Learning from a Piece of Pie

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1. Introduction

Economic applications of the Nash solution:

The bargaining game between the management and the workers, possibly represented by a union (de Menil, 1971; Hamermesh, 1973);

The employment contracts in search models (Moscarini, 2005; Postel-Vinay and Robin 2006);

The international cooperation for fiscal and trade policies (Chari and Kehoe, 1990);

The negotiations in joint venture operations (Svejnar and Smith, 1984);

The sharing of profit in cartels (Harrington, 1991) and oligopoly (Fershtman and Muller, 1986);

The household behavior (Manser and Brown, 1980; McElroy and Horney, 1981; Lundberg and Pollak, 1993; Kotlikoff, Shoven and Spivak, 1986).

Is Nash Bargaining empirically relevant?

Consider a game in which two players, 1 and 2, bargain about a pie of size y.

If the players agree on some sharing (ρ_1, ρ_2) with $\rho_1 + \rho_2 = y$, it is implemented.

The bargaining environment is described by a vector x of n variables.

An agreement is reached if and only if there exists a sharing (ρ_1, ρ_2), with $\rho_1 + \rho_2 = y$, such that

$$U^1(\rho_1, x) \geq T^1(y, x) \quad \text{and} \quad U^2(\rho_2, x) \geq T^2(y, x).$$

In that case, the sharing ρ $(\rho_1 = \rho, \rho_2 = y - \rho)$ solves:

$$\max_{\rho} \left(U^1(\rho, x) - T^1(y, x)\right) \cdot \left(U^2(y - \rho, x) - T^2(y, x)\right).$$

The Objectives

This raises two questions.

1. Is it possible derive testable restrictions on the bargaining outcomes *without previous knowledge of individual utilities*? In other words, what does this structure imply (if anything) on the function ρ? On the domain of ρ?

2. Can the utility players derive from the consumption of either their share of the pie or their reservation payment be recovered from the sole observation of the bargaining outcomes?

The econometrician's *prior information* will be described by some classes to which the utility or threat functions are known to belong.

The identification of a *cardinal* representation of preferences can be renvisaged.

Deterministic versus stochastic models

In deterministic models, the econometrician has access to ideal data: individual shares are observed as *deterministic functions* of the variables of the game.

The problem is the counterpart, in a bargaining context, of well known results in consumer theory.

Economic models are, in general, stochastic because of unobserved heterogeneity and measurement errors.

In stochastic models, the econometrician observes a *joint distribution* of incomes and outcomes.

The Main findings

We first consider the deterministic version of the model and show that:

1. In its most general version, Nash bargaining is not testable: any Pareto efficient rule can be rationalized as the outcome of a Nash bargaining process.

2. If some exclusion restrictions on U^s and T^s are supposed, the Nash model generates strong, testable restrictions, that take the form of a PDE on the function ρ.

3. If further exclusion restrictions on U^s and T^s are supposed, generically, both individual utility and threat functions can be cardinally identified.

The Main findings (continued)

We then consider a stochastic version of the model:

$$\max_{\rho}\left(U^1(\rho,x)-T^1(x)+\epsilon_1\right)\cdot\left(U^2(y-\rho,x)-T^2(x)+\epsilon_2\right)$$

an show that:

4. Under the same exclusion restrictions as in (2) and (3), testable restrictions are generated, and individual utility and threat functions are cardinally identified.

Note: The approach is differentiable.

2. The Deterministic Model

The framework

Consider a game in which two players, 1 and 2, bargain about a pie of size y.

An agreement is reached if and only if there exists a sharing (ρ_1, ρ_2), with $\rho_1 + \rho_2 = y$, such that

$$U^s(\rho_s, x) \geq T^s(y, x), \quad s = 1, 2. \tag{1}$$

In that case, the observed sharing $(\rho_1 = \rho, \rho_2 = y - \rho)$ solves:

$$\max_{0 \leq \rho \leq y} \left(U^1(\rho, x) - T^1(y, x)\right) \cdot \left(U^2(y - \rho, x) - T^2(y, x)\right). \tag{2}$$

The set of all functions $U^s(\rho_s, x)$ (resp. $T^s(y, x)$) that are compatible with the a priori restrictions is denoted by $\mathcal{U}^s$ (resp. $\mathcal{T}^s$).

Let $\mathcal{N}$ denote the subset of $\mathcal{S}$ on which no agreement is reached, and $\mathcal{M}$ the subset on which an agreement is reached, with $\mathcal{S} = \mathcal{M} \cup \mathcal{N}$.

Remarks

1. What we can recover is (at best) a *cardinal representation* of the functions under consideration: if we replace (U^s, T^s) in the program:

$$\max_{0 \leq \rho \leq y} \left(U^1(\rho, x) - T^1(y, x)\right) \cdot \left(U^2(y - \rho, x) - T^2(y, x)\right),$$

with the affine transforms $(\alpha_s U^s + \beta_s, \alpha_s T^s + \beta_s)$, the solution ρ is not modified.

2. The present framework cannot be used to test Pareto optimality. Indeed, efficiency is automatically imposed.

Proposition 1.

Let $\rho(y,x)$ *be some function defined over* $\mathcal{M}$.

Then, for any pair of utility functions U^1, U^2, *there exist two threat functions* T^1, T^2 *such that the agents' behavior is compatible with Nash bargaining.*

Proof.

Given any pair of functions U^1, U^2, it is possible to define T^1, T^2 as:

$$\begin{aligned} T^s(y,x) &= U^s(\rho_s(y,x),x) \text{ if } (y,x)\in\mathcal{M}, \\ T^s(y,x) &> U^s(y,x) \qquad\qquad \text{if } (y,x)\in\mathcal{N}. \end{aligned}$$

Remarks

1. When threat points are unknown, Nash bargaining has no empirical content (beyond Pareto efficiency at least).

2. The observation of the outcome brings no information on preferences (and in particular the concavity of the utility functions).

3. These negative results are by no means specific to Nash bargaining.

The bargaining structure

We first restrict the sets $\mathcal{U}^s$ of the players' utility functions and the sets $\mathcal{T}^s$ of the players' threat functions.

Assumption U.1. For $s = 1, 2$,

(a) the functions $U^s(\rho_s, x)$ are sufficiently smooth, strictly increasing and concave in ρ_s;

(b) there exists a partition $x = (x_1, x_2)$ such that $U^s(\rho_s, x) = U^s(\rho_s, x_s)$.

Assumption T.1. For $s = 1, 2$,

(a) the function $T^s(y, x)$ is sufficiently smooth;

(b) $T^s(y, x) = T^s(x_s)$.

Assumption S.1. For any $(y, x) \in \mathcal{M}$, the sharing (ρ_1, ρ_2) is interior; i.e., $\rho_s > 0$, with $s = 1, 2$.

3. Testability: The Deterministic Case

The general agreement case

Assumption S.2. For any $(y, x) \in \mathcal{S}$, there exists a sharing (ρ_1, ρ_2), with $\rho_s \geq 0$, and $\rho_1 + \rho_2 = y$, such that $U^s(\rho_s, x) - T^s(y, x) > 0$ for $s = 1, 2$.

The Nash bargaining solution solves:

$$\max_{0 \leq \rho \leq y} \left(U^1(\rho, x_1) - T^1(x_1)\right) \times \left(U^2(y - \rho, x_2) - T^2(x_2)\right).$$

The first order condition is:

$$R^1(\rho, x_1) = R^2(y - \rho, x_2)$$

where

$$R^s(\rho_s, x_s) \equiv \frac{\partial U^s(\rho_s, x_s)/\partial \rho_s}{U^s(\rho_s, x_s) - T^s(x_s)}.$$

Proposition 2.

Suppose that U.1, T.1, S.1 and S.2 hold.

Then:

$$0 < \frac{\partial \rho}{\partial y}(y, x) < 1.$$

Moreover, there exist functions $(\phi_1, \ldots, \phi_{n_1})$ *of* (ρ_1, x_1) *and* $(\psi_1, \ldots, \psi_{n_1})$ *of* (ρ_2, x_2) *such that, for any* $(y, x) \in \mathcal{M}$,

$$\begin{aligned}
\left(1 - \frac{\partial \rho}{\partial y}(y, x)\right)^{-1} \left(\frac{\partial \rho}{\partial x_{1i}}(y, x)\right) &= \phi_i(\rho, x_1), \\
\left(\frac{\partial \rho}{\partial y}(y, x)\right)^{-1} \left(\frac{\partial \rho}{\partial x_{2j}}(y, x)\right) &= \psi_j\left(y - \rho, x_2\right).
\end{aligned}$$

Proposition 2 (continued).

The functions $\phi_i(\rho, x_1)$ *and* $\psi_j\,(y-\rho, x_2)$ *satisfies*

$$\begin{aligned}
\frac{\partial \phi_i}{\partial x_{1i'}} + \phi_{i'} \frac{\partial \phi_i}{\partial \rho_1} &= \frac{\partial \phi_{i'}}{\partial x_{1i}} + \phi_i \frac{\partial \phi_{i'}}{\partial \rho_1}, \\
\frac{\partial \psi_j}{\partial x_{2j'}} + \psi_{j'} \frac{\partial \psi_j}{\partial \rho_2} &= \frac{\partial \psi_{j'}}{\partial x_{2j}} + \psi_j \frac{\partial \psi_{j'}}{\partial \rho_2},
\end{aligned}$$

Conversely, any sharing rule satisfying these conditions can be rationalized as the Nash bargaining solution of a model satisfying U.1, T.1, S.1 and S.2; that is, conditions listed above are sufficient as well.

Intuition.

1) The threat-point is independent of y.

2) Differentiating the first order condition

$$R^1(\rho, x_1) = R^2(y - \rho, x_2)$$

gives:

$$\begin{aligned}
\left(\frac{\partial R^1}{\partial \rho_1} + \frac{\partial R^2}{\partial \rho_2}\right)\left(1 - \frac{\partial \rho}{\partial y}(y, x)\right) &= \frac{\partial R^1}{\partial \rho_1}, \\
\left(\frac{\partial R^1}{\partial \rho_1} + \frac{\partial R^2}{\partial \rho_2}\right)\frac{\partial \rho}{\partial x_{1i}}(y, x) &= -\frac{\partial R^1}{\partial x_{1i}}.
\end{aligned}$$

Intuition (continued).

3) The system of PDE

$$\frac{\partial R^1/\partial x_{1i}}{\partial R^1/\partial \rho_1} = -\phi^i(\rho, x_1),$$

can be solved with respect to R^1 up to a transform. That is: $R^1 = G(\bar{R}^1)$.

4) The cross derivative restrictions that garantee integration.

5) Integration of

$$\frac{\partial \log(U^s(\rho_s, x_s) - T^s(x_s))}{\partial \rho_s} = R^s(\rho_s, x_s)$$

gives:

$$U^s(\rho_s, x_s) = K_s(x_s) \exp\left(\int_0^{\rho_s} R^s(u_s, x_s) du_s\right) + T^s(x_s).$$

Remarks

1. When the information about the game is described by U.1 and T.1, the Nash bargaining solution can be falsified (in Popper's terms).

2. Conversely, these conditions are sufficient. If they are satisfied, one can construct a bargaining model for which the solution coincides with the sharing rule.

3. Some conditions implies:

$$\frac{\partial \rho}{\partial x_{1i}} \left(\frac{\partial^2 \rho}{\partial x_{2j} \partial y} \frac{\partial \rho}{\partial y} - \frac{\partial^2 \rho}{\partial y^2} \frac{\partial \rho}{\partial x_{2j}} \right) + \left(1 - \frac{\partial \rho}{\partial y} \right) \left(\frac{\partial^2 \rho}{\partial x_{1i} \partial x_{2j}} \frac{\partial \rho}{\partial y} - \frac{\partial^2 \rho}{\partial x_{1i} \partial y} \frac{\partial \rho}{\partial x_{2j}} \right) = 0.$$

4. Any sharing function which can be rationalized by the maximization of an additively separable index such as $f^1(\rho_1, x_1) + f^2(\rho_2, x_2)$ will satisfy the conditions.

5. If a solution satisfies IIA (+PO and CO) then it can be described by the maximization of $F(\rho_1, \rho_2, x_1, x_2)$.

6. The set of solutions described by a maximization such as $f^1(\rho_1, x_1) + f^2(\rho_2, x_2)$ includes the Egalitarian solution and the Utilitarian solution. Technically:

$$f^s = \begin{cases} \delta_s\left(\left(U_s - T_s\right)^\gamma / \gamma\right) & \text{if } \gamma \neq 0 \\ \delta_s \log\left(U_s - T_s\right) & \text{if } \gamma = 0 \end{cases}.$$

Parametric example 1.

Consider the following specification for the sharing function:

$$\rho = y \cdot \mathcal{L}\left(a_{00} + a_{01}x_1 + a_{02}x_2 + a_{11}x_1^2 + a_{22}x_2^2 + a_{12}x_1x_2\right)$$

where $\mathcal{L}(x) = 1/(1+\exp(x))$ is the logistic distribution function.

Conditions in Proposition 2 require that $a_{12} = 0$.

If this restriction is satisfied, the first order condition is:

$$G(\rho_1 g_1(x_1)) = G(\rho_2 g_2(x_2)),$$

where

$$g_1(x_1) = \exp\left(a_{00} + a_{01}x_1 + a_{11}x_1^2\right),$$

$$g_2(x_2) = \exp\left(-\left(a_2 x_{02} + a_{22}x_2^2\right)\right).$$

Outside and along the agreement frontier

When $(y, x) \in \mathcal{N}$, the econometrician can learn next to nothing about the underlying structure.

The agreement frontier $\mathcal{F}$ is defined by the points that belong to the intersection of the closure of the agreement set $\mathcal{M}$ and the closure of the non-agreement set $\mathcal{N}$, that is,

$$\mathcal{F} = \mathrm{cl}(\mathcal{M}) \cap \mathrm{cl}(\mathcal{N}).$$

If $(y, x) \in \mathcal{F}$, then each agent is indifferent between her share of the pie and her reservation payment, i.e.,

$$(y, x) \in \mathcal{F} \Rightarrow U^1(\rho, x) = T^1(x), \; U^2(y - \rho, x) = T^2(x).$$

Proposition 3.

Suppose that U.1 and T.1 hold.

If the agents' behavior $(\{\mathcal{M}, \mathcal{N}\}, \rho)$ *is compatible with Nash bargaining, then there exists a function* $\sigma(x)$*, such that* $y = \sigma(x)$ *iff* $(y, x) \in \mathcal{F}$*, and*
(i) if $(y, x) \in \mathcal{M}$*, then* $y \geq \sigma(x)$*,*
(ii) if $(y, x) \in \mathcal{N}$*, then* $y \leq \sigma(x)$*.*
Moreover, the function $\sigma(x)$ *is additive in the sense that* $\sigma(x) = \sigma_1(x_1) + \sigma_2(x_2)$ *for some functions* $\sigma_1(x_1)$ *and* $\sigma_2(x_2)$*.*

Proof.

Definition of the frontier:

$$U^1(\rho, x_1) = T^1(x_1) \Rightarrow \rho = \sigma^1(x_1)$$

and

$$U^2(y - \rho, x_2) = T^2(x_2) \Rightarrow y - \rho = \sigma^2(x_2)$$

so that

$$y = \sigma_1(x_1) + \sigma_2(x_2) = \sigma(x).$$

Proposition 4.

Suppose U.1, T.1 and S.1 hold.

If the agents' behavior $(\{\mathcal{M}, \mathcal{N}\}, \rho)$ *is compatible with Nash bargaining, then for any* (y, x) *in* $\mathcal{F}$*,*

$$\frac{\partial \sigma}{\partial x_{1i}} = \frac{\partial \rho / \partial x_{1i}}{1 - \partial \rho / \partial y}, \qquad \frac{\partial \sigma}{\partial x_{2j}} = -\frac{\partial \rho / \partial x_{2j}}{\partial \rho / \partial y},$$

for every $i = 1, \ldots, n_1$ *and* $j = 1, \ldots, n_2$.

Proof.

$$\sigma_1(x_1) = \rho(y, x_1, x_2) = \rho(\sigma_1(x_1) + \sigma_2(x_2), x_1, x_2)$$

$$\Rightarrow \frac{\partial \sigma_1}{\partial x_{1i}} = \frac{\partial \rho}{\partial y} \frac{\partial \sigma_1}{\partial x_{1i}} + \frac{\partial \rho}{\partial x_{1i}}.$$

4. Identifiability: the deterministic case

Proposition 5.

Let $\rho(y, x)$ be some function that satisfies conditions in Proposition 2.

Then there exists a continuum of different utility functions U^1, U^2 and threat functions T^1, T^2, such that U.1 and T.1 are satisfied and the agents' behavior is compatible with Nash bargaining.

Intuition.

The function G is not identified.

In this proposition, utility functions U^s and $\bar{U}^s$ are different if and only if there does not exist functions $\alpha(x_s) > 0$ and $\beta(x_s)$ such that

$$U^s = \alpha(x_s)\,\bar{U}^s + \beta(x_s).$$

Parametric example 2.

Coming back to our numerical example:

$$\rho = y \cdot \mathcal{L}\left(a_{00} + a_{01}x_1 + a_{02}x_2 + a_{11}x_1^2 + a_{22}x_2^2\right).$$

Then, the functions R^s are identified up to a transform. For example,

$$\frac{\partial U^1(\rho, x_1)/\partial\rho}{U^1(\rho, x_1) - T^1(x_1)} = G(\rho g_1(x_1))$$

where $g_1(x_1) = \exp\left(a_{00} + a_{01}x_1 + a_{11}x_1^2\right)$.

If $G(x) = x$,

$$U^1(\rho, x_1) = K(x_1)\exp\left(\tfrac{1}{2}g_1(x_1)\rho^2\right) + T^1(x_1).$$

If $G(x) = x^{-1}$,

$$U^1(\rho, x_1) = K(x_1)\,\rho^{\exp - g_1(x_1)} + T^1(x_1).$$

Identifying assumptions: x_s–independent utility functions

Assumption U.2. For $s = 1, 2$, $U^s(\rho_s, x_s) = U^s(\rho_s)$.

Under U.1, U.2 and T.1, the sharing function $\rho\,(y, x_1, x_2)$ solves the problem:

$$\max_{0 \le \rho \le y} \left(U^1\,(\rho) - T^1\,(x_1)\right) \cdot \left(U^2\,(y - \rho) - T^2\,(x_2)\right).$$

The first order condition is

$$R^1\,(\rho_1, x_1) = R^2\,(\rho_2, x_2)\,,$$

where

$$R^s\,(\rho_s, x_s) = \frac{\partial U^s\,(\rho_s)\,/\partial \rho_s}{U^s\,(\rho_s) - T^s\,(x_s)}.$$

Proposition 6.

Assume U^s *is not exponential (i.e.,* $U^s(\rho_s)$ *is not of the form* $\alpha e^{\mu\rho_s} + \beta$ *for some* α, β, μ*). Then, under U.1, U.2, T.1, S.1 and S.2,*

(a) the functions U^s *and* T^s *are identified up to an affine, increasing transform;*

(b) the sharing function must satisfy additional testable restrictions.

Intuition.

The functions R^s are known to be identified up to a unique transform G, that is, $R^s = G(\bar{R}^s)$, where $\bar{R}^s$ is a known function.

From the additional assumption U.2, the functions R^s must be of the form:

$$G(\bar{R}^s\left(\rho_s, x_s\right)) = \frac{\partial U^s\left(\rho_s\right)/\partial \rho_s}{U^s\left(\rho_s\right) - T^s\left(x_s\right)},$$

which determines the function G.

The utility functions can be identified up to an affine transform: any particular solution for the utility function $\bar{U}^s$ must be independent of x_s for ρ fixed.

Parametric example 3.

The logistic-quadratic form is not compatible with U.2.

Indeed,

$$G(yg_1(x_1)) = \frac{\partial U^1(\rho_1)/\partial \rho_1}{U^1(\rho_1) - T^1(x_1)}$$

where

$$g_1(x_1) = \exp\left(a_{00} + a_{01}x_1 + a_{11}x_1^2\right).$$

Conclusion: An empirical model of bargaining that is using either the logistic-quadratic specification *must* assume (at least implicitly) that individual utilities in case of an agreement depend on the threat point payments.

Remark

The restrictions here are generally not satisfied by the family of bargaining solutions that can be described by the maximization of $f^1(\rho_1, x_1) + f^2(\rho_2, x_2)$.

One exception: these restrictions hold when f^s is given by

$$f^s = \begin{cases} \delta_s \left((U_s - T_s)^\gamma / \gamma \right) & \text{if } \gamma \neq 0 \\ \delta_s \log (U_s - T_s) & \text{if } \gamma = 0 \end{cases}.$$

with $\delta_1 = \delta_2$.

5. Identifiability: the stochastic case

The bargaining model with unobserved heterogeneity

The model depends on variables ϵ, unobservable to the econometrician.

The unobservables induce a nondegenerate distribution of (m, ρ) given (y, x).

Suppose that the players always reach an agreement.

Then the agreed sharing function solves:

$$\max_{0 \leqslant \rho \leqslant y} \left(U^1(\rho) - T^1(x_1) + \epsilon_1\right) \cdot \left(U^2(y - \rho) - T^2(x_2) + \epsilon_2\right).$$

More assumptions

Assumption D.1: $(\epsilon_1, \epsilon_2) \perp (x_1, x_2) \,|\, (y)$.

Assumption D.2: The conditional distribution $F_{\epsilon_1,\epsilon_2|\, y}$ of (ϵ_1, ϵ_2) given (y) is continuous and has full support on $\mathbb{R}^2_+$.

Normalization conditions.

(i) for known ρ_s^0 and k^s, $U^s(\rho_s^0) = k^s$,

(ii) for known x_s^0 and c^s, $T^s(x_s^0) = c^s$,

(iii) for known ρ_s^* and $K^s > 0$, $\partial U^s(\rho_s^*)/\partial\rho_s = K^s$.

where the values ρ_s^0, x_s^0 and the functions k^s, c^s and K^s can be arbitrarily chosen.

(iv) $\mathsf{E}[\epsilon_s |\, y] = 0$.

Proposition 7.

Suppose U.1, U.2, T.1, D.1, and D.2 hold. Suppose that the normalization conditions (i)-(iv) hold.

Then, (U^1, U^2, T^1, T^2) *are identified from* $F_{\rho|\,y,x}$.

Intuition.

Start from:

$$\frac{\partial U^1(\rho)/\partial\rho}{U^1(\rho) - T^1(x_1) + \varepsilon_1} = \frac{\partial U^2(y-\rho)/\partial\rho_2}{U^2(y-\rho) - T^2(x_2) + \varepsilon_2}$$

where $(\varepsilon_1, \varepsilon_2)$ is independent of (x_1, x_2, y).

This gives:

$$\begin{aligned}
\varepsilon_1 \frac{\partial U^2(y-\rho)}{\partial\rho} - \varepsilon_2 \frac{\partial U^1(\rho)}{\partial\rho} \quad = \quad & \frac{\partial U^1(\rho)}{\partial\rho}\left[U^2(y-\rho) - T^2(x_2)\right] \\
& - \frac{\partial U^2(y-\rho)}{\partial\rho}\left[U^1(\rho) - T^1(x_1)\right]
\end{aligned}$$

Intuition (continued).

Therefore if $\Phi(\rho, y, x_1, x_2)$ is the conditional cdf:

$$\begin{aligned}\Phi(r, y, x_1, x_2) &= \Pr(\rho \leq r \mid y, x_1, x_2) \\ &= \Pr(E(y,\rho) \leq S(x_1, x_2))\end{aligned}$$

where

$$\begin{aligned}E(y,\rho) &= \varepsilon_1 \frac{\partial U^2(y-\rho)}{\partial \rho} - \varepsilon_2 \frac{\partial U^1(\rho)}{\partial \rho}, \\ S(x_1, x_2) &= \frac{\partial U^1(r)}{\partial r}\left[U^2(y-r) - T^2(x_2)\right] \\ &\quad - \frac{\partial U^2(y-r)}{\partial r}\left[U^1(r) - T^1(x_1)\right]\end{aligned}$$

Intuition (continued).

It follows that:

$$\frac{\partial \Phi\left(r,y,x_1,x_2\right)/\partial x_1^k}{\partial \Phi\left(r,y,x_1,x_2\right)/\partial x_2^s} = \frac{U'^2\left(y-r\right)}{U'^1\left(r\right)} \frac{\partial T^1\left(x_1\right)/\partial x_1^k}{\partial T^2\left(x_2\right)/\partial x_2^s}$$

therefore

$$\begin{aligned} \log\left(\frac{\partial \Phi\left(r,y,x_1,x_2\right)/\partial x_1^k}{\partial \Phi\left(r,y,x_1,x_2\right)/\partial x_2^s}\right) &= \log U'^2\left(y-r\right) - \log U'^1\left(r\right) \\ &\quad + \log \frac{\partial T^1\left(x_1\right)}{\partial x_1^k} - \log \frac{\partial T^2\left(x_2\right)}{\partial x_2^s} \end{aligned}$$

6. Conclusion

The methodology developed in this paper can be used in family economics.

It opens new and interesting directions for future research in experimental economics.

A cardinal representation of each agent's utility function can be identified from

$$\max_{\rho}\left(U^{1}(\rho)-T^{1}(x_{1})\right)\cdot\left(U^{2}(y-\rho)-T^{2}(x_{2})\right).$$

Identification does not require any form of uncertainty.

Suggestion: One could first face individuals of a given sample with menus of lotteries, in order to assess their level of risk aversion from their choices; then match the agents by pairs and let them play a two-sided bargaining problem to recover their bargaining-relevant utility functions. A comparison is then possible.