agents. If costs are non-linear (contrary to our case), the possibility of using the concept of ratio equilibrium[5] (Kaneko, 1977) as an alternative to the Lindahl equilibrium could be explored.

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[5]An axiomitization of the ratio equilibrium is proposed by van den Nouweland, Tijs and Wooders (2002), In a ratio equilibrium, personalised prices are replaced by personalised cost-sharing ratios for each public good. Both concepts coincide in the linear cost case (our case).