Private versus public consumption within groups: testing the nature of goods from aggregate data

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The collective consumption model

We focus on testable restrictions of the well known *collective consumption model* introduced by Browning and Chiappori, *Econometrica* (1998) with

- one household,
- two intra-household members,
- private and public consumption within the household.

Household' behavior. Under the badget constraint, the outcome of the household decision problem is a Pareto efficient allocation.

Testable restrictions: Two different methodologies

As it is well known, there are two different methodologies to check whether or not a model is testable.

Both methodologies consist in determining conditions associated with testability, commonly known as testable restrictions.

1. The **parametric approach** is based on comparative statics properties.

2. The **nonparametric approach** is based on revealed preferences theory (Afriat' inequalities and/or GARP).

Both approaches have been used in consumer theory and in collective consumption models.

Our basic references

Parametric Approach

• Browning, M. and P.-A. Chiappori, (1998). "Efficient Intra-Household Allocations: A General Characterization and Empirical Tests", *Econometrica* 66, 1241-1278.

• Chiappori, P.-A. and I. Ekeland, (2006). "The micro economics of group behavior: General Characterization", *Journal of Economic Theory* 130, 1-26.

Nonparametric Approach

• Cherchye, L., De Rock, B. and Vermeulen, F., (2007). "The Collective Model of Household Consumption: A Nonparametric Characterization and Empirical Test", *Econometrica* 75, 553-574.

Outline

1. The collective consumption model.
2. Nonparametric restrictions.
3. Two benchmark cases.
4. Main result: The private or public nature of consumption within the household is testable.

The model

We present the classical *collective consumption model*.

- n is the number of goods.
- One household with two intra-household members $i = 1, 2$.
- $x^i \in \mathbb{R}^n_+$ is the **consumption privately consumed** by the intra-household member $i = 1, 2$.
- $g \in \mathbb{R}^n_+$ is the **consumption publicly consumed** by the household.
- U^i is the utility function of the intra-household member i. $U^i(x^1, x^2, g)$ is the utility level associated to (x^1, x^2, g).

Household maximization problem

Given a price system $p \in \mathbb{R}^n_{++}$, wealth $y \in \mathbb{R}_+$ and weight $\mu \in \mathbb{R}_{++}$,

$$\begin{aligned} &\max_{(x^1,x^2,g)} \ \mu U^1(x^1,x^2,g) + (1-\mu)U^2(x^1,x^2,g) \\ &\text{subject to } p \cdot (x^1 + x^2 + g) \leq y \end{aligned}$$

That is, in a *collective consumption model* à la Browning and Chiappori, the household problem is a Pareto optimal decision problem under the budget constraint.

$q = x^1 + x^2 + g$ denotes the **aggregate consumption demand** of the household.

The price system p and the aggregate demand q are the **observable variables**.

Nonparametric approach

Using the *parametric* approach, Browning and Chiappori (1998), and Chiappori and Ekeland (2006) provide testable restrictions of the collective consumption model.

Following Cherchye, De Rock and Vermeulen, *Econometrica* (2007), we focus on a *nonparametric* approach and on testable restrictions of the model in the case of positive externalities.

The central tool is an appropriate version of the **Generalized Axiom of Revealed Preference (GARP)** which involves *personalized prices* and *personalized consumption*.

So, to provide the main result of Cherchye, De Rock and Vermeulen (2007), we introduce the notation for the personalized prices.

Personalized prices

We remind that p and q are observable.

• $p^{ij} \in \mathbb{R}^n_+$ is the **personalized price** payed by the member i for the consumption **privately consumed** by the member j.

• $p^{ig} \in \mathbb{R}^n_+$ is the **personalized price** payed by the member i for the consumption **publicly consumed** by the household.

$$p^1 := (p^{11}, p^{12}, p^{1g}) \text{ and } p^2 := (p^{21}, p^{22}, p^{2g})$$

The personalized prices p^1 and p^2 are **feasible** if

1. $\forall i$ and $\forall j,\ p^{ij} \leq p \quad$ and $\quad \forall i,\ p^{ig} \leq p$

2. In the spirit of Lindahl conditions,

$$p^{11} + p^{21} = p, \qquad p^{12} + p^{22} = p, \qquad p^{1g} + p^{2g} = p$$

The consumptions x^1, x^2, and g are **feasible** if $x^1 + x^2 + g = q$.

Nonparametric restrictions

Let $S = \{(p_t, q_t); t = 1, \ldots, T\}$ be a dataset of prices and aggregate demands at different dates.

From now on, U^i is assumed to be continuous, concave and increasing with respect to all variables (positive externalities).

Theorem (Cherchye, De Rock and Vermeulen, 2007). There exists a pair of utility functions U^1 and U^2 that provide a collective rationalization of the dataset S if and only if there exist feasible personalized prices and quantities such that

$$\{(p_t^1, (x_t^1, x_t^2, g_t)); t = 1, \ldots, T\} \text{ and } \{(p_t^2, (x_t^1, x_t^2, g_t)); t = 1, \ldots, T\}$$

simultaneously satisfy GARP.

Importantly, this result **does not require the observability** of personalized prices (p_t^1, p_t^2) and personalized quantities (x_t^1, x_t^2, g_t).

Two benchmark cases

Chiappori and Ekeland, *Journal of Economic Theory* (2006) also focus on two benchmark cases, that is,

Case 1. The collective model in which all goods are **only publicly consumed**.

Case 2. The collective model in which all goods are **only privately consumed** (no externalities, no public consumption).

Negative result. Using a parametric approach, Chiappori and Ekeland (2006) show that the general collective model has exactly the same testability implications as the two benchmark cases.

So, it seems that the private or public nature of consumption is **not testable**.

Our main contributions

Using GARP, differently from Chiappori and Ekeland (2006), we show that the previous benchmark cases are distinguishable. In particular,

1) We provide an example of a dataset which is consistent with Case 2 (all goods are privately consumed) but not with case 1 (all goods are publicly consumed).

2) We prove that any dataset with three observations that is consistent with the Case 1, it is also consistent with Case 2. Consequently, one needs at least four observations to provide an example of a dataset which is consistent with Case 1 but not with Case 2.

3) Using the result above, we provide an example with four observations that is consistent with the Case 1 but not with Case 2.

Differences between Case 1 and Case 2

Case 1

- The preferences of the intra-household member i depend only on goods that are publicly consumed, that is $u^i(g) := U^i(0, 0, g)$
- We observe the aggregate demand $q = g$.
- But, we do not observe the personalized prices of g such that

$$p^{1g} + p^{2g} = p$$

Case 2

- The intra-household member $i = 1, 2$ only cares for his private consumption, $u^1(x^1) := U^1(x^1, 0, 0)$ and $u^2(x^2) := U^2(0, x^2, 0)$
- We observe $q = x^1 + x^2$. But, we do not observe x^1 and x^2.
- We observe the personalized price payed by member $i = 1, 2$ for his private consumption, i.e. $p^{ii} = p$.

A first example

Three observations and three goods.

Consider the dataset $S = \{(p_t, q_t); t = 1, 2, 3\}$ defined by

$$\begin{array}{lll} t = 1 & p_1 = (4, 1, 1), & q_1 = (5, 2, 2) \\ t = 2 & p_2 = (1, 4, 1), & q_2 = (2, 5, 2) \\ t = 3 & p_3 = (1, 1, 4), & q_3 = (2, 2, 5) \end{array}$$

Consider the following personalized consumptions and prices.

$$\begin{array}{lllll} x_1^1 = q_1, & x_1^2 = 0, & g_1 = 0, & p_1^1 = (p_1, 0, p_1), & p_1^2 = (0, p_1, 0) \\ x_2^1 = \frac{1}{2} q_2, & x_2^2 = \frac{1}{2} q_2, & g_2 = 0, & p_2^1 = (p_2, 0, p_2), & p_2^2 = (0, p_2, 0) \\ x_3^1 = 0, & x_3^2 = q_3, & g_3 = 0, & p_3^1 = (p_3, 0, p_3), & p_3^2 = (0, p_3, 0) \end{array}$$

The personalized consumptions and personalized prices given above satisfy GARP for Case 2 (all goods are privately consumed).

But, the dataset S is not consistent with Case 1 (all goods are publicly consumed), i.e.

$$x_t^1 = x_t^2 = 0 \text{ and } g_t = q_t, \quad \text{for all } t = 1, 2, 3$$

Why?

It is possible to show that for a dataset with the following property, for all $t, s, = 1, 2, 3$ with $t \neq s$,

$$p_t \cdot q_t > p_t \cdot q_s$$

any feasible personalized prices and personalized consumptions which are consistent with GARP must satisfy the following direct revealed preferences.

For all $t, s, z = 1, 2, 3$ with $t \neq s \neq z$,

Member 1

(x_t^1, x_t^2, g_t) is directly revealed preferred to (x_s^1, x_s^2, g_s)

(x_s^1, x_s^2, g_s) is directly revealed preferred to (x_z^1, x_z^2, g_z)

Member 2

(x_z^1, x_z^2, g_z) is directly revealed preferred to (x_s^1, x_s^2, g_s)

(x_s^1, x_s^2, g_s) is directly revealed preferred to (x_t^1, x_t^2, g_t)

Importantly, these direct revealed preferences with

$$x_t^1 = x_t^2 = 0 \text{ and } g_t = q_t, \quad \text{for all } t = 1, 2, 3$$

are not consistent GARP. So, the dataset S is not consistent with the case in which all goods are publicly consumed (i.e., Case 1).

Are the two cases distinguishable?

So, we have provided an example for which GARP is satisfied for Case 2 (all goods are privately consumed) but not for Case 1 (all goods are publicly consumed).

Are we able to find another dataset for which GARP is satisfied for Case 1 but not Case 2?

We first provide the following property for every dataset with **three observations**.

Proposition

Let $S = \{(p_t; q_t); t = 1, 2, 3\}$ be a dataset that satisfies GARP associated with the general collective model. Then, the dataset satisfies GARP associated with Case 2 (all goods are privately consumed).

So, any dataset with three observations that is consistent with Case 1 (all goods are publicly consumed) it is also consistent with Case 2 (all goods are privately consumed).

Consequently, one needs at least **four observations** to reject GARP for the Case 2

Using a similar strategy as in the previous example we provide another dataset with **four observations** for which GARP is satisfied for Case 1 but not Case 2.

Conclusions. Using GARP, differently from Chiappori and Ekeland (2006), we find that the public or private nature of consumption have testable implications even if one only observes market prices and aggregate household consumptions.

Thanks !

GARP

Denote with x the vector of the feasible personalized consumptions, that is

$$x = (x^1, x^2, g)$$

For member i, the set of feasible personalized prices and consumptions $\{(p_t^i, x_t)\,;\, t = 1, ..., T\}$ satisfies GARP if there exist relations R_0^i and R^i such that

(1) if $p_s^i \cdot x_s \geq p_s^i \cdot x_t \Longrightarrow x_s R_0^i x_t$

(2) if $x_s R_0^i x_u$, $x_u R_0^i x_v$, ..., $x_z R_0^i x_t$ for some (possibly empty) sequence $(u, v, ..., z) \Longrightarrow x_s R^i x_t$

(3) if $x_s R^i x_t \Longrightarrow p_t^i \cdot x_t \leq p_t^i \cdot x_s$.

Proof of the Proposition

In Cherchye et al. (2007), it is proved that a dataset with the following property does not satisfy GARP for the general collective model.

For all $t, s, z = 1, 2, 3$ with $t \neq s \neq z$,

$$p_t \cdot q_t \geq p_t \cdot (q_s + q_z)$$

Without loosing of generality, we assume that

$$p_2 \cdot q_2 < p_2 \cdot (q_1 + q_3)$$

Consider the following personalized quantities and prices.

$$\begin{array}{lll} (q_1, 0, 0), & p_1^1 = (p_1, 0, p_1), & p_1^2 = (0, p_1, 0) \\ (\alpha q_2, (1-\alpha) q_2, 0), & p_2^1 = (p_2, 0, p_2), & p_2^2 = (0, p_2, 0) \\ (0, q_3, 0), & p_3^1 = (p_3, 0, p_3), & p_3^2 = (0, p_3, 0) \end{array}$$

with $\alpha \in [0, 1]$. These feasible prices and quantities are consistent with Case 2.