Learning From a Piece of Pie[*]

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Abstract

We investigate the empirical content of the Nash solution to two-player bargaining games. The bargaining environment is described by a set of variables that may affect agents' preferences over the agreement sharing, the status quo outcome, or both. The outcomes (i.e., whether an agreement is reached, and if so the individual shares) and the environment (including the size of the pie) are known, but neither the agents' utilities nor their threat points. We consider both a deterministic version of the model (in which the econometrician

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observes the shares as deterministic *functions* of the variables under consideration) and a stochastic one (in which, because of unobserved heterogeneity or measurement errors, only the joint distribution of incomes and outcomes is recorded). We show that, in the most general framework, any outcome can be rationalized as a Nash solution. However, under mild exclusion restrictions that are precisely described, the Nash bargaining assumption generates strong testable restrictions on observable behavior. Moreover, the underlying structure of the bargaining, i.e., the players' utility and threat point functions, can be recovered under slightly more demanding conditions. Finally, our methods allow to recover cardinal representations of individual preferences in the absence of uncertainty; we discuss the implications of this finding. We conclude that empirical works entailing Nash bargaining could (and should) use much more general and robust versions than they usually do.

I passed by his garden, and marked, with one eye,

How the Owl and the Panter were sharing a pie:

The Panther took pie-crust, and gravy, and meat,

While the Old had the dish as its share of the treat.

When the pie was all finished, the Owl, as a boon,

Was kindly permitted to pocket the spoon:

While the Panther received knife and fork with a growl,

And concluded the banquet by...

Lewis Caroll (Alice's Adventures in Wonderland, 1866)

1 Introduction

Is Nash Bargaining empirically relevant?   The axiomatic theory of bargaining, originated in a fundamental paper by John F. Nash (1950), has provided a simple, elegant framework to resolve the indeterminateness of the terms of bargaining. As such, it has been widely applied in economics. For instance, several approaches model the firm's decision process as a bargaining game between the management and the workers, represented by a union. Most of the time, such models assume a Nash solution (see de Menil, 1971, and Hamermesh, 1973); and several papers have tried to test this assumption (Bognanno and Dworkin, 1975; Bowlby and Schriver, 1978; Sevjnar (1986), for instance). Another example of application is provided by household behavior. During the last two decades, several models accounting for the fact that spouses' goals may differ—and, therefore, that the decision process at stake has crucial consequences on the outcomes—have emerged. While the collective approach pioneered by Chiappori (1988a, 1992) relies on the sole assumption that the intrahousehold decision process is efficient,

Manser and Brown (1980), McElroy and Horney (1981), Lundberg and Pollak (1993), who consider couples, and Kotlikoff, Shoven and Spivak (1986), who concentrate on negotiations between parents and children, explicitly refer to a Nash-bargaining equilibrium concept.[1] An interesting problem is whether (and under what conditions) the additional structure provided by Nash-bargaining results in either additional testable predictions on behavior, or a more accurate identification of individual preferences and decision processes.[2] Still other domains of applications of Nash bargaining include international cooperation for fiscal and trade policies (Chari and Kehoe, 1990) or the sharing of profit in cartels (Harrington, 1991) and oligopoly (Fershtman and Muller, 1986). Finally, the modern analysis of employment contracts is often based on search models, in which, once a meeting results in an employment contract, the parties bargain over the distribution of the surplus (see Postel-Vinay and Robin, 2006, for a recent survey).

The Nash solution is used in the large majority of these contributions, and can be considered as the main solution concept to the bargaining problem, at least when the later arises under a general form. From an empirical perspective, a few contributions consider specific examples in which players follow an explicit bargaining protocol, the details of which are moreover known to the econometrician; then non cooperative bargaining theory (and more precisely a non cooperative bargaining model constructed to exactly mimic the rules under consideration) is a natural tool to be taken to data. Most of the time, however, the bargaining environment is not known, or even not prop-

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[1]Note that since Nash-bargaining generates efficient outcomes, the Nash-bargaining approach is at any rate a particular case of the 'collective' framework, see Browning, Chiappori and Weiss (2010).

[2]See Chiappori (1988b, 1991) and McElroy and Horney (1990) for an early exchange on this issue.

erly defined ex ante.[3] Then the Nash solution is regularly employed (Budd and Slaughter, 2004; Bughin, 1995; Coles and Hildreth, 2000; Moscarini, 2005; Svejnar, 1986; Svejnar and Smith, 1984) as the reduced form of a more complicated strategic bargaining process.

Benchmark Example While Nash bargaining is an elegant and convenient tool for approaching an old and important problem, its empirical relevance has not received the attention it deserves. As a benchmark example, consider for instance a game in which two players, 1 and 2, bargain about a pie of size y. If the players agree on some sharing (ρ_1, ρ_2) with $\rho_1 + \rho_2 = y$, it is implemented. If not, each agent i receives some reservation payment x_i. The setting of the process (i.e., the size y and the reservation payments x_i), as well as its outcome (the individual shares ρ_i) are typically observable by an outside econometrician; however, individual utilities are not. Let us now assume that agents use a Nash bargaining solution. What is the empirical content of this assumption? Specifically, if $\rho_1 = \rho$ and $\rho_2 = y - \rho$, a Nash-bargaining agreement exists if and only if one can find some $\rho \in [0, y]$ such that $U^1(\rho) \geq T^1(x_1)$ and $U^2(y - \rho) \geq T^2(x_2)$; then the Nash solution ρ solves a program of the type:

$$\max_{\rho} \left(U^1(\rho) - T^1(x_1) \right) \cdot \left(U^2(y - \rho) - T^2(x_2) \right) \tag{1}$$

for some functions U^1, U^2, T^1, T^2 where i's utility function if an agreement is reached is denoted U^i, and T^i in the opposite case.[4] What does this structure imply (if anything) on the the relationship between (y, x_1, x_2) and ρ?

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[3]Intrahousehold bargaining is a prime example of this situation; while the idea that spouses bargain over decisions to be made is natural (and has been used by many contributors), the bargaining game is mostly informal, and cannot be described by fixed rules.

[4]Note that we allow in principle these utilities to differ. For instance, individuals may have different preferences, say, different marginal rates of substitution between consump-

The traditional approach arbitrarily assumes a specific (usually linear) form for individual utilities.[5] Under such an assumption, the sharing function solves the program:

$$\max_{\rho} (\rho - x_1) \cdot (y - \rho - x_2),$$

giving the simple, linear form $\rho = \frac{1}{2}(y + x_1 - x_2)$ whenever $y \geq x_1 + x_2$. While this prediction is indeed testable, it totally relies on the linearity assumption; since the Nash bargaining outcome depends on the cardinal representation of individual preferences, any deviation from linear utilities will give a different form for the resulting shares.[6] It follows that any test based on the above program is a joint test of two assumptions, one general (Nash bargaining), the other very specific (linear utilities). A rejection is likely to be considered as inconclusive, since the burden of rejection can always be put on the specific and often ad hoc linearity assumption.

From a methodological perspective, assuming linear forms a priori contradicts the generally accepted rule in empirical economics, whereby preferences should be recovered from the data rather than assumed a priori. This remark, in turn, raises two questions. First, is it possible to test the Nash bargaining assumption *without previous knowledge of individual utilities*? And second, can the utility players derive from the consumption of either their share of the pie or their reservation payment be recovered from the sole observation of the bargaining outcomes?

In the present paper, we address these two questions—the testability of Nash bargaining models and the identifiability of the underlying structure

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tion and leisure, when married than when single.

[5]This approach is explained and discussed by Svejnar (1980). The household literature is an obvious exception.

[6]In the example above, for instance, if the utility of agent 1 is $U(\rho) = \sqrt{\rho}$ instead of $U(\rho) = \rho$, the solution becomes $\rho = \frac{1}{3}y + \frac{2}{9}x_1 - \frac{1}{3}x_2 + \frac{2}{9}\sqrt{x_1(x_1 + 3y - 3x_2)}$.

from observed behavior—in a general framework. In our setting, the environment is described by a set of variables that may affect agents' preferences over the agreement sharing, the status quo outcome, or both. A key role will be played by the econometrician's *prior information* on the structure of the model at stake. In a non-parametric spirit, this information will be described by some (broad) classes to which the utility or threat point functions are known to belong. We are mainly interested in situations in which this prior information is limited. We thus do not assume that the econometrician knows the parametric form of the utility and threat point functions, but simply that these functions satisfy some exclusion restrictions.[7] Our basic question can thus be precisely restated in the following way: *what is the minimum prior information needed to achieve (i) testability of the Nash bargaining theory, and (ii) identifiability of the underlying structural model.*

Regarding the identifiability issue, an interesting aspect is that Nash solutions are not invariant to monotonic transformations of utility functions. It follows that one may, in principle, retrieve a *cardinal* representation of preferences. While the identification of cardinal preferences is a standard problem in economics, the present situation is original in that it does not involve uncertainty. Whether concavity of utility functions matter in bargaining because of risk aversion (as might be suggested by (some of) the non cooperative foundations of Nash bargaining) or for unrelated reasons is an interesting conceptual problem, on which our findings shed a new light. This issue will be further discussed in the concluding Section.

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[7]To put it in a Popperian perspective (Popper, 1959), we do not want the falsifiability of Nash bargaining to be entirely driven by ad hoc auxiliary hypotheses such as particular functional forms of individual utility functions.

Deterministic versus stochastic models Economic models are, in general, stochastic. As noted by Marschak (1950), however, important distinct properties of the models can be brought out even if we assume all the variables to be measured exactly and to conform exactly to the predictions of economic theory. In particular, important insights into the problem of testability and identifiability of Nash bargaining models can be obtained even if we assume the econometrician has access to 'ideal data', i.e., she observes individual shares as *deterministic functions* of the variables entering the game (in our introductory example, our observer would know ρ_1 and ρ_2 as functions of (y, x_1, x_2)). Thus stated, the problem is the counterpart, in a bargaining context, of well known results in consumer theory—namely, that a smooth demand function can be derived from utility maximization under linear budget constraint if and only if it satisfies homogeneity, adding up and the Slustky conditions, and that the underlying utility can then be recovered up to an increasing transform. In other words, the first perspective can be summarized as follows: Find an equivalent, for the Nash bargaining setting, of Slutsky relationships in consumer theory.

The perspective just sketched, however, is largely hypothetical; it relies on the availability of 'ideal data', in which a smooth relationship between the fundamentals of the bargaining process and its outcomes is not affected by any disturbance, and the variables are moreover observed without any error. In practice, unobserved individual heterogeneity is paramount which affects the output of the game; in addition, the econometrician observes are data sets in which each observation is affected by various measurement errors. Such features can only be accommodated in a stochastic version of the bargaining model. In this case, we typically observe a *joint distribution* of incomes and outcomes: in our example, of $(y, x_1, x_2, \rho_1, \rho_2)$. Note that

the first (deterministic) perspective is but a statistically degenerate version of the second (stochastic), in the sense that it is relevant only when the distribution is degenerate (i.e., when its support is born by the graph of some deterministic function mapping (y, x_1, x_2) to (ρ_1, ρ_2)). In the second context, we may again ask whether the Nash bargaining context imposes restrictions on the observed distribution, and whether, conversely, the distribution allows to identify the underlying structure. The answer, however, now depends not only on the bargaining framework but also on the stochastic structure attached to it.[8]

In the present paper, we successively address the two problems. We first consider the deterministic version of the model. We show that, in its most general version, Nash bargaining is not testable, in the sense that any Pareto efficient rule can be rationalized as the outcome of a Nash bargaining process (and as a matter of fact of any predetermined bargaining process, provided the latter is individually rational and generates Pareto efficient outcomes).[9] We then introduce simple, exclusion restrictions; namely, we assume that (i) threat point utilities do not depend on the size of the surplus over which agents bargain, and (ii) for each agent i, there exists one variable (at least), say x_i, that only affect this agent's utility and threat point functions (i.e., if $j \neq i$, neither U^j nor T^j depend on x_i). Then the Nash model generates strong, testable restrictions, that take the form of a Partial Differential Equa-

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[8]The key remark, here, is that identifiability of the deterministic model is a necessary, although not sufficient, condition for identifiability of its stochastic version; obviously, if two different structural models generate the same 'ideal' demand function, there is no hope whatsoever of empirically distinguishing them.

[9]That Pareto efficiency by itself may generate testable restrictions is a classic finding of 'collective' literature; see for instance Donni and Chiappori (2009) and Browning, Chiappori and Weiss (forthcoming) for a general presentation.

tion (PDE) on the function ρ. In addition, if either one of the pairs of functions (U^i, T^i) is known or if there exists, for each agent, a variable that enters the agent's threat point but not her utility when an agreement is reached, then generically both individual utility and individual threat point functions can be cardinally identified (i.e., identified up to an affine transform). In particular, in the benchmark example (1) given above, the conditions are satisfied; therefore both testability and identifiability are achieved. Note, however, that the result is only generic; it holds for all utilities functions except exponential ones.

We then move to a stochastic version of the model, in which each individual surplus is the sum of a deterministic component (equal, as above, to the difference between the utility of the share and the reservation utility) and an individual-specific random term. It follows that the Nash solution now involves two random terms. Coming back to our benchmark example, the program would now become:

$$\max_{\rho} \left(U^1(\rho) - T^1(x_1) + \epsilon_1 \right) \cdot \left(U^2(y - \rho) - T^2(x_2) + \epsilon_2 \right)$$

Here, the ϵ_i's can be seen either as measurement errors or, more interestingly, as latent variables reflecting some unobserved heterogeneity between couples. In particular, we want to allow these random terms to be correlated with each other in an arbitrary way—reflecting the fact that the initial match between the negotiating partners is typically not random, but reflects some degree of (positive or negative) assortativeness. What the econometrician observes is the joint distribution of $(y, x_1, x_2, \rho_1, \rho_2)$ over the population under consideration—and we now assume this distribution is not degenerate.

In this stochastic setting, we can show the following result: under the same exclusion restrictions as before plus an independence assumption, testable

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restrictions are generated, and individual utility and threat point functions are cardinally identified. In other words, the deterministic results do extend to the stochastic framework, even though the stochastic structure under consideration involves an unknown bivariate error distribution.

Related works The empirical content of game theory is undoubtedly a topical issue as illustrated by several recent contributions. For example, Sprumont (2001) considers, from the revealed preferences viewpoint, a non-cooperative game played by a finite number of players, each of whom can choose a strategy from a finite set. Ray and Zhou (2001) adopt a similar set-up but focus on extensive-form games. Other related papers include Bossert and Sprumont (2002, 2003), Carvajal, Ray and Snyder (2004), Ray and Snyder (2003), Xu and Zhou (2007). Nonetheless, our contribution differs in many respects from what is generally done. Firstly, our subject matter—the Nash solution—has never been investigated in spite of the various applications of bargaining models in economics. Secondly, our methodology is not based on revealed preferences. The inspiration of the present paper, in fact, is more closely related to the work of Chiappori (1988, 1992) and its numerous sequels (Chiappori and Ekeland, 2006 and 2009) on the empirical implications of Pareto efficiency. This methodology is probably more appropriate for the empirical implementation of theoretical results. Thirdly, the emphasis of this paper is largely on the identification problem, which is generally ignored by the authors cited above (with the exception of Chiappori and Ekeland, 2009).

The organization of the paper is as follows. In the next Section, we develop the general model and show that neither testability, nor identification obtain without a priori information on utility and threat point functions.

In Section 3, then, we introduce additional structure into the model, and show that testability obtains under mild assumptions on utility and threat point functions. In Section 4, we note that identification requires stronger assumptions, of which several examples are given. Section 5 presents the stochastic version of the model and the main results. In the concluding Section, we discuss the potential applications of the results.

2 The deterministic model

2.1 The framework

We consider a bargaining game in which two players, 1 and 2, share a pie of size y. The bargaining environment is described by a vector x of n variables and we assume that (y, x) vary within some convex, compact subset $\mathcal{S}$ of $\mathbb{R}_+ \times \mathbb{R}^n$ with non empty interior. We let $\mathcal{N}$ denote the subset of $\mathcal{S}$ on which no agreement is reached (so that agents receive their reservation payment), and $\mathcal{M}$ the subset on which an agreement is reached, with $\mathcal{S} = \mathcal{M} \cup \mathcal{N}$. Over $\mathcal{M}$, a sharing is observed. For notational convenience, we define the *sharing function* ρ as the share of the pie allocated to player 1, i.e., $\rho(y, x) = \rho_1(y, x)$ (then $y - \rho(y, x) = \rho_2(y, x)$). Let $U^i(\rho_i, x)$ denote i's utility when an agreement is reached and the sharing ρ is implemented. Similarly, let $T^i(y, x)$ denote i's threat point, i.e., utility when no agreement is reached and the reservation payments are made. The functions $U^i(\rho_i, x)$ and $T^i(y, x)$ may in general be different.

The Nash bargaining model is then defined as follows: an agreement is reached if and only if there exists a sharing (ρ_1, ρ_2), with $\rho_1 + \rho_2 = y$, such

that

$$T^i(y, x) \leq U^i(\rho_i, x), \quad i = 1, 2, \tag{2}$$

i.e., the allocation $(T^1(y, x), T^2(y, x))$ lies within the Pareto frontier; in that case, the observed sharing $(\rho_1 = \rho, \rho_2 = y - \rho)$ solves:

$$\max_{0 \leq \rho \leq y} \left(U^1(\rho, x) - T^1(y, x)\right) \cdot \left(U^2(y - \rho, x) - T^2(y, x)\right). \tag{3}$$

The set of all functions $U^i(\rho_i, x)$ (resp. $T^i(y, x)$) that are compatible with the a priori restrictions is denoted by $\mathcal{U}^i$ (resp. $\mathcal{T}^i$).

The utilities $U^i(\rho_i, x)$ and the threat functions $T^i(y, x)$ are assumed to be unknown to the econometrician. We assume, here, that the econometrician observes the 'ideal data' referred to in Introduction; namely, she observes the variables y and x, whether or not an agreement is reached, and if so, the player i's share, ρ_i, as a function $\rho_i(y, x)$ of the relevant variables (with $\rho_1(y, x) + \rho_2(y, x) = y$). Two questions then arise: are the observables compatible with the model in (2)-(3)? and second, do they allow the econometrician to uncover the functions $U^i(\rho_i, x)$ and $T^i(y, x)$ that generated them? More formally, we shall use the following definitions.

Definition 1 *The observables (y, x, ρ) and $\{\mathcal{M}, \mathcal{N}\}$ are compatible with Nash bargaining if and only if there exist two utility functions $U^i \in \mathcal{U}^i$ and two threat point functions $T^i \in \mathcal{T}^i$, with $i = 1, 2$, that satisfy (2) and (3). If the functions (U^i, T^i) are unique up to a common affine transform, then the Nash bargaining model (2)-(3) is said to be identified.*

Note that what we can recover is, at best, a *cardinal representation* of the functions under consideration: if we replace (U^i, T^i) with the affine transforms $(\alpha_i U^i + \beta_i, \alpha_i T^i + \beta_i)$, program (3) is not modified. Moreover, the α and β can themselves be functions of (some of) the variables at stake—an issue that will be clarified below.

2.2 A negative result

The answers to the two questions raised above, testability and identifiability, obviously depends on the prior information one is willing to exploit in the framework at stake. Our first result is that the fully general setting, in which the form of threat point functions is not restricted, is simply too general. The answer to both testability and identifiability questions is negative: Nash bargaining cannot generate testable predictions on observed outcomes, and the observation of the outcome does not allow to recover preferences. This is stated formally in the following proposition.

Proposition 2 *Let $\rho(y, x)$ be some function defined over $\mathcal{M}$, and whose range is included in $[0, y]$. Then, for any pair of utility functions U^1, U^2 there exist two threat point functions T^1, T^2 such that the agents' behavior is compatible with Nash bargaining.*

The proof of Proposition 2 is simple: given any pair of functions U^1, U^2 one can define T^1, T^2 by:

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\begin{aligned}
T^i(y, x) &= U^i(\rho_i(y, x), x) \text{ if } (y, x) \in \mathcal{M}, \\
T^i(y, x) &> U^i(y, x) \qquad \text{if } (y, x) \in \mathcal{N}.
\end{aligned}
$$

Then for any (y, x) in $\mathcal{N}$, no agreement can be reached, whereas for any (y, x) in $\mathcal{M}$, the sharing $(\rho_1(y, x), \rho_2(y, x))$ is the only one compatible with individual rationality; thus it is obviously the Nash bargaining allocation.

The intuition behind this result is straightforward: it is always possible to chose the status quo utilities (T^1, T^2) equal to the agents' respective utilities at the observed outcome whenever an agreement is reached (so that, in practice, the chosen point is the only feasible point compatible with individual

rationality), while making sure that (T^1, T^2) is outside the Pareto frontier when agents are observed to disagree. Simple as it may seem, this argument still conveys two important messages. One is that when threat points are unknown, Nash bargaining has no empirical content (beyond Pareto efficiency); any efficient outcome can be reconciled with Nash bargaining. Secondly, the observation of the outcome brings no information on preferences (and in particular the concavity of the utility functions): any utilities can be made compatible with observed outcomes, using ad hoc threat points. Finally, it is important to stress that these negative results are by no means specific to Nash bargaining. The proof applies to any bargaining concept satisfying individual rationality—a very mild requirement indeed.

2.3 Bargaining structure

The negative result above does not mean that Nash bargaining (or, for that matter, bargaining theory altogether) cannot be tested, but simply that more structure is needed to achieve that goal. To this end, we first restrict the sets $\mathcal{U}^i$ of the players' utility functions.

Assumption U.1 *(a) For $i = 1, 2$, the functions $U^i(\rho_i, x)$, defined over $\mathcal{S}$, are sufficiently smooth, strictly increasing and concave in ρ_i.*
(b) There exists a partition $x = (x_1, x_2, \bar{x})$, with $x_1 = (x_{11}, \ldots, x_{1n_1})$, $x_2 = (x_{21}, \ldots, x_{2n_2})$, $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_{\bar{n}})$ and $n_1 \geq 1, n_2 \geq 1$, $n = n_1 + n_2 + \bar{n}$, such that U^i does not depend on x_j, where $i, j = 1, 2$ and $i \neq j$; i.e., $U^i(\rho_i, x) = U^i(\rho_i, x_i, \bar{x})$.

Essentially, Assumption U.1 says that some variables are excluded from the arguments of utility functions. For instance, the utility function of one

player may depend on her own characteristics (such as age and education) but not on those of the other player.

We further restrict the sets $\mathcal{T}^i$ of the players' threat functions.

Assumption T.1 *(a) For $i = 1, 2$, the functions $T^i(y, x)$ are sufficiently smooth.*
(b) For $i = 1, 2$, T^i does not depend on either x_j or the size of the pie y; i.e.,
$T^i(y, x) = T^i(x_i, \bar{x})$.

The differentiability of T^i is sufficient to obtain some restrictions on the sharing function. However, our interpretation of testability is more demanding and we introduce, in addition, some exclusion restrictions. The additional structure given by this assumptions should a priori increase significantly the empirical content of the bargaining game. The exclusion of y is standard; it is typical, for instance, of situations where the opportunity at stake in the bargaining game (i.e., the pie) is lost in the absence of an agreement. The exclusion of x_j provides the key structure needed for testability.[10] The smoothness of T^i, together with that of U^i, guarantees us that the sharing function is sufficiently differentiable for our purpose.

Finally, for the sake of simplicity, we concentrate on the case for which the solution of the Nash program is interior. This is formalized by the next assumption.

Assumption S.1 *For any $(y, x) \in \mathcal{M}$, the sharing (ρ_1, ρ_2) is interior; i.e., $\rho_i > 0$, with $i = 1, 2$.*

This condition will be automatically satisfied, for instance, if $\lim_{\rho_i \to 0} \partial U^i / \partial \rho_i = \infty$. In the benchmark example, if $x_i > 0$ for $i = 1, 2$, then a

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[10]Indeed testable restrictions can be obtained without the exclusion of y provided that x_1 and x_2 are multi-dimensional vectors.

sufficient condition for S.1 is that the utilities be strictly increasing and that the normalization $U^i(0) = T^i(0) = 0$ holds for $i = 1, 2$.

3   Testability: the deterministic case

In this section we study the properties of the Nash bargaining model under Assumptions U.1 and T.1. It is straightforward to check that the negative result of Proposition 2 continues to hold even if the utility functions satisfy Assumption U.1. However, under the additional restrictions given in T.1, the answer to the testability question is now positive: there exist strong testable restrictions on ρ generated by the Nash-bargaining approach. For the sake of presentation, we separately consider two cases depending on whether or not disagreements are observed for some $(y, x) \in \mathcal{S}$.

3.1   The general agreement case

We first leave aside the situations in which the players either disagree or are indifferent between agreeing and disagreeing, and make the simplifying assumption (which will be relaxed in the next subsection) that an agreement is always reached. Formally:

Assumption S.2 *For any $(y, x) \in \mathcal{S}$, there exists a sharing (ρ_1, ρ_2), with $\rho_i \geq 0$, with $i = 1, 2$ and $\rho_1 + \rho_2 = y$, such that $U^i(\rho_i, x) - T^i(y, x) > 0$ for $i = 1, 2$.*

Under Assumptions U.1, T.1, S.1 and S.2, the sharing function ρ is then defined as a function of (y, x) over the entire space $\mathcal{S}$, and solves the problem:

$$\max_{0 \leq \rho \leq y} \left(U^1\left(\rho, x_1, \bar{x}\right) - T^1\left(x_1, \bar{x}\right)\right) \cdot \left(U^2\left(y - \rho, x_2, \bar{x}\right) - T^2\left(x_2, \bar{x}\right)\right).$$

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In addition, the first order condition of this program is:

$$R^1 \left(\rho, x_1, \bar{x} \right) = R^2 \left(y - \rho, x_2, \bar{x} \right) \tag{4}$$

where we have let

$$R^i(\rho_i, x_i, \bar{x}) \equiv \frac{\partial U^i(\rho_i, x_i, \bar{x})/\partial \rho_i}{U^i(\rho_i, x_i, \bar{x}) - T^i(x_i, \bar{x})}. \tag{5}$$

We now proceed to derive the Nash bargaining restrictions. The first result is the following:

Proposition 3 *Suppose that Assumptions U.1, T.1, S.1 and S.2 hold. If the observables are compatible with Nash bargaining, then the function $\rho(y, x)$ has a range included in $]\,0,\, y\,[$, and satisfies:*

$$0 < \frac{\partial \rho}{\partial y}(y, x) < 1. \tag{6}$$

Moreover, there exist functions $(\phi, \Phi_1, \ldots, \Phi_{n_1})$ of $(\rho_1, x_1, \bar{x})$, and functions $(\psi, \Psi_1, \ldots, \Psi_{n_2})$ of $(\rho_2, x_2, \bar{x})$, such that for any $(y, x) \in \mathcal{M}$:

$$\left(1 - \frac{\partial \rho}{\partial y}(y, x) \right)^{-1} \left(\frac{\partial \rho}{\partial x_{1i}}(y, x) \right) = \Phi_i(\rho, x_1, \bar{x}) \tag{7}$$

$$\left(\frac{\partial \rho}{\partial y}(y, x) \right)^{-1} \left(\frac{\partial \rho}{\partial x_{2j}}(y, x) \right) = \Psi_j(y - \rho, x_2, \bar{x}), \tag{8}$$

for every $i = 1, \ldots, n_1$ and $j = 1, \ldots, n_2$, and the functions Φ_i and Ψ_j satisfy (Slutsky) conditions:

$$\frac{\partial \Phi_i}{\partial x_{1k}} + \Phi_k \frac{\partial \Phi_i}{\partial \rho_1} = \frac{\partial \Phi_k}{\partial x_{1i}} + \Phi_i \frac{\partial \Phi_k}{\partial \rho_1} \tag{9}$$

$$\frac{\partial \Psi_j}{\partial x_{2k}} + \Psi_k \frac{\partial \Psi_j}{\partial \rho_2} = \frac{\partial \Psi_k}{\partial x_{2j}} + \Psi_j \frac{\partial \Psi_k}{\partial \rho_2} \tag{10}$$

Conversely, any sharing rule satisfying these conditions can be rationalized as the Nash solution of a model satisfying U.1, T.1, S.1 and S.2; that is, conditions (6), (7), (8), (9) and (10) are sufficient as well.

A proof of Proposition 3 is in Appendix. Put in words, Proposition 3 shows that when the econometrician's information about the structure of the game is described by U.1 and T.1, the Nash bargaining solution can be falsified (in Popper's, 1959, terms) by observable behavior. Specifically, condition (6) states that any increase in the size of the pie must benefit both agents; it is a direct consequence of the exclusion of the size of the pie from the arguments of threat point functions. Moreover, condition (8) translates the particular functional structure of the first order condition (4) which defines the sharing function into a partial differential equation. The intuition is the following. Let us consider a simultaneous variation in y and x_1 such that the share of player 2 does not change. Because of the exclusion restrictions imposed by U.1 and T.1, the marginal surplus of player 2 is not affected. Then condition (8) says that, when ρ_1 is fixed, the direction of the simultaneous variation in y and x_1 is not affected by a change in x_2 (and the same argument applies, *mutatis mutandis*, to (7).

Conditions (7) and (8) can equivalently be stated in terms of higher order partial derivatives of ρ. Indeed, differentiating again the expressions in (7) and (8), respectively, with respect to y and x_{2j}, and y and x_{1i}, respectively, gives:

$$\frac{\partial \rho}{\partial x_{1i}} \left(\frac{\partial^2 \rho}{\partial x_{2j} \partial y} \frac{\partial \rho}{\partial y} - \frac{\partial^2 \rho}{\partial y^2} \frac{\partial \rho}{\partial x_{2j}} \right) +$$
$$\left(1 - \frac{\partial \rho}{\partial y} \right) \left(\frac{\partial^2 \rho}{\partial x_{1i} \partial x_{2j}} \frac{\partial \rho}{\partial y} - \frac{\partial^2 \rho}{\partial x_{1i} \partial y} \frac{\partial \rho}{\partial x_{2j}} \right) = 0, \qquad (11)$$

for every $i = 1, \ldots, n_1$ and $j = 1, \ldots, n_2$. Note that condition (11) is equivalent to both (7) and (8).

Finally, conditions (6) and (11) are not specific to the Nash solution. Indeed, from the first order equation (4) one can see that any sharing function which can be rationalized by the maximization of an additively separable

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index such as $f^1\left(\rho_1, x_1, \bar{x}\right) + f^2\left(\rho_2, x_2, \bar{x}\right)$ for some functions f^1, f^2 that are smooth, increasing and concave in ρ_i, will satisfy conditions (6) and (11). This includes, as possible solution concepts, the Weighted Nash solution with constant bargaining weights, the Utilitarian solution and the Egalitarian solution (but excludes the Kalai-Smorodinsky solution).[11] In particular, if $f^i = -\left(1/\gamma\right)\left(U_i - T_i\right)^{-\gamma}$ for some $\gamma \neq 0$, the Utilitarian solution is obtained when $\gamma = -1$ and the Egalitarian solution, as a limit case, when $\gamma \to \infty$.[12] If a solution concept can be described by the maximization of such an additive index, then it satisfies the Independence of Irrelevant Alternatives (IIA) but the converse is not true (Peters and Wakker, 1991). For this reason, the conditions stated in the proposition above cannot be interpreted as a test of IIA.

The preceding result can be exploited to test whether players make use of the Nash solution. The simplest way is to translate conditions (6) and (11) into constraints on the parameters of a functional form. An illustration is provided by the following example.

Parametric example 1. For the sake of notational simplicity, we omit $\bar{x}$ and assume that the vectors x_i are one dimensional. We then choose the following, 'semi-parametric' specification for the sharing function:

$$\rho = y \cdot \mathcal{L}\left(a_{00} + a_{01}x_1 + a_{02}x_2 + a_{11}x_1^2 + a_{22}x_2^2 + a_{12}x_1x_2\right) \qquad (12)$$

---

[11]For a precise definitions of the axioms and a taxonomy of the solutions, the reader is referred to Thompson (1994). Lensberg (1987) characterizes the axioms that are necessary and sufficient to describe all the bargaining solutions that can be represented by the maximization of such an additively separable index. This characterization, unfortunately, requests a variable number of players, which limits the applicability of this result here.

[12]The first order condition in the Egalitarian case is simply given by $U_1 - T_1 = U_2 - T_2$.

where

$$\mathcal{L}(x) = \frac{1}{1 + \exp(x)}$$

is the logistic distribution function; in words, the respective shares ρ/y are taken to be logistic transformations of a general second order approximation in (x_1, x_2). This form implies, as expected, that $\rho(y, x_1, x_2)$ is necessarily between 0 and y. Moreover, condition (6) is globally satisfied and condition (11) requires that:

$$a_{12} = 0.$$

If this restriction is satisfied, the first order condition is:

$$\rho \exp\left(a_{00} + a_{01}x_1 + a_{11}x_1^2\right) = (y - \rho)\exp\left(-\left(a_{02}x_2 + a_{22}x_2^2\right)\right),$$

which has exactly the form of condition (4). Hence an econometric test of the Nash solution, under U.1–T.1, boils down to testing that $a_{12} = 0$.

Finally, note that this example can be generalized with an approximation of any arbitrary order.

3.2   Outside and along the agreement frontier

In the previous subsection, it is assumed that cooperation always generates a positive surplus that can be shared between the players. From now on, we consider a more general case: $\mathcal{M} \subseteq \mathcal{S}$ so that the possibility of a disagreement between the players, or an agreement along the boundary of $\mathcal{M}$, can no longer be excluded. To begin with, it is worth noting that, when $(y, x) \in \mathcal{N}$, i.e., the players do not agree about the sharing of the pie, the outside econometrician can learn next to nothing about the underlying structure of the bargaining. The econometrician can only infer from the observation of a disagreement that the status quo point must lie outside the Pareto frontier.

The study of the agreement frontier—the locus where the players are indifferent whether the agreement is reached or not—is much more interesting. Formally, the agreement frontier $\mathcal{F}$ is defined by the points that belong to the intersection of the closure of the agreement set $\mathcal{M}$ and the closure of the non-agreement set $\mathcal{N}$, that is, $\mathcal{F} = \mathrm{cl}(\mathcal{M}) \cap \mathrm{cl}(\mathcal{N})$. In what follows, we assume that $\mathcal{F} \neq \varnothing$. Along this frontier, the econometrician observes the sharing of the pie, as a function of the size of the pie and the set of environmental variables, and knows that, by definition, the bargaining surplus is exactly equal to zero. If $(y, x) \in \mathcal{F}$, then each agent is indifferent between her share of the pie and her reservation payment, i.e.,

$$(y, x) \in \mathcal{F} \Rightarrow U^1 (\rho, x) = T^1 (x) , \, U^2 (y - \rho, x) = T^2 (x) ,$$

for some ρ, with $0 \leq \rho \leq y$.[13] The agreement frontier $\mathcal{F}$, if non-empty, is observable, by construction. It should have some features that can be tested. Indeed, the following proposition presents a set of testable restrictions which are based on the observation of the sole agreement frontier.

Proposition 4 *Suppose that Assumptions U.1 and T.1 hold. If the agents'*
behavior $(\{\mathcal{M}, \mathcal{N}\}, \rho)$ is compatible with Nash bargaining, then there exists a
subset $\mathcal{B}$ in $\mathbb{R}^l$, and a sufficiently smooth function $\sigma(x)$ defined over $\mathcal{B}$, such
that $y = \sigma(x)$ if and only if $(y, x) \in \mathcal{F}$, and

(i) if $(y, x) \in \mathcal{M}$ and $x \in \mathcal{B}$, then $y \geq \sigma(x)$,

(ii) if $(y, x) \in \mathcal{N}$ and $x \in \mathcal{B}$, then $y \leq \sigma(x)$.

Moreover, the function $\sigma(x)$ is additive in the sense that $\sigma(x) = \sigma_1(x_1, \bar{x}) +$
$\sigma_2(x_2, \bar{x})$ for some functions $\sigma_1(x_1, \bar{x})$ and $\sigma_2(x_2, \bar{x})$.

---

[13]Technically, the converse is not necessarily true. Indeed, it is possible that, for some (y, x) that do not belong to $\mathcal{F}$, the surplus of the players is exactly equal to zero. This will be the case if these points belong to $\mathcal{N}$ but are not in the neighborhood of $\mathcal{M}$.

A proof of Proposition 4 is in Appendix. The first part of the proposition states that the equation characterizing the agreement frontier can be written as:

$$y = \sigma(x)$$

whereby players agree about the sharing of a pie if and only if its size exceeds some reservation value $\sigma(x)$. The second part of the proposition yields a very strong, testable restriction on the form of the agreement frontier.

The conditions at stake here involve only the agreement frontier. Thus, when $\mathcal{F} \neq \varnothing$ the econometrician can test Nash bargaining even without observing the sharing of the pie. However, stronger conditions obtain when the sharing of the pie is observed. This is formally stated as follows:

Proposition 5 *Suppose Assumptions U.1, T.1 and S.1 hold. If the agents' behavior $(\{\mathcal{M}, \mathcal{N}\}, \rho)$ is compatible with Nash bargaining, then for any (y, x) in $\mathcal{F}$,*

$$\frac{\partial \sigma}{\partial x_{1i}} = \frac{\partial \rho / \partial x_{1i}}{1 - \partial \rho / \partial y}, \qquad \frac{\partial \sigma}{\partial x_{2j}} = -\frac{\partial \rho / \partial x_{2j}}{\partial \rho / \partial y},$$

for every $i = 1, \ldots, n_1$ and $j = 1, \ldots, n_2$.

4 Identifiability: the deterministic case

4.1 A non identifiability result

We now consider the identification problem; i.e., we ask whether the utility and threat point functions can be retrieved from the observation of the sharing function. We put the emphasis on what happens inside the agreement frontier (outside this frontier, identification cannot be reached). It is clear from the form of the problem that Nash bargaining is invariant by affine

transformation of individual utilities. Moreover, the affine transformation of U^i may actually depend on the common variables $(x_i, \bar{x})$ in an arbitrary way. Therefore, we say that utility functions U^i and $\bar{U}^i$ (resp. threat-point functions T^i and $\bar{T}^i$) are different if and only if there does not exist functions $a(x_i, \bar{x}) > 0$ and $b(x_i, \bar{x})$ such that $U^i = a(x_i, \bar{x})\bar{U}^i + b(x_i, \bar{x})$ (resp. $T^i = a(x_i, \bar{x})\bar{T}^i + b(x_i, \bar{x})$).

The main conclusion, then, is that the model is not identified. Formally, we have the following result:

Proposition 6 *Let $\rho(y, x)$ be some twice continuously differentiable function defined over $\mathcal{S}$, that satisfies conditions (6) through (10), and whose range is included in $]0, y[$. Then there exists a continuum of different utility functions U^1, U^2 and threat point functions T^1, T^2, such that Assumptions U.1 and T.1 are satisfied and the agents' behavior is compatible with Nash bargaining. Specifically, under Assumptions S.1 and S.2, the functions $R^1(\rho_1, x_1, \bar{x})$ and $R^2(\rho_2, x_2, \bar{x})$ defined in (5) are determined up to the same transform $G(\cdot, \bar{x})$, increasing in its first argument. For any choice of G, U^1 and U^2 are determined up to an increasing, affine transform.*

A proof of Proposition 6 is in Appendix. Several remarks are in order. As announced above, a by-product of the proof is that when there is always an agreement, the conditions stated in Proposition 3 are sufficient as well, in that any sharing rule satisfying these conditions can be rationalized as a Nash bargaining solution for well-chosen utilities and threat points satisfying Assumptions U.1 and T.1. Another consequence of this result, however, is that along the agreement frontier, the conditions in Propositions 4 and 5 are *not* sufficient. Indeed, as previously shown, the functions $R^i(\rho_i, x_i, \bar{x})$ for

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$i = 1, 2$ are defined up to some increasing function G. Remember now that

$$R^i(\rho_i, \lambda_i, \bar{x}) = \frac{\partial U^i / \partial \rho_i}{U^i(\rho_i, x_i, \bar{x}) - T^i(x_i, \bar{x})}.$$

Hence, any particular solution $\bar{R}^i(\rho_i, x_i, \bar{x})$ has to satisfy a boundary condition, i.e., $\lim_{y \to \sigma(x)} \bar{R}^i(\rho_i(y, x), x_i, \bar{x}) = \infty$. Finally, the framework falls short of providing a uniqueness result; identification requires still more information.

The intuition of Proposition 6 is that, at best, the functions R^1 and R^2 in expression (4) are defined up to some (common) mapping G. This is illustrated below.

Parametric example 2. Coming back to our numerical example, with the semi-parametric specification for the sharing function:

$$\rho = y \cdot \mathcal{L}\left(a_{00} + a_{01}x_1 + a_{02}x_2 + a_{11}x_1^2 + a_{22}x_2^2\right).$$

Let

$$
\begin{aligned}
g_1(x_1) &= \exp\left(a_{00} + a_{01}x_1 + a_{11}x_1^2\right), \\
g_2(x_2) &= \exp\left(-\left(a_2 x_{02} + a_{22}x_2^2\right)\right).
\end{aligned}
$$

Then, one can see that the functions R^i are given by:

$$
\begin{aligned}
R^1(\rho_1, x_1) &= G(\rho_1 g_1), \\
R^2(\rho_2, x_2) &= G(\rho_2 g_2),
\end{aligned}
$$

where G is an arbitrary function. For any choice of G, one can recover the utility functions for arbitrary choices of the threat points. For instance, for $G(x) = x$, we have that:

$$
\begin{aligned}
U^1(\rho_1, x_1) &= K^1(x_1) \exp\left(\tfrac{1}{2}g_1\rho_1^2\right) + T^1(x_1), \\
U^2(\rho_2, x_2) &= K^2(x_2) \exp\left(\tfrac{1}{2}g_2\rho_2^2\right) + T^2(x_2).
\end{aligned}
$$

where $K^1(x_1)$ and $K^2(x_2)$ are some positive functions. However, this transform is not convenient because the resulting utility functions are not concave. For $G(x) = x^{-1}$, we have that:

$$U^1(\rho_1, x_1) = K^1(x_1)\rho_1^{1/g_1} + T^1(x_1),$$
$$U^2(\rho_2, x_2) = K^2(x_2)\rho_2^{1/g_2} + T^2(x_2).$$

Then, these expressions correspond to CRRA utility functions if $g_1, g_2 > 1$.

We now provide two examples of additional assumptions that enable to recover the underlying structural model from observed behavior.

4.2   Case 1: one known utility function

We first assume that one pair of utility functions say, (U^2, T^2) is known from other sources. Then stronger identifiability results can be derived. We first present the general argument, then specialize it to the empirically useful case where U^2 is in fact affine.

Assume, first, that both U^2 is some arbitrary, *known* function. Equation (4) becomes:

$$\frac{\partial U^1/\partial \rho_1}{U^1(\rho, x_1, \bar{x}) - T^1(x_1, \bar{x})} = \frac{\partial U^2/\partial \rho_2}{U^2(y - \rho, x_2, \bar{x}) - T^2(x_2, \bar{x})} \tag{13}$$

In the neighborhood of a point where $\partial \rho/\partial y \neq 0$, consider the change in variables $(y, x_1, x_2, \bar{x}) \rightarrow (\rho, x_1, x_2, \bar{x})$. It is locally invertible, giving y as some function $\theta(\rho, x_1, x_2, \bar{x})$. The previous equation becomes:

$$\frac{\partial U^1/\partial \rho_1(\rho, x_1, \bar{x})}{U^1(\rho, x_1, \bar{x}) - T^1(x_1, \bar{x})} = \frac{\partial U^2/\partial \rho_2(\theta(\rho, x_1, x_2, \bar{x}) - \rho, x_2, \bar{x})}{U^2(\theta(\rho, x_1, x_2, \bar{x}) - \rho, x_2, \bar{x}) - T^2(x_2, \bar{x})} \tag{14}$$
$$\equiv V(\theta(\rho, x_1, x_2, \bar{x}) - \rho, x_2, \bar{x}) \tag{15}$$

where both V and θ are known functions. This can then be integrated as:

$$U^1(\rho, x_1, \bar{x}) - T^1(x_1, \bar{x}) = K(x_1, x_2, \bar{x}) \cdot \exp \int_0^\rho V(\theta(s, x_1, x_2, \bar{x}) - s, x_2, \bar{x}) \, ds$$

for some function K. In addition, K must be such that the right-hand side of the previous equation does not depend on x_2, which pins down $\partial K/\partial x_2$; we conclude that the difference $U^1 - T^1$ is identified up to a multiplicative function of $(x_1, \bar{x})$.

We now consider the particular case of an affine ('risk neutral') utility:

$$U^2\left(y - \rho, x_2, \bar{x}\right) = \alpha\left(x_2, \bar{x}\right) + \beta\left(x_2, \bar{x}\right) \cdot \left(y - \rho\right).$$

This may be the case, for instance, if agent 2 represents a risk-neutral employer who bargains with a risk averse worker (or trade union). If so, the V function just defined becomes:

$$V\left(\theta\left(\rho, x_1, x_2, \bar{x}\right) - \rho, x_2, \bar{x}\right) = \frac{1}{\theta\left(\rho, x_1, x_2, \bar{x}\right) - \rho - \gamma\left(x_2, \bar{x}\right)}$$

where $\gamma = \left(T^2 - \alpha\right)/\beta$, showing that only the ratio γ is relevant in the maximization program. Therefore:

$$U^1\left(\rho, x_1, \bar{x}\right) - T^1\left(x_1, \bar{x}\right) = K\left(x_1, x_2, \bar{x}\right).\exp \int_0^\rho \frac{ds}{\theta\left(\rho, x_1, x_2, \bar{x}\right) - \rho - \gamma\left(x_2, \bar{x}\right)}$$

where K must satisfy:

$$\frac{\partial K/\partial x_{j2}}{K} = \int_0^\rho \frac{\partial\theta/\partial x_{j2} - \partial\gamma/\partial x_{j2}}{\left(\theta\left(\rho, x_1, x_2, \bar{x}\right) - \rho - \gamma\left(x_2, \bar{x}\right)\right)^2} ds \qquad (16)$$

for all $j = 1, ..., n_2$. When $n_2 \geq 2$, these equations generate cross-derivative conditions. If the later are satisfied, then K is defined up to a multiplicative function $K'\left(x_1, \bar{x}\right)$:

$$K\left(x_1, x_2, \bar{x}\right) = K'\left(x_1, \bar{x}\right).\bar{K}\left(x_1, x_2, \bar{x}\right)$$

where $\bar{K}$ is a particular solution of (16), so finally:

$$U^1\left(\rho, x_1, \bar{x}\right) - T^1\left(x_1, \bar{x}\right) = K'\left(x_1, \bar{x}\right).\bar{K}\left(x_1, x_2, \bar{x}\right).\exp \int_0^\rho \frac{ds}{\theta\left(\rho, x_1, x_2, \bar{x}\right) - \rho - \gamma\left(x_2, \bar{x}\right)}$$

where θ and $\bar{K}$ are known functions.

4.3   Case 2: x_i-independent utility functions

An alternative, identifying assumption is that x_1 and x_2 are only relevant for the threat points; they have no direct impact on utilities. We now proceed to show that, in this context, not only additional restrictions are generated on the shape of the sharing function, but both individual utilities and threat points are uniquely recovered (up to the same affine transform).

Formally, we thus introduce the following assumption:

Assumption U.2 *For $i = 1, 2$, the individual utilities U^i are independent of x_i; i.e., $U^i(\rho_i, x_i, \bar{x}) = U^i(\rho_i, \bar{x})$.*

Under U.1, U.2 and T.1, the sharing function $\rho(y, x_1, x_2, \bar{x})$ thus solves the problem:

$$\max_{0 \leq \rho \leq y} \left(U^1(\rho, \bar{x}) - T^1(x_1, \bar{x}) \right) \cdot \left(U^2(y - \rho, \bar{x}) - T^2(x_2, \bar{x}) \right). \qquad (17)$$

In what follows, for the sake of notational simplicity, we disregard the variable $\bar{x}$, and we take x_1 and x_2 to be one dimensional; the extension to the general case is straightforward (although it requires tedious notations) and is left to the reader. We can now state the main result:

Proposition 7 *Assume U^i is not exponential (i.e., $U^i(\rho_i)$ is not of the form $\alpha e^{\mu \rho_i} + \beta$ for some α, β, μ). Then, under Assumptions U.1, U.2, T.1, S.1 and S.2, U^i and T^i are identified up to an affine, increasing transform that may depend on $\bar{x}$.*

A proof of Proposition 7 is in Appendix. Again, additional testable restrictions are generated by this particular form. These conditions are complicated but their intuition is relatively simple. First, let us note that utility

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functions can be identified up to an affine transform. Take any particular solution for the utility function $\bar{U}^i$. Then, it must be independent of x_i for ρ fixed, that is, $\partial \bar{U}^i / \partial x_i = 0$, which implies restrictions on the sharing function under the form of partial differential equations. These restrictions are formally derived in the Appendix.

In particular, it can easily be seen that the logistic-quadratic form used in the empirical example above is not compatible with this setting, because there exist no function G such that

$$G\left(\rho_i g_i\left(x_i\right)\right) = \frac{\partial U^i / \partial \rho_i}{U^i\left(\rho_i\right) - T^i\left(x_i\right)},$$

for some functions U^i and T^i, with $i = 1, 2$, where g_i is defined as previously. In other words, an empirical model of bargaining that is using the logistic-quadratic specification must assume (at least implicitly) that individual utilities in case of an agreement depend on the threat point payment—a strong assumption indeed. This remark illustrates the relevance of a preliminary, theoretical investigation. An empirical specification based on the logistic-quadratic form may be quite appealing (and fit the data); but it is internally inconsistent with the model at stake, at least if one assumes (as it seems natural) that agents care about their threat point utility only insofar as it affects the bargaining outcome.

5   Identifiability: the stochastic case

Finally, let us consider the stochastic version of the model. We now assume that not all of the variables entering the model are observed by the econometrician. Specifically, the model also depends on variables ϵ that are unobservable (to the econometrician) in addition to the observed preference

and payoff variables x. In the presence of latent variables ϵ (such as unobserved individual heterogeneity or measurement errors) the knowledge of the size of the pie to be shared y and of the observed preference and status quo payoff variables x no longer fully determines the agreement event m and the agreed sharing rule ρ. Instead, the unobservables induce a nondegenerate distribution of (m, ρ) given (y, x). The question of identification then is whether upon observing this distribution, it is possible for the econometrician to uniquely recover the agents' utilities U^i and threat functions T^i.

Specifically, we now consider the situation in which the players always reach an agreement and the agreed sharing function solves:

$$\max_{0 \leqslant \rho \leqslant y} \left(U^1(\rho, \bar{x}) - T^1(x_1, \bar{x}) + \epsilon_1 \right) \cdot \left(U^2(y - \rho, \bar{x}) - T^2(x_2, \bar{x}) + \epsilon_2 \right). \quad (18)$$

Here, the econometrician observes the joint distribution of the variables $y \in \mathbb{R}_+$, $x = (x_1, x_2, \bar{x}) \in \mathbb{R}^n$, and the resulting share $\rho \in]0, y[$, while $(\epsilon_1, \epsilon_2) \in \mathbb{R}_+^2$ remain latent.

We shall maintain the following assumptions:

Assumption D.1 $(\epsilon_1, \epsilon_2) \perp (x_1, x_2) \,|\, (y, \bar{x})$.

Assumption D.2 *The conditional distribution $F_{\epsilon_1, \epsilon_2 | y, \bar{x}}$ of (ϵ_1, ϵ_2) given $(y, \bar{x})$ is absolutely continuous and has full support on $\mathbb{R}_+^2$.*

Assumption T.2 *The threat functions T^i are strictly monotonic in x_{i1}, i.e., $\partial T^i(x_i, \bar{x})/\partial x_{1i} \neq 0$ for every $(x^i, \bar{x})$. Moreover, T^i is proper in x_{i1}, i.e., $\lim_{|x_{i1}| \to \infty} |T^i(x_1, \bar{x})| = \infty$.*

Assumption D.1 states that the model unobservables (ϵ_1, ϵ_2) are conditionally independent of the agent specific variables (x_1, x_2) given the size of

the pie y and the common variables $\bar{x}$. This conditional independence property takes the form of exclusion restrictions which will be shown to drive our identification results. Assumption D.2 is a support restriction; we use it to guarantee that the conditional distribution of the share ρ given the observables (y, x) is nondegenerate. Finally, if one is interested in identifying the agents' utilities U^i and threat functions T^i alone then Assumption T.2 can be omitted; it only plays a role in the identification of the conditional distribution $F_{\epsilon_1, \epsilon_2 | y, \bar{x}}$ of the unobservables.

Before proceeding, we recall several useful definitions. Following the related literature (Koopmans and Reiersøl, 1950; Brown, 1983; Roehrig, 1988; Matzkin, 2003), we call *structure* a particular value of the quintuplet $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$. Note that the model (18) simply corresponds to the set of all structures $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$ that satisfy the a priori restrictions given by Assumptions U.1, T.1, T.2, D.1 and D.2. Each structure in the model induces a conditional distribution $F_{\rho | y, x}$ of the observables, and two structures $(\tilde{U}^1, \tilde{U}^2, \tilde{T}^1, \tilde{T}^2, \tilde{F}_{\tilde{\epsilon}_1, \tilde{\epsilon}_2 | y, \bar{x}})$ and $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$ are *observationally equivalent* if they generate the same $F_{\rho | y, x}$. The model (18) said to be *identified*, if the set of structures that are observationally equivalent to $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$ reduces to a singleton. More formally, the structure $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$ is globally identified if any observationally equivalent structure $(\tilde{U}^1, \tilde{U}^2, \tilde{T}^1, \tilde{T}^2, \tilde{F}_{\tilde{\epsilon}_1, \tilde{\epsilon}_2 | y, \bar{x}})$ satisfies:

$$\tilde{U}^i(\rho_i, \bar{x}) = U^i(\rho_i, \bar{x}), \quad \tilde{T}^i(x_i, \bar{x}) = T^i(x_i, \bar{x}), \text{ and}$$
$$\tilde{F}_{\tilde{\epsilon}_1, \tilde{\epsilon}_2 | y, \bar{x}}(t | y, \bar{x}) = F_{\epsilon_1, \epsilon_2 | y, \bar{x}}(t | y, \bar{x}),$$

for every $(\rho_1, \rho_2, x_1, x_2, t)$, a.e. $(y, \bar{x})$, and $i = 1, 2$.

Our main result is as follows:

Proposition 8 *Suppose that Assumptions U.1, T.1, as well as D.1, D.2, and T.2 hold. Then the structures $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$ and $(\tilde{U}^1, \tilde{U}^2, \tilde{T}^1, \tilde{T}^2, \tilde{F}_{\tilde{\epsilon}_1, \tilde{\epsilon}_2 | y, \bar{x}})$ are observationally equivalent if and only if there exist functions $A^1(\bar{x}) > 0$, $A^2(\bar{x}) > 0$, $B^1(\bar{x}) > 0$ and $\alpha^1(\bar{x})$, $\alpha^2(\bar{x})$, $\beta^1(\bar{x})$, $\beta^2(\bar{x})$, such that for every (r, y, x) the agents' utilities and threat functions satisfy:*

$$\tilde{U}^1(\rho_1, \bar{x}) = A^1(\bar{x}) \cdot U^1(\rho_1, \bar{x}) + \alpha^1(\bar{x}), \quad A^1(\bar{x}) > 0, \tag{19}$$

$$\tilde{U}^2(\rho_2, \bar{x}) = A^2(\bar{x}) \cdot U^2(\rho_2, \bar{x}) + \alpha^2(\bar{x}), \quad A^2(\bar{x}) > 0, \tag{20}$$

$$\tilde{T}^1(x_1, \bar{x}) = B^1(\bar{x}) \cdot T^1(x_1, \bar{x}) + \beta^1(\bar{x}), \quad B^1(\bar{x}) > 0, \tag{21}$$

$$\tilde{T}^2(x_2, \bar{x}) = A^2(\bar{x})(A^1(\bar{x}))^{-1} B^1(\bar{x}) \cdot T^2(x_2, \bar{x}) + \beta^2(\bar{x}), \tag{22}$$

and the conditional distributions of the unobservables $F_{\epsilon_1, \epsilon_2 | y, \bar{x}}$ and $\tilde{F}_{\tilde{\epsilon}_1, \tilde{\epsilon}_2 | y, \bar{x}}$ are such that for every $t \in \mathbb{R}$, every $r \in\,]0, y[$, and almost every (y, x),

$$\frac{\partial \tilde{U}^2(y - r, \bar{x})}{\partial \rho_2} \left[\tilde{U}^1(r, \bar{x}) + \tilde{\epsilon}_1 \right] - \frac{\partial \tilde{U}^1(r, \bar{x})}{\partial \rho_1} \left[\tilde{U}^2(y - r, \bar{x}) + \tilde{\epsilon}_2 \right] \underset{|(y,x)}{\sim}$$

$$A^2(\bar{x}) B^1(\bar{x}) \left\{ \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \left[U^1(r, \bar{x}) + \epsilon_1 + \frac{\beta^1(\bar{x})}{B^1(\bar{x})} \right] \tag{23} \right.$$

$$\left. - \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \left[U^2(y - r, \bar{x}) + \epsilon_2 + \frac{A^1(\bar{x})}{A^2(\bar{x})} \frac{\beta^2(\bar{x})}{B^1(\bar{x})} \right] \right\},$$

where $\underset{|(y,x)}{\sim}$ denotes equality in conditional distribution given (y, x).

A proof of Proposition 8 is in Appendix. Proposition 8 is essentially an identification result. Its main implication is that the joint distribution of the observable variables determines individual preferences and threat points (the U^i and T^i) up to affine transforms (which, obviously, can depend on $\bar{x}$). More precisely, in any two observationally equivalent structures, agents' utilities and treat functions need to be related by simple strictly increasing affine transformations involving unknown functions of $\bar{x}$ (those transformations are then "undone" at the level of the conditional distributions of the

disturbances). Moreover, there exists a relationship between the affine trans-forms (see the form of (22)). Lastly, the conditional distribution of the random terms is not identified, but if two models are observationally equivalent their respective distributions are related by condition (23).

The previous result can be clarified if we first discuss the normalizations that are needed in our framework. Clearly, if we let $\tilde{U}^1(\rho_1, \bar{x}) = U^1(\rho_1, \bar{x}) + \mu^1(\bar{x})$, $\tilde{T}^1(x_1, \bar{x}) = T^1(x_1, \bar{x}) + \nu^1(\bar{x})$ and $\tilde{\epsilon}_1 = \epsilon_1 - \mu_1(\bar{x}) - \nu_1(\bar{x})$, then the structure $(\tilde{U}^1, U^2, \tilde{T}^1, T^2, F_{\tilde{\epsilon}_1, \epsilon_2 | y, \bar{x}})$ is observationally equivalent to $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$ for any choice of functions $\mu^1(\bar{x})$ and $\nu_1(\bar{x})$. Analogous result obtains if instead of modifying the utility and threat function of player 1 we do so with player 2. Similarly, if for any $\lambda^1(\bar{x}) > 0$ we let $\bar{U}^1(\rho_1, \bar{x}) = U^1(\rho_1, \bar{x}) \lambda^1(\bar{x})$, $\bar{T}^1(x_1, \bar{x}) = T^1(x_1, \bar{x})^1 \lambda(\bar{x})$, $\bar{\epsilon}_1 = \epsilon_1 \lambda^1(\bar{x})$, then the structure $(\bar{U}^1, U^2, \bar{T}^1, T^2, \bar{F}_{\bar{\epsilon}_1, \epsilon_2 | y, \bar{x}})$ is again observationally equivalent to $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$; the same holds for player 2. We therefore impose that any U^i, T^i, and ϵ_i $(i = 1, 2)$ in (18) satisfy the following normalization conditions:

$$
\begin{array}{rl}
\text{(i) for known } \rho_i^0 \text{ and } k^i(\bar{x}), & U^i(\rho_i^0, \bar{x}) = k^i(\bar{x}), \\
\text{(ii) for known } x_i^0 \text{ and } c^i(\bar{x}), & T^i(x_i^0, \bar{x}) = c^i(\bar{x}), \\
\text{(iii) for known } \rho_i^* \text{ and } K^i(\bar{x}) > 0, & \partial U^i(\rho_i^*, \bar{x}) / \partial \rho_i = K^i(\bar{x}).
\end{array}
\tag{24}
$$

Note that, here, the values ρ_i^0, x_i^0 and the functions $k^i(\bar{x}), c^i(\bar{x})$ and $K^i(\bar{x})$ can be arbitrarily chosen.

There is a total of six normalization conditions in (24); we then obtain the following Corollary to Proposition 8.

Corollary 9 *Let the assumptions of Proposition 8 hold and assume in addition that the normalization condition (24) is satisfied. Then the struc-*

tures $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 | y, \bar{x}})$ *and* $(\tilde{U}^1, \tilde{U}^2, \tilde{T}^1, \tilde{T}^2, \tilde{F}_{\tilde{\epsilon}_1, \tilde{\epsilon}_2 | y, \bar{x}})$ *are observationally equivalent if and only if for every* $(\rho_i, \bar{x})$:

$$\tilde{U}^i(\rho_i, \bar{x}) = U^i(\rho_i, \bar{x}),$$

and there exists a function $B(\bar{x}) > 0$, *such that for every* $(x_i, \bar{x})$,

$$\tilde{T}^i(x_i, \bar{x}) - c^i(\bar{x}) = B(\bar{x}) \cdot \left[T^i(x_i, \bar{x}) - c^i(\bar{x}) \right],$$

and for every $t \in \mathbb{R}$, *every* $r \in \,]0, y[$ *and almost every* $(r, y, x_i, \bar{x})$,

$$\frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \left[U^1(r, \bar{x}) - c^1(\bar{x}) + \tilde{\epsilon}_1 \right] - \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \left[U^2(y - r, \bar{x}) - c^2(\bar{x}) + \tilde{\epsilon}_2 \right] \underset{|(y,x)}{\sim}$$
$$B(\bar{x}) \left\{ \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \left[U^1(r, \bar{x}) - c^1(\bar{x}) + \epsilon_1 \right] - \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \left[U^2(y - r, \bar{x}) - c^2(\bar{x}) + \epsilon_2 \right] \right\}.$$

It is clear from Corollary 9 that under normalization (24) alone, it is not possible to nonparametrically identify the Nash bargaining model (18). To achieve identification we still need one more restriction to pin down the function $B(\bar{x})$. For instance, we can impose an additional restriction on the disturbances ϵ_1 and ϵ_2 which would require that for $i = 1, 2$:

$$E[\epsilon_i | y, \bar{x}] = 0. \tag{25}$$

Letting $\epsilon(r, y, \bar{x}) \equiv \partial U^2(y - r, \bar{x}) / \partial \rho_2 \cdot [U^1(r, \bar{x}) - c^1(\bar{x}) + \epsilon_1] - \partial U^1(r, \bar{x}) / \partial \rho_1 \cdot [U^2(y - r, \bar{x}) - c^2(\bar{x}) + \epsilon_2]$ it then follows that

$$E[\epsilon(r, y, \bar{x}) | y, \bar{x}] = \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \left[U^1(r, \bar{x}) - c^1(\bar{x}) \right]$$
$$- \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \left[U^2(y - r, \bar{x}) - c^2(\bar{x}) \right],$$

which is a known quantity. Now consider $\tilde{\epsilon}(r, y, \bar{x}) = B(\bar{x}) \cdot \epsilon(r, y, \bar{x})$. Provided there exists a value of (r, y) for which the right hand side in the

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above equation is non-zero for all $\bar{x}$, it then follows that $E[\epsilon(r, y, \bar{x})|\, y, \bar{x}] = E[\tilde{\epsilon}(r, y, \bar{x})|\, y, \bar{x}]$, only if $B(\bar{x}) = 1$. Thus we have another Corollary to Proposition 8:

Corollary 10 *Let the assumptions of Proposition 8 hold and assume in addition that the normalization condition* (24) *and moment condition* (25) *are satisfied. Then,* (U^1, U^2, T^1, T^2) *and the conditional distribution of* $\epsilon(r, y, \bar{x})$ *given* (y, x) *are identified.*

It remains to be shown when the knowledge of the conditional distribution of $\epsilon(r, y, \bar{x})$ given (y, x) is sufficient to uniquely determine the joint distribution of ϵ_1 and ϵ_2. Note that when (r, y, x) is fixed, then $\epsilon(r, y, \bar{x})$ is simply a linear combination of ϵ_1 and ϵ_2, in which all the coefficients are known. Identification of $F_{\epsilon_1, \epsilon_2 |\, y, \bar{x}}$ from the conditional distribution of $\epsilon(r, y, \bar{x})$ can then be obtained under an additional conditional independence restriction: $\epsilon_1 \perp \epsilon_2 |\, y, \bar{x}$. This is the so-called deconvolution problem whose solution is well understood (Bonhomme and Robin, 2010). We thus obtain a final Corollary to Proposition 8:

Corollary 11 *Let the assumptions of Proposition 8 hold and assume in addition that the normalization condition* (24) *and moment condition* (25) *are satisfied, and that moreover* $\epsilon_1 \perp \epsilon_2 |\, y, \bar{x}$. *Then,* $(U^1, U^2, T^1, T^2, F_{\epsilon_1, \epsilon_2 |\, y, x})$ *is identified.*

6 Conclusion

Our main results leads to a significant qualification of the widely accepted views that "bargaining theory contains very few interesting propositions that

can be tested empirically," to quote Hamermesh (1973, p.1146). Admittedly, testability and identifiability do not obtain in the most general model. If the econometrician knows nothing about the form of utility and threat point functions, any (efficient) sharing of the pie is compatible with Nash bargaining. Nevertheless, whenever utility and threat point functions satisfy specific exclusion properties, Nash bargaining generates strong restrictions on observed behavior. Clearly, the relevance of the exclusion conditions cannot be assessed a priori but depends on the bargaining context.

Our results have potentially important consequences for basically all applications listed in introduction. Considering for instance the negotiations between a firm and its employees (or a trade union representing them), the case in which one utility function is known to be linear (as analyzed in Subsection 4.2) seems quite relevant, because the linearity assumption makes often sense on the firm's side. Profit maximization is a standard theoretical axiom, and the firm's risk neutrality can be derived from specific assumptions on, say, complete financial markets. On the contrary, workers' risk aversion is often viewed as a driving force in the design of employment contracts, so a risk neutrality assumption on the worker's side would be quite debatable. Our results suggest that such an assumption is by not needed. Not only can Nash bargaining be tested without this assumption, but the worker's preferences (and in particular her risk aversion) can in principle be identified from the outcome of the negotiation.

Similarly, regarding household decision making, our results imply that the Nash bargaining assumption, per se, implies very little beyond efficiency—a conclusion already conjectured by Chiappori (1991). More surprisingly, however, Proposition 3 suggests that mild assumptions may be sufficient to reverse this conclusion. For instance, in a model with purely private

consumption, the form of the intrahousehold sharing rule may indeed be constrained by the Nash bargaining context, even when the threat points are not explicitly specified, provided that some exclusion restrictions can be assumed to hold.[14]

Perhaps one of the most promising directions of research opened by these results regards experimental economics. The investigation of bargaining theory in experimental economics dates back to the seminal works by Siegel and Fouraker (1960). A standard problem with experiments of this type is that the observer does not know the players' preferences. As we said in Introduction, assuming linear preferences may unduly restrict the scope of the test: a joint test of Nash bargaining and linear preferences is likely to be rejected just because preferences fail to be linear—and then the rejection tells very little about the status of the Nash bargaining hypothesis. A possible solution, introduced by Roth and Malouf (1979), is to consider players who bargain about probabilities of a lottery. The idea, here, is that linearity immediately follows from the expected utility hypothesis. Note, however, that once again one jointly tests Nash bargaining and expected utility. Given that expected utility tends to be rejected in experiments, once again the status of the test (as a test of Nash bargaining) is ambiguous at best.

From this point of view, the methodology developed in this paper opens new and interesting directions for future research in this area. Consider again the simple experiment discussed in Introduction. Our main conclusion is that a cardinal representation of each agent's utility function can be identified from it. This identification does not require any form of uncertainty; in

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[14]As an example of natural exclusion restrictions, one may consider the assumption that a spouse's threat point does not depend on the spouse's wage (or does so only through some specific function - say, the form of the divorce settlement).

particular, it does not rely on the assumption that preferences under uncertainty are of VNM type. Moreover, the Nash bargaining structure generates strong testable properties for the sharing function.

The possibility of identifying a cardinal representation of individual utilities in the absence of uncertainty raises interesting perspectives. The mere fact that Nash bargaining involves cardinal representations of individual utilities (i.e., concavity matters) even in the absence of uncertainty can be given various interpretations. One of these relies on the non-cooperative foundations of Nash bargaining, which do involve randomness.[15] While interesting, this interpretation raises however several problems. First, the non-cooperative interpretation provided by Binmore, Rubinstein and Wolinsky relies on expected utility. This requirement is somewhat problematic: it is hard to see why the use of Nash bargaining should be restricted to preferences compatible with expected utility maximization, rather than more general preferences under uncertainty. Recent progress have been made in this direction by Rubinstein, Safra and Thomson (1992), who extend the interpretation of Nash-bargaining to a family of non-expected utility preferences. Still, why the definition of Nash bargaining should rely at all on preferences on lotteries is not clear. After all, non cooperative models are not the only justification of Nash bargaining, and possibly not the most convincing one. The initial definition of Nash bargaining was axiomatic; and none of the axioms used by Nash in his original contribution did rely on decision under uncertainty in any manner. In practice, Nash bargaining is used in a variety of situations, most of which involve no uncertainty.[16]

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[15]See Binmore, Rubinstein and Wolinsky(1986), and Myerson (1990, Chapter 8) for a very pedagogical presentation.

[16]Moreover, even in situations where the bargaining game is indeed non cooperative and involves uncertainty, the game at stake may fail to fit the formal structure referred to by

In other words, the interpretation just described, based on the idea that concavity of the utility function matters in Nash bargaining because Nash bargaining should be viewed as a reduced form for some non cooperative game that does involve randomness, needs not be the ultimate one. It should in particular be put in perspective with a standard claim, made by (some) tenants of non expected utility approaches, that concavity of utility has little to do with risk aversion. Decreasing marginal utility of income, it is argued, relates to psychological patterns of individual satisfaction that can be understood independently of any risk. Risk aversion, in this perspective, is a completely different issue, which is (at least in some versions) related to transformations of the probability distribution.

The theoretical discussion is stimulating, challenging and intricate. However, a very interesting question is whether some *empirical* light can be shed on the debate. In other words, could there be a way of directly testing the relationship between decreasing marginal utility of income and risk aversion? The obvious problem with such a program is that the decreasingness of marginal utility of income is hard (or impossible, it is often argued) to assess in a context of certainty. Our suggestion is that Nash bargaining may actually provide such an assessment. We believe, in other words, that it may be worth trying to take the theory literally and trying to recover the concavity of individual utilities from the observation of negotiations between agents in the absence of uncertainty. From this perspective, the tools provided by this paper may be useful precisely because they show how individual utilities can be retrieved (up to an affine transform) in a bargaining context.

Whether the level of concavity implicit in the Nash bargaining outcome

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the Binmore-Rubinstein-Wolinsky framework (alternative offers, exogenous termination probabilities, etc.)

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is correlated with the individuals' attitude toward risk is an interesting empirical question. After all, the same person may in principle be a tough negotiator and a risk averse decision maker. At any rate, an experiment should be easy to perform. It should go along the following lines:

1. face each individual of a given group with menus of lotteries, in order to assess her level of risk aversion from her choices.

2. match randomly the agents by pairs, and let them play a two-sided bargaining problem identical to the one discussed in Introduction; use the theoretical approach described in this paper to recover their utility functions.

3. compare the two sets of results. According to the standard interpretation, more risk averse individuals, being characterized by more concave VNM utilities, should perform poorly in the bargaining stage; an empirical check of this prediction would be quite illuminating.

4. Interestingly enough, this approach has various by-products. For instance, the idea that risk aversion has more to do with probability transformation than with decreasing marginal utility of consumption could be taken to data in a systematic way: if one believes that the concavity retrieved from the second phase has general relevance, then it can be plugged into the first stage to recover possible probability transformations.

Experiments of this kind will be the topic of future work.

A Proofs of Propositions

Proof of Proposition 3. The proof of Proposition 3 is in two steps. The first establishes the necessity and the second shows the sufficiency of the conditions (6)-(10).

Step 1. Necessity: If the first order condition (4) is differentiated with respect to y and x_{1i}, $i = 1, \ldots, n_1$, after rearrangement, one gets:

$$\left(\frac{\partial R^1}{\partial \rho_1} + \frac{\partial R^2}{\partial \rho_2} \right) \left(1 - \frac{\partial \rho}{\partial y} \right) = \frac{\partial R^1}{\partial \rho_1}, \qquad (26)$$

$$\left(\frac{\partial R^1}{\partial \rho_1} + \frac{\partial R^2}{\partial \rho_2} \right) \frac{\partial \rho}{\partial x_{1i}} = -\frac{\partial R^1}{\partial x_{1i}}. \qquad (27)$$

Firstly, it is easily shown that $\partial R^i / \partial \rho_i < 0$ for $i = 1, 2$ since the functions U^i are strictly increasing and concave in ρ_i. Hence, from (26),

$$\frac{\partial \rho}{\partial y} = \frac{\partial R^2 / \partial \rho_2}{\partial R^1 / \partial \rho_1 + \partial R^2 / \partial \rho_2} \in \,]0, 1[,$$

which demonstrates the necessity of the condition (6). Secondly, from (26) and (27) together, one also gets:

$$\frac{\partial \rho / \partial x_{1i}}{1 - \partial \rho / \partial y} = -\frac{\partial R^1 / \partial x_{1i}}{\partial R^1 / \partial \rho_1},$$

where the right-hand side is a function Φ_i of $(\rho, x_1, \bar{x})$ alone. This shows that the condition (7) is necessary as well. Working out cross derivative restrictions gives (9).

Finally, differentiating the first order condition (4) with respect to y and x_{2j}, $j = 1, \ldots, n_2$, then taking ratios, one gets:

$$\frac{\partial \rho / \partial x_{2j}}{\partial \rho / \partial y} = \frac{\partial R^2 / \partial x_{2j}}{\partial R^2 / \partial \rho_2},$$

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where the right-hand side is now a function Ψ_j of $(y - \rho, x_2, \bar{x})$ alone. This demonstrates that condition (8) is also necessary. Again, working out cross derivative restrictions gives (10)

Step 2. Sufficiency: Consider first the case of agent 1. If condition (7) is fulfilled, each ratio

$$\frac{\partial \rho / \partial x_{1i}}{1 - \partial \rho / \partial y},$$

for every $i = 1, \ldots, n_1$, can be written as some function Φ^i of $(\rho, x_1, \bar{x})$. Then define a function $R^1(\rho_1, x_1, \bar{x})$ as a solution to the system of partial differential equations

$$\frac{\partial R^1 / \partial x_{1i}}{\partial R^1 / \partial \rho_1} = -\Phi^i(\rho, x_1, \bar{x}), \tag{28}$$

for $i = 1, \ldots, n_1$, with $\partial R^1 / \partial \rho_1 \neq 0$. If (9) is satisfied, the system of equations (28) defines $R^1(\rho_1, x_1, \bar{x})$ up to an increasing transform $G(\cdot, \bar{x})$, i.e. for some known function $\bar{R}^1(\rho_1, x_1, \bar{x})$ that solves (28) we have

$$R^1(\rho_1, x_1, \bar{x}) = G\big(\bar{R}^1(\rho_1, x_1, \bar{x}), \bar{x}\big).$$

Now, by construction, $\bar{R}^1$ (and therefore any transform of $\bar{R}^1$) can be written as a function of $(\rho_2, x_2, \bar{x})$ - say, $R^2(\rho_2, x_2, \bar{x})$.

Finally, consider the equation:

$$R^i(\rho_i, x_i, \bar{x}) \equiv \frac{\partial U^i(\rho_i, x_i, \bar{x}) / \partial \rho_i}{U^i(\rho_i, x_i, \bar{x}) - T^i(x_i, \bar{x})}$$

as a PDE in $U^i - T^i$. It can readily be integrated into:

$$U^i(\rho_i, x_i, \bar{x}) - T^i(x_i, \bar{x}) = K_i \exp\left(\int_0^{\rho_i} R^i(s_i, x_i, \bar{x}) ds_i\right) \tag{29}$$

where K_i is an arbitrary constant. One can pick up an arbitrary $T^i(x_i, \bar{x})$, then U^i is defined by the previous equation. ∎

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Proof of Proposition 4. Consider the system of equations, which is satisfied for any $(y, x) \in \mathcal{F}$,

$$U^1 (\rho, x_1, \bar{x}) - T^1 (x_1, \bar{x}) = 0, \tag{30}$$

$$U^2 (y - \rho, x_2, \bar{x}) - T^2 (x_2, \bar{x}) = 0, \tag{31}$$

which implicitly defines ρ and y as a function of x. Inverting (30) with respect to ρ yields:

$$\rho = \sigma^1 (x_1, \bar{x}). \tag{32}$$

Hence, the sharing function is independent of x_2 and y along the agreement frontier. Similarly, inverting (31) with respect to $y - \rho$ yields:

$$y - \rho = \sigma^2 (x_2, \bar{x}). \tag{33}$$

Then, substituting Equation (32) into Equation (33) proves that $\sigma(x)$ is additive in the sense of the proposition. ∎

Proof of Proposition 5. The proof of Proposition 5 is a direct consequence of equations (32). We have $\sigma_1(x_1, x) = \rho(\sigma_1(x_1, \bar{x}) + \sigma_2(x_2, \bar{x}), x_1, x_2, \bar{x})$ so differentiating this expression with respect to x_{1i} and x_{2j} immediately gives the desired conditions. ∎

Proof of Proposition 6. The proof directly follows from that of Proposition 3. Indeed, we have seen that the G function in that proof can be arbitrarily chosen. For each particular choice, R^1 and R^2 are exactly determined, and so are U^1 and U^2 (up to an affine transform) by (29). ∎

Proof of Proposition 7. From the previous results, we just need to show that under the additional assumption U2, the function G is uniquely determined, leading to the identification of of U^i and T^i up to an affine transform. This is a consequence of the following lemma:

Lemma 1 *Let $F(x, y)$ be a given function, and assume that for some functions B, C, G the following equation is satisfied:*

$$G(F(x, y)) = \frac{B'(x)}{B(x) - C(y)}$$

Assume that F is such that $\partial F(x, y) / \partial x \neq 0, \partial F(x, y) / \partial y \neq 0$, and $B(x)$ is not exponential. Then B and C are identified from F up to the same affine transform.

Proof of Lemma 1. In this proof, the notation f_x stands for the differential of function f with respect to variable x; the notation f' is used when f has only one argument. Note first that $G(F(x, y))$ is of the form:

$$G(F(x, y)) = \frac{A(x)}{B(x) - C(y)},$$

where $A(x) = B'(x)$. It follows that

$$\frac{F_x(x, y)}{F_y(x, y)} = \frac{A'(x)(B(x) - C(y)) - A(x)B'(x)}{A(x)C'(y)}. \tag{34}$$

Define:

$$\phi(x, y) = \frac{F_x(x, y)}{F_y(x, y)}.$$

Note that ϕ is a known function, i.e., it does not depend on G, and is such that

$$\log \phi(x, y) = \log\left(\frac{\partial F(x, y)}{\partial x}\right) - \log\left(\frac{\partial F(x, y)}{\partial y}\right).$$

Consider Equation (34) as an equation in $A = B', B, C$. We now show that generically on ϕ, this equation identifies B, C up to an affine transform. We now distinguish two cases, depending on whether $\log \phi(x, y) = \log(F_x(x, y)) - \log(F_y(x, y))$ is additively separable in x and y or not.

Case 1 (general case): $\log \phi(x, y)$ **is not additively separable in** x **and** y**.**

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The proof goes in 3 steps.

Step 1: Define $v(x) = A'(x)/A(x)$ and $w(x) = v(x)B(x) - B'(x)$, then Equation (34) becomes:

$$\phi(x,y) C'(y) + v(x) C(y) = w(x). \tag{35}$$

Differentiating with respect to x yields:

$$\phi_x(x,y) C'(y) + v'(x) C(y) = w'(x). \tag{36}$$

If

$$\phi_x(x,y) v(x) - \phi(x,y) v'(x) = 0,$$

then $\phi(x,y) = D(y) v(x)$ for some function D and $\log \phi(x,y)$ is additively separable in x and y which contradicts the assumption. Hence the expression is non zero, and from equations (35) and (36) one gets:

$$C(y) = \frac{\phi_x(x,y) w(x) - \phi(x,y) w'(x)}{\phi_x(x,y) v(x) - \phi(x,y) v'(x)}, \tag{37}$$

$$C'(y) = \frac{v(x) w'(x) - v'(x) w(x)}{\phi_x(x,y) v(x) - \phi(x,y) v'(x)}. \tag{38}$$

A first necessary condition expresses the fact that the derivative of the right-hand-side of Equation (37) equals the right-hand-side of Equation (38). This gives either

$$w'(x) v(x) - v'(x) w(x) = 0$$

or

$$\phi_x(x,y) v(x) + \phi_y(x,y) \phi_x(x,y) - \phi_{xy}(x,y) \phi(x,y) - \phi(x,y) v'(x) = 0.$$

If $(w'(x) v(x) - v'(x) w(x)) = 0$, then $C'(y) = 0$ and $F_y(x,y) = 0$, which is excluded by assumption. Hence

$$\phi_x(x,y) v(x) - \phi(x,y) v'(x) = \phi_{xy}(x,y) \phi(x,y) - \phi_y(x,y) \phi_x(x,y). \tag{39}$$

Step 2: Differentiating Equation (39) with respect to y gives

$$\phi_{xy}(x,y)\, v(x) - \phi_y(x,y)\, v'(x) = \phi_{xyy}(x,y)\, \phi(x,y) - \phi_{yy}(x,y)\, \phi_x(x,y).$$
(40)

If

$$\phi_y(x,y)\, \phi_x(x,y) = \phi_{xy}(x,y)\, \phi(x,y),$$

then $\phi(x,y)$ is of the form $D(x) \cdot E(y)$ and again $\log \phi(x,y)$ is additively separable in x and y which contradicts the assumption. Hence the system (39) and (40) allows to recover v:

$$v(x) = \frac{\phi_{xyy}\phi^2 + \phi_y^2\phi_x - \phi\phi_{xy}\phi_y - \phi\phi_{yy}\phi_x}{\phi_{xy}\phi - \phi_y\phi_x}.$$
(41)

This defines $A(x)$ up to some multiplicative constant a, and generate testable conditions since the right hand side cannot depend on y.

Step 3: In our case, $A(x) = B'(x)$, so $B(x)$ is identified from $A(x)$ up to some additive constant b:

$$B(x) = a\bar{B}(x) + b$$

where $\bar{B}(x)$ is known; $B(x)$ is thus identified up to an affine transform. Finally, since $w(x) = v(x)\, B(x) - B'(x) = a\left(\bar{B}(x)\, v(x) - \bar{B}'(x)\right) + bv(x)$, Equation (37) becomes

$$C(y) = a\frac{\phi_x\left(\bar{B}v - \bar{B}'\right) - \phi\left(\bar{B}'v + \bar{B}v' - \bar{B}''\right)}{\phi_x v - \phi v'} + b$$

and $C(y)$ is identified up to the same affine transform.

Case 2 (particular case): $\log \phi(x,y)$ **is additively separable in** x **and** y**.**

Then $\frac{\partial^2 \log \phi(x,y)}{\partial x \partial y} = 0$; since

$$\phi(x,y) = \frac{A'(x)B(x) - A(x)B'(x)}{A(x)} \frac{1}{C'(y)} - \frac{A'(x)}{A(x)} \frac{C(y)}{C'(y)}$$

this implies either

$$\frac{C(y)}{C'(y)} = \frac{N}{C'(y)}$$

or

$$A'(x)B(x) - A(x)B'(x) = 0$$

or

$$A'(x) = 0$$

or

$$\frac{A'(x)B(x) - A(x)B'(x)}{A(x)} = N\frac{A'(x)}{A(x)}$$

for some constant N. The first relation implies that $C(y) = N$, hence $F_y(x,y) = 0$, a contradiction. The second, with the fact that $A(x) = B'(x)$, implies that B is exponential or linear. The third implies that $A(x)$ is constant and $B(x)$ linear. Finally, the fourth case gives

$$A'(x)(B(x) - N) = A(x)B'(x)$$

hence

$$A(x) = a(B(x) - N)$$

for some constant a. Since $A(x) = B'(x)$, finally

$$B(x) = \mu e^{ax} + N$$

and $B(x)$ is exponential. Note that, in that case, identification does not hold. Indeed, if $F(x) = e^x / (e^x + C(y))$ and $G(u) = a/\left(1 + \left(\frac{1-u}{u}\right)^a\right)$, then

$$G\left(\frac{e^x}{e^x + C(y)}\right) = \frac{ae^{ax}}{e^{ax} + C(y)^a}$$

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and the right-hand-side is also of the required form with $B(x)$ exponential with a different coefficient. ∎

Proof of Proposition 7 continued. We now show that the lemma implies Proposition 7. As before, define R^1 and R^2 by

$$R^i\left(\rho_i, x_i\right) = \frac{\partial U^i/\partial \rho_i}{U^i\left(\rho_i\right) - T^i\left(x_i\right)}.$$

From Proposition 6, we know that R^i is identified up to some increasing transform; i.e., there exists some known function $\bar{R}^i$ such that:

$$G\left(\bar{R}^i\left(\rho_i, x_i\right)\right) = R^i\left(\rho_i, x_i\right) = \frac{\partial U^i/\partial \rho_i}{U^i\left(\rho_i\right) - T^i\left(x_i\right)}$$

for some G. It remains to be shown that given the particular form at stake, the knowledge of R^i up to an increasing transform is sufficient to identify U^i and T^i. Clearly, Lemma 1 immediately implies the conclusion. Moreover, if B and C are identified up to the same affine transform, then $G\left(R\left(x,y\right)\right)$ is exactly identified, hence G as well. Finally, note that all the arguments presented above are valid for any given values of the $\bar{x}$ variables; in particular, the affine transform may depend on $\bar{x}$. ∎

Proof of Proposition 8. The first order condition is:

$$R^1(\rho, x_1, \epsilon_1, \bar{x}) = R^2(y - \rho, x_2, \epsilon_2, \bar{x}) \tag{42}$$

where

$$R^i(\rho_i, x_i, \epsilon_i, \bar{x}) \equiv \frac{\partial U^i(\rho_i, \bar{x})/\partial \rho_i}{U^i(\rho_i, \bar{x}) - T^i(x_i, \bar{x}) + \epsilon_i}.$$

Since $\partial R^i/\partial \rho_i < 0$, Equation (42) implicitly defines a unique solution

$$\rho = \rho(y, x_1, x_2, \epsilon_1, \epsilon_2, \bar{x}).$$

Consider any $r \in \,]0, y\,[$ and note that we have $\rho(y, x_1, x_2, \epsilon_1, \epsilon_2, \bar{x}) \leqslant r$ if and only if

$$R^1(r, x_1, \epsilon_1, \bar{x}) - R^2(y - r, x_2, \epsilon_2, \bar{x}) \geqslant 0,$$

that is

$$\frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \left[U^1(r, \bar{x}) + \epsilon_1 \right] - \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \left[U^2(y - r, \bar{x}) + \epsilon_2 \right] \leqslant$$
$$- \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} T^2(x_2, \bar{x}) + \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} T^1(x_1, \bar{x}). \quad (43)$$

Now let

$$\Phi(r, y, x_1, x_2, \bar{x}) \equiv \Pr \left\{ \rho \leqslant r \,\middle|\, y, x_1, x_2, \bar{x} \right\}$$

be the conditional distribution of the shares ρ, observed for given $(y, x_1, x_2, \bar{x})$. Moreover, let

$$G(r, y, x_1, x_2, \bar{x}) \equiv -\frac{\partial U^1(r, \bar{x})}{\partial \rho_1} T^2(x_2, \bar{x}) + \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} T^1(x_1, \bar{x}) \quad (44)$$

$$F(t, r, y, \bar{x}) \equiv \Pr \left\{ \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \left[U^1(r, \bar{x}) + \epsilon_1 \right] \right.$$
$$\left. - \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \left[U^2(y - r, \bar{x}) + \epsilon_2 \right] \leqslant t \,\middle|\, y, x_1, x_2, \bar{x} \right\}. \quad (45)$$

Note that the probability $F(t, r, y, \bar{x})$ does not depend on (x_1, x_2) because (i) (x_1, x_2) do not enter U^1 nor U^2, and (ii) (ϵ_1, ϵ_2) is conditionally independent of (x_1, x_2) (Assumption D.1). Then, we have that:

$$\Phi(r, y, x_1, x_2, \bar{x}) = F\big(G(r, y, x_1, x_2, \bar{x}), r, y, \bar{x}\big). \quad (46)$$

In particular, if $x_1 = (x_{11}, \ldots, x_{1n_1})$, then for every $1 \leqslant i \leqslant n_1$, we have:

$$\frac{\partial \Phi(r, y, x_1, x_2, \bar{x})}{\partial x_{1i}} = \frac{\partial T^1(x_1, \bar{x})}{\partial x_{1i}} \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \frac{\partial F\big(G(r, y, x_1, x_2, \bar{x}), r, y, \bar{x}\big)}{\partial t}.$$
$$(47)$$

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Similarly, if $x_2 = (x_{21}, \ldots, x_{2n_2})$, then for every $1 \leqslant j \leqslant n_2$, we have:

$$\frac{\partial \Phi(r, y, x_1, x_2, \bar{x})}{\partial x_{2j}} = -\frac{\partial T^2(x_2, \bar{x})}{\partial x_{2j}} \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \frac{\partial F\big(G(r, y, x_1, x_2, \bar{x}), r, y, \bar{x}\big)}{\partial t}. \quad (48)$$

In particular, we focus on (47) when $i = 1$ and on (48) when $j = 1$. Note that under Assumption T.2 we have

$$\frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} \neq 0 \text{ and } \frac{\partial T^2(x_2, \bar{x})}{\partial x_{21}} \neq 0 \quad \text{for all } x = (x_1, x_2, \bar{x}),$$

while Assumption D.2 ensures that $\partial F(t, r, y, \bar{x})/\partial t > 0$ for every $t \in \mathbb{R}$. Taking ratios of (47) and (48) obtained for $i = j = 1$ we then have:

$$\frac{\partial \Phi(r, y, x_1, x_2, \bar{x})/\partial x_{11}}{\partial \Phi(r, y, x_1, x_2, \bar{x})/\partial x_{21}} = -\frac{\partial T^1(x_1, \bar{x})/\partial x_{11}}{\partial T^2(x_2, \bar{x})/\partial x_{21}} \frac{\partial U^2(y - r, \bar{x})/\partial \rho_2}{\partial U^1(r, \bar{x})/\partial \rho_1}$$

so

$$\log \left| \frac{\partial \Phi(r, y, x_1, x_2, \bar{x})/\partial x_{11}}{\partial \Phi(r, y, x_1, x_2, \bar{x})/\partial x_{21}} \right| =$$
$$\log \left| \frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} \right| - \log \left| \frac{\partial T^2(x_2, \bar{x})}{\partial x_{21}} \right| + \log \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} - \log \frac{\partial U^1(r, \bar{x})}{\partial \rho_1}$$

for every (y, r) and every $x = (x_1, x_2, \bar{x})$. Now consider the change of variables $\rho_1 \equiv r$ and $\rho_2 \equiv y - r$. We then obtain that for every (ρ_1, ρ_2) and every $x = (x_1, x_2, \bar{x})$,

$$s(\rho_1, \rho_2, x_1, x_2, \bar{x}) = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (49)$$
$$\log \left| \frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} \right| - \log \left| \frac{\partial T^2(x_2, \bar{x})}{\partial x_{21}} \right| + \log \frac{\partial U^2(\rho_2, \bar{x})}{\partial \rho_2} - \log \frac{\partial U^1(\rho_1, \bar{x})}{\partial \rho_1}$$

where we have let

$$s(\rho_1, \rho_2, x_1, x_2, \bar{x}) \equiv \log \left| \frac{\partial \Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})/\partial x_{11}}{\partial \Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})/\partial x_{21}} \right|.$$

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Step 1: Identification of U^1. Differentiating (49) with respect to ρ_1 gives:

$$\frac{\partial s(\rho_1, \rho_2, x_1, x_2, \bar{x})}{\partial \rho_1} = -\frac{\partial}{\partial \rho_1} \log \frac{\partial U^1(\rho_1, \bar{x})}{\partial \rho_1}.$$

Integrating from some $\rho_1^* \in]0, y[$ we then obtain:

$$\frac{\partial U^1(\rho_1, \bar{x})}{\partial \rho_1} = K^1(\bar{x}) \exp\left[-\int_{\rho_1^*}^{\rho_1} \frac{\partial s(u, \rho_2, x_1, x_2, \bar{x})}{\partial \rho_1} du\right],$$

where $K^1(\bar{x}) \equiv \partial U^1(\rho_1^*, \bar{x})/\partial \rho_1 > 0$ is an unknown function. We can again integrate from some $\rho_1^0 \in]0, y[$ which gives:

$$U^1(\rho_1, \bar{x}) = K^1(\bar{x}) \int_{\rho_1^0}^{\rho_1} \exp\left[-\int_{\rho_1^*}^{v} \frac{\partial s(u, \rho_2, x_1, x_2, \bar{x})}{\partial \rho_1} du\right] dv + k^1(\bar{x})$$

where $k^1(\bar{x}) \equiv U^1(\rho_1^0, \bar{x})$ is unknown. Hence, the utility of agent 1 is determined up to a strictly increasing affine transformation (in $\bar{x}$) of a known utility function $\bar{U}^1(\rho_1, \bar{x})$:

$$U^1(\rho_1, \bar{x}) = K^1(\bar{x}) \cdot \bar{U}^1(\rho_1, \bar{x}) + k^1(\bar{x}), \quad K^1(\bar{x}) > 0, \tag{50}$$

where we have let

$$\bar{U}^1(\rho_1, \bar{x}) \equiv \int_{\rho_1^0}^{\rho_1} \exp\left[-\int_{\rho_1^*}^{v} \frac{\partial s(u, \rho_2, x_1, x_2, \bar{x})}{\partial \rho_1} du\right] dv.$$

Step 2: Identification of U^2. Differentiating (49) with respect to ρ_2 gives:

$$\frac{\partial s(\rho_1, \rho_2, x_1, x_2, \bar{x})}{\partial \rho_2} = \frac{\partial}{\partial \rho_2} \log \frac{\partial U^2(\rho_2, \bar{x})}{\partial \rho_2}.$$

Following the same reasoning as above and integrating twice from some $(\rho_2^*, \rho_2^0) \in]0, y[^2$, we have:

$$U^2(\rho_2, \bar{x}) = K^2(\bar{x}) \int_{\rho_2^0}^{\rho_1} \exp\left[\int_{\rho_2^*}^{v} \frac{\partial s(\rho_1, u, x_1, x_2, \bar{x})}{\partial \rho_2} du\right] dv + k^2(\bar{x})$$

where the functions $k^2(\bar{x}) \equiv U^2(\rho_2^0, \bar{x})$ and $K^2(\bar{x}) \equiv \partial U^2(\rho_2^*, \bar{x})/\partial \rho_2 > 0$ are unknown. Hence, the utility of agent 2 is also determined up to a strictly increasing affine transformation (in $\bar{x}$) of a known utility function $\bar{U}^2$,

$$U^2(\rho_2, \bar{x}) = K^2(\bar{x}) \cdot \bar{U}^2(\rho_2, \bar{x}) + k^2(\bar{x}), \quad K^2(\bar{x}) > 0, \tag{51}$$

where we have let

$$\bar{U}^2(\rho_2, \bar{x}) \equiv \int_{\rho_2^0}^{\rho_1} \exp\left[\int_{\rho_2^*}^{v} \frac{\partial s(\rho_1, u, x_1, x_2, \bar{x})}{\partial \rho_2} du\right] dv.$$

Step 3: Identification of T^1. Differentiating (49) with respect to x_{1k} $(1 \leqslant k \leqslant n_1)$, we get that:

$$\frac{\partial s(\rho_1, \rho_2, x_1, x_2, \bar{x})}{\partial x_{1k}} = \frac{\partial}{\partial x_{1k}} \log \left| \frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} \right|.$$

In particular, consider the partial derivative with respect to x_{11}, i.e. $k = 1$. For some c_{11} define:

$$t_1(\bar{x}, x_1) \equiv \int_{c_{11}}^{x_{11}} \frac{\partial s(\rho_1, \rho_2, u, x_{12}, \ldots, x_{1n_1}, x_2, \bar{x})}{\partial x_{11}} du.$$

Then, we have that:

$$\log \left| \frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} \right| = t_1(x_1, \bar{x}) + g_1(x_{12}, \ldots, x_{1n_1}, \bar{x}), \tag{52}$$

where $g_1(x_{12}, \ldots, x_{1n_1}, \bar{x})$ is an unknown function. Differentiating with respect to x_{12} gives:

$$\frac{\partial}{\partial x_{12}} \log \left| \frac{\partial T^1(\bar{x}, x_1)}{\partial x_{11}} \right| - \frac{\partial t_1(\bar{x}, x_1)}{\partial x_{12}} = \frac{\partial}{\partial x_{12}} g_1(x_{12}, \ldots, x_{1n_1}, \bar{x}),$$

that is

$$\frac{\partial}{\partial x_{12}} g_1(x_{12}, \ldots, x_{1n_1}, \bar{x}) = \frac{\partial s(\rho_1, \rho_2, x_1, x_2, \bar{x})}{\partial x_{12}} - \int_{c_{11}}^{x_{11}} \frac{\partial^2 s(\rho_1, \rho_2, u, x_{12}, \ldots, x_{1n_1}, x_2, \bar{x})}{\partial x_{11} \partial x_{12}} du$$

$$\equiv \sigma_2(x_{12}, \ldots, x_{1n_1}, \bar{x}).$$

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Note that the function $\sigma_2(x_{12}, \ldots, x_{1n_1}, \bar{x})$ on the right hand side of the above equality is known; hence, we can integrate with respect to x_{12} to get:

$$g_1(x_{12}, \ldots, x_{1n_1}, \bar{x}) = \int_{c_{12}}^{x_{12}} \sigma_2(u, x_{13}, \ldots, x_{1n_1}, \bar{x})du + g_2(x_{13}, \ldots, x_{1n_1}, \bar{x})$$

$$\equiv t_2(x_{12}, \ldots, x_{1n_1}, \bar{x}) + g_2(x_{13}, \ldots, x_{1n_1}, \bar{x}).$$

Plugging back into (52) we get that:

$$\log\left|\frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}}\right| = t_1(x_{11}, \ldots, x_{1n_1}, \bar{x}) + t_2(x_{12}, \ldots, x_{1n_1}, \bar{x}) + g_2(x_{13}, \ldots, x_{1n_1}, \bar{x}).$$

Repeating the same reasoning as above for x_{13} etc all the way to x_{1n_1} we get that:

$$\log\left|\frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}}\right| = t^1(x_1, \bar{x}) + g^1(\bar{x}),$$

where the known function $t^1(x_1, \bar{x})$ is defined as the sum of the recursively computed functions $t_k(x_{1k}, \ldots, x_{1n_1}, \bar{x})$, and $g^1(\bar{x})$ is the unknown residual function. Since from (47) we know that the sign of $\partial T^1(x_1, \bar{x})/\partial x_{11}$ is the same as that of $\partial \Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})/\partial x_{11}$, the above implies:

$$\frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} = C^1(\bar{x}) \cdot \mathrm{sgn}\left(\frac{\Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})}{\partial x_{11}}\right) \cdot \exp\left[t^1(x_1, \bar{x})\right],$$

where $C^1(\bar{x}) = \exp[g^1(\bar{x})] > 0$ is an unknown function. This gives that:

$$\frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} = C^1(\bar{x}) \cdot \tau(x_1, \bar{x}) \tag{53}$$

where

$$\tau(x_1, \bar{x}) \equiv \mathrm{sgn}\left(\frac{\Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})}{\partial x_{11}}\right) \cdot \exp\left[t^1(x_1, \bar{x})\right].$$

Integrating (53) with respect to x_{11} from some constant d_{11} then gives:

$$T^1(x_1, \bar{x}) = C^1(\bar{x}) \cdot \int_{d_{11}}^{x_{11}} \tau(u, x_{12}, \ldots, x_{1n_1}, \bar{x})du + D_1(x_{12}, \ldots, x_{1n_1}, \bar{x}),$$

for some unknown function $D_1(x_{12}, \ldots, x_{1n_1}, \bar{x})$. Differentiating the above with respect to x_{12} then gives that:

$$\frac{\partial}{\partial x_{12}} D_1(x_{12}, \ldots, x_{1n_1}, \bar{x}) = \tag{54}$$

$$\frac{\partial T^1(x_1, \bar{x})}{\partial x_{12}} - C^1(\bar{x}) \cdot \int_{d_{11}}^{x_{11}} \frac{\partial \tau(u, x_{12}, \ldots, x_{1n_1}, \bar{x})}{\partial x_{12}} du.$$

Now consider again (47); taking the ratio of the expression obtained for $i = 2$ and $i = 1$ gives:

$$\frac{\partial T^1(x_1, \bar{x})}{\partial x_{12}} = \frac{\partial T^1(x_1, \bar{x})}{\partial x_{11}} \cdot \frac{\partial \Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})/\partial x_{12}}{\partial \Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})/\partial x_{11}}. \tag{55}$$

Combining (55) with (53) then shows that $\partial T^1(x_1, \bar{x})/\partial x_{12}$ is known up to a multiplication by the same function $C^1(\bar{x})$; this means that the right hand side of (54) is known up to a multiplication by the unknown function $C^1(\bar{x}) > 0$. We can then integrate with respect to x_{12}. Following the same recursive reasoning as before, it follows that:

$$T^1(x_1, \bar{x}) = C^1(\bar{x}) \cdot \bar{T}^1(x_1, \bar{x}) + c^1(\bar{x}), \quad C^1(\bar{x}) > 0, \tag{56}$$

where the function $\bar{T}^1(x_1, \bar{x})$ is known (and defined recursively), while $C^1(\bar{x}) > 0$ and $c^1(\bar{x})$ are unknown. This means that agent 1's threat function T^1 is determined up to an increasing affine transformation in $\bar{x}$.

Step 4. Identification of T^2. Now, consider again (49) and combine it with the expressions for T^1, U^1 and U^2 obtained in (56), (50), (51), respectively. It follows that for some known function $t^2(x_2, \bar{x})$, we have:

$$\left| \frac{\partial T^2(x_2, \bar{x})}{\partial x_{21}} \right| = \frac{C^1(\bar{x}) K^2(\bar{x})}{K^1(\bar{x})} \cdot \exp\left[t^2(x_2, \bar{x}) \right],$$

so using again (48) we get,

$$\frac{\partial T^2(x_2, \bar{x})}{\partial x_{21}} = \frac{C^1(\bar{x}) K^2(\bar{x})}{K^1(\bar{x})} \cdot \text{sgn}\left(-\frac{\Phi(\rho_1, \rho_1 + \rho_2, x_1, x_2, \bar{x})}{\partial x_{21}} \right) \cdot \exp\left[t^2(x_2, \bar{x}) \right]$$

Following the same steps as in Step 3 it then follows that:

$$T^2(x_2, \bar{x}) = \frac{C^1(\bar{x})K^2(\bar{x})}{K^1(\bar{x})} \bar{T}^2(x_2, \bar{x}) + c^2(\bar{x}), \tag{57}$$

in which the function $\bar{T}^2(x_2, \bar{x})$ is known, the function $c^2(\bar{x})$ is unknown, and $C^1(\bar{x}) > 0$, $K^1(\bar{x}) > 0$ and $K^2(\bar{x}) > 0$ are the same unknown functions obtained in (56), (50), (51), respectively. In particular, this means that agent 2's threat function is determined up to an unknown increasing affine transformation (in $\bar{x}$) whose slope depends on those obtained for T^1, U^1, and U^2.

Step 5. Identification of F. Combining the inequality in (43) with the expressions for T^2, T^1, U^2, U^1 obtained in (57), (56), (51) and (50), respectively, we get that $\rho(y, x_1, x_2, \epsilon_1, \epsilon_2, \bar{x}) \leqslant r$ if and only if:

$$\frac{K^1(\bar{x})}{C^1(\bar{x})} \left\{ \frac{\partial \bar{U}^2(y - r, \bar{x})}{\partial \rho_2} \left[\bar{U}^1(r, \bar{x}) + \frac{k^1(\bar{x}) - c^1(\bar{x}) + \epsilon_1}{K^1(\bar{x})} \right] \right.$$
$$\left. - \frac{\partial \bar{U}^1(r, \bar{x})}{\partial \rho_1} \left[\bar{U}^2(y - r, \bar{x}) + \frac{k^2(\bar{x}) - c^2(\bar{x}) + \epsilon_2}{K^2(\bar{x})} \right] \right\} \leqslant$$
$$- \frac{\partial \bar{U}^1(r, \bar{x})}{\partial \rho_1} \bar{T}^2(x_2, \bar{x}) + \frac{\partial \bar{U}^2(y - r, \bar{x})}{\partial \rho_2} \bar{T}^1(x_1, \bar{x}).$$

Let then

$$\bar{G}(y, r, x_1, x_2, \bar{x}) \equiv - \frac{\partial \bar{U}^1(r, \bar{x})}{\partial \rho_1} \bar{T}^2(x_2, \bar{x}) + \frac{\partial \bar{U}^2(y - r, \bar{x})}{\partial \rho_2} \bar{T}^1(x_1, \bar{x}) \tag{58}$$

be the quantity on the right hand side of the above inequality; note that the function $\bar{G}(y, r, x_1, x_2, \bar{x})$ is known. Similar to previously, let also

$$\bar{F}(t, r, y, \bar{x}) \equiv \Pr \left\{ \frac{K^1(\bar{x})}{C^1(\bar{x})} \left(\frac{\partial \bar{U}^2(y - r, \bar{x})}{\partial \rho_2} \left[\bar{U}^1(r, \bar{x}) + \frac{k^1(\bar{x}) - c^1(\bar{x}) + \epsilon_1}{K^1(\bar{x})} \right] \right. \right. \tag{59}$$
$$\left. \left. - \frac{\partial \bar{U}^1(r, \bar{x})}{\partial \rho_1} \left[\bar{U}^2(y - r, \bar{x}) + \frac{k^2(\bar{x}) - c^2(\bar{x}) + \epsilon_2}{K^2(\bar{x})} \right] \right) \leqslant t \, \middle| \, y, x_1, x_2, \bar{x} \right\}.$$

Then, we have that for every $r \in \ ]0, y[$ and every (y, x):

$$\Phi(r, y, x_1, x_2, \bar{x}) = \bar{F}\big(\bar{G}(y, r, x_1, x_2, \bar{x}), r, y, \bar{x}\big), \qquad (60)$$

where $\Phi(r, y, x_1, x_2, \bar{x}) = \Pr\{\rho \leqslant r \,|\, y, x_1, x_2, \bar{x}\}$ as before. We now show that the above equality determines $\bar{F}(t, r, y, \bar{x})$ for all $t \in \mathbb{R}$. For this, fix $(r, y, x_{12}, \ldots, x_{1n_1}, x_2, \bar{x})$ and note that under Assumption T.2 $\bar{G}$ is strictly increasing in x_{11}. Moreover, we have that $\lim_{|x_{11}| \to \infty} |\bar{G}(y, r, x_1, x_2, \bar{x})| = \infty$. This means that for any $t \in \mathbb{R}$, we have $\bar{G}(y, r, x_1, x_2, \bar{x}) = t$ if and only if

$$x_{11} = (\bar{T}^1)^{-1} \left(\left[\frac{\partial \bar{U}^2 (y - r, \bar{x})}{\partial \rho_2} \right]^{-1} \left[t + \frac{\partial \bar{U}^1(r, \bar{x})}{\partial \rho_1} \bar{T}^2(x_2, \bar{x}) \right], x_{12}, \ldots, x_{1n_1}, \bar{x} \right) \equiv x_{11}(t).$$

Now, we can invert (60) to show that for any $t \in \mathbb{R}$,

$$\bar{F}(t, r, y, \bar{x}) = \Phi(r, y, x_{11}(t), x_{12}, \ldots, x_{1n_1}, x_2, \bar{x}), \qquad (61)$$

which is a known function.

Step 6. Observational Equivalence. We now use the results obtained in (50), (51), (56), (57) and (61) to characterize any two observationally equivalent structures. We start with agent 1's utilities: from (50) two observationally equivalent utilities U^1 and $\tilde{U}^1$ must satisfy

$$U^1(\rho_1, \bar{x}) = K^1(\bar{x}) \cdot \bar{U}^1(\rho_1, \bar{x}) + k^1(\bar{x}), \quad K^1(\bar{x}) > 0,$$
$$\tilde{U}^1(\rho_1, \bar{x}) = \tilde{K}^1(\bar{x}) \cdot \bar{U}^1(\rho_1, \bar{x}) + \tilde{k}^1(\bar{x}), \quad \tilde{K}^1(\bar{x}) > 0.$$

Let then

$$A^1(\bar{x}) \equiv \frac{\tilde{K}^1(\bar{x})}{K^1(\bar{x})} > 0, \quad \alpha^1(\bar{x}) \equiv \tilde{k}^1(\bar{x}) - A^1(\bar{x}) \cdot k^1(\bar{x}).$$

It follows that

$$\tilde{U}^1(\rho_1, \bar{x}) = A^1(\bar{x}) \cdot U^1(\rho_1, \bar{x}) + \alpha^1(\bar{x}), \quad A^1(\bar{x}) > 0, \qquad (62)$$

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where the functions $A^1(\bar{x}) > 0$ and $\alpha^1(\bar{x})$ are unknown. Analogously, using (51) (resp. (56)), any two observationally equivalent utilities for agent 2 (resp. threat functions for agent 1) must satisfy:

$$\tilde{U}^2(\rho_2, \bar{x}) = A^2(\bar{x}) \cdot U^2(\rho_2, \bar{x}) + \alpha^2(\bar{x}), \quad A^2(\bar{x}) > 0, \tag{63}$$

$$\tilde{T}^1(x_1, \bar{x}) = B^1(\bar{x}) \cdot T^1(x_1, \bar{x}) + \beta^1(\bar{x}), \quad B^1(\bar{x}) > 0, \tag{64}$$

where

$$A^2(\bar{x}) \equiv \frac{\tilde{K}^2(\bar{x})}{K^2(\bar{x})} > 0, \quad \alpha^2(\bar{x}) \equiv \tilde{k}^2(\bar{x}) - A^2(\bar{x}) \cdot k^2(\bar{x}),$$

$$B^1(\bar{x}) \equiv \frac{\tilde{C}^1(\bar{x})}{C^1(\bar{x})} > 0, \quad \beta^1(\bar{x}) \equiv \tilde{c}^1(\bar{x}) - B^1(\bar{x}) \cdot c^1(\bar{x}).$$

Now, for agent 2's threat functions, using (57) we have

$$\tilde{T}^2(x_2, \bar{x}) = B^1(\bar{x}) \frac{A^2(\bar{x})}{A^1(\bar{x})} \cdot T^2(x_2, \bar{x}) + \beta^2(\bar{x}), \tag{65}$$

where

$$\beta^2(\bar{x}) \equiv \tilde{c}^2(\bar{x}) - B^1(\bar{x}) \frac{A^2(\bar{x})}{A^1(\bar{x})} \cdot c^2(\bar{x}).$$

Finally, combining all of the above with (61), we have that conditional on (y, x) the unobservables (ϵ_1, ϵ_2) and $(\tilde{\epsilon}_1, \tilde{\epsilon}_2)$ must satisfy:

$$\frac{\partial \tilde{U}^2(y - r, \bar{x})}{\partial \rho_2} \left[\tilde{U}^1(r, \bar{x}) + \tilde{\epsilon}_1 \right] - \frac{\partial \tilde{U}^1(r, \bar{x})}{\partial \rho_1} \left[\tilde{U}^2(y - r, \bar{x}) + \tilde{\epsilon}_2 \right] \underset{|(y,x)}{\sim}$$

$$A^2(\bar{x}) B^1(\bar{x}) \left\{ \frac{\partial U^2(y - r, \bar{x})}{\partial \rho_2} \left[U^1(r, \bar{x}) + \epsilon_1 + \frac{\beta^1(\bar{x})}{B^1(\bar{x})} \right] \right.$$

$$\left. - \frac{\partial U^1(r, \bar{x})}{\partial \rho_1} \left[U^2(y - r, \bar{x}) + \epsilon_2 + \frac{A^1(\bar{x})}{A^2(\bar{x})} \frac{\beta^2(\bar{x})}{B^1(\bar{x})} \right] \right\},$$

where $\underset{|(y,x)}{\sim}$ denotes equality in conditional distribution given (y, x). ∎

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