On revealed preferences in oligopoly games

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Introduction

Suppose we make a finite set of observations $T = \{1, ..., m\}$, $m \geq 1$, of a perfectly homogeneous-good oligopoly market.

There is a finite number of firms $N = \{1, ..., n\}$, $n \geq 2$, which compete in the market.

In each period we observe the price of each firm, P_{it}, their output, Q_{it}, and possibly their costs incurred, C_{it}.

The total set of observations can then be summarized as $(P_{it}, Q_{it}, C_{it})_{i \in N, t \in T}$.

Given this information how should we go about checking whether these observations are consistent/inconsistent with firms playing a Nash equilibrium in prices?

Related literature

Sprumont (2000) considered the case where we observe players choices from all possible subsets of strategy choices, all action spaces were finite, and identified two conditions, expansion and contraction consistency, which are necessary and sufficient to be able to rationalize the observed choices as Nash equilibria.

Zhou (2005) considered two-player games where players' strategy sets were the unit interval and assumed that we observe a finite subset of choices.

Carvajal and Quah (2009) analyzed the Cournot oligopoly model- this was my motivation for studying this problem.

Anticipating this work, Sprumont (2000, p.221) noted at the end of his paper that it would be interesting to characterize observable restrictions in games with more "economic flesh" such as oligopoly games.

Equilibrium concepts

First consider a perfectly homogeneous-good market with $N = \{1, ..., n\}$, $n \geq 2$, firms.

Each firm has a cost function $C_i : \Re_+ \rightarrow \Re_+$ which is strictly increasing, continuous and satisfies $C_i(0) = 0$.

he market demand $D : \Re_+ \rightarrow \Re_+$ is continuous and strictly decreasing whenever $D > 0$.

We shall let the function $F : \Re_+ \rightarrow \Re_+$ denote the inverse market demand in what follows.

Bertrand equilibrium

Each firm simultaneously and independently chooses a $P_i \in \Re_+$.

Firms in the market commit to supplying all the demand forthcoming at any price.

If a firm posts the unique minimum price in the market then it obtains all the market demand and its payoff is $P_i D(P_i) - C_i(D(P_i))$.

If a firm ties with $m - 1$ firms at the minimum price then they share the demand equally between themselves and the payoff of each firm is given by $\frac{1}{m} P_i D(P_i) - C_i(\frac{1}{m} D(P_i))$.

If a firm is undercut by any other firm in the market then it obtains zero demand, and given the assumption that the cost function passes through the origin, its payoff is zero.

Bertrand equilibrium

These payoffs are summarized below.

$$\pi_i(P_i, P_{-i}) = \begin{cases} P_i D(P_i) - C_i(D(P_i)) & \text{if } P_i < P_j \ \forall j \neq i; \\ \frac{1}{m} P_i D(P_i) - C_i(\frac{1}{m} D(P_i)) & i \text{ ties with } m-1 \text{ firms;} \\ 0 & \text{if } P_i > P_j \text{ for some } j. \end{cases}$$

Definition

A pure strategy Bertrand equilibrium is a Nash equilibrium of the game with payoffs defined above. That is, a vector of prices (P_i^B, P_{-i}^B) such that $\pi_i(P_i^B, P_{-i}^B) \geq \pi_i(P_i, P_{-i}^B)$ for all $P_i \in \Re_+$ and $i \in N$.

Cournot equilibrium

Each firm simultaneously and independently chooses a $Q_i \in \Re_+$. Given the total output chosen by the firms the market demand clears this output and sends back a single market-clearing price.

The payoff which any firm receives, given the vector of chosen outputs is (Q_i, Q_{-i}), is given below.

$$\pi_i(Q_i, Q_{-i}) = F(\sum_{j=1}^{n} Q_j)Q_i - C_i(Q_i)$$

Definition

A pure strategy Cournot equilibrium is a Nash equilibrium of the game with payoffs defined above. That is, a vector of outputs (Q_i^C, Q_{-i}^C) such that $\pi_i(Q_i^C, Q_{-i}^C) \geq \pi_i(Q_i, Q_{-i}^C)$ for all $Q_i \in \Re_+$ and $i \in N$.

Revealed Nash equilibria in oligopoly games

Now return to the issue of what observable outcomes the equilibrium concepts impose on the set $(P_{it}, Q_{it}, C_{it})_{i \in N, t \in T}$.

To organize the observations let $P_t^* = \min_{i \in N} P_{it}$.

Let $Q_t^* = \sum_{i \in N} Q_{it}$ denote the aggregate output produced in observation t.

The set of firms which tie at the minimum price, what we shall informally call the set of 'active firms', is given by $A_t = \{i \in N : P_{it} = P_t^*\}$.

Definition

A set of oligopoly observations is a generic homogeneous-good market data set if it satisfies the following conditions:

i) $P_{it} > 0$, $Q_{it} \geq 0$, $C_{it} \geq 0$, $P_{it}Q_{it} \geq C_{it}$ and $Q_{it} \neq Q_{it'}$ whenever $t \neq t'$.

ii) If $P_{it} > P_{jt}$ then $Q_{it} = 0$.

iii) If $P_{it} = P_{jt} = P_t^*$ then $Q_{it} = Q_{jt}$.

iv) $|A_t| \geq 2$.

We observe an oligopoly market where 'the law of one price' holds and tieing firms split the market demand equally.

A special type of data set is what we shall term a *single-price data set*: this is a generic homogeneous-good data set with the additional property that $P_{it} = P_t^*$ for all $i \in N$ and $t \in T$.

Rationalizable observations

Definition

A set of generic homogeneous-good observations, $(P_{it}, Q_{it}, C_{it})_{i \in N, t \in T}$, is Bertrand rationalizable if there exist C^2 functions, $\bar{C}_i : \Re_+ \rightarrow \Re_+$ for each $i \in N$, $\bar{D}_t : \Re_+ \rightarrow \Re_+$ for each $t \in T$, such that:

i) $\bar{C}_i(0) = 0$ and $\bar{C}_i'(x) > 0$ for all $x > 0$.

ii) $\bar{D}_t(x) \geq 0$ and $\bar{D}_t'(x) \leq 0$ with the latter inequality holding strictly whenever $\bar{D}_t(x) > 0$.

iii) $\bar{C}_i(Q_{it}) = C_{it}$ and $\bar{D}_t(P_t^*) = Q_t^*$.

iv) The set of observed prices $(P_{1t}, ..., P_{nt})$ is a Bertrand equilibrium in pure strategies for each $t \in T$.

Definition

A single-price data set, $(P_t^*, Q_{it}, C_{it})_{i \in N, t \in T}$, is Cournot rationalizable if there exist C^2 functions, $\bar{C}_i : \Re_+ \to \Re_+$ for each $i \in N$, $\bar{F}_t : \Re_+ \to \Re_+$ for each $t \in T$, such that:

i) $\bar{C}_i(0) = 0$ and $\bar{C}_i'(x) > 0$ for all $x > 0$.

ii) $\bar{F}_t(x) \geq 0$ and $\bar{F}_t'(x) \leq 0$ with the latter inequality holding strictly whenever $\bar{F}_t(x) > 0$.

iii) $\bar{C}_i(Q_{it}) = C_{it}$ and $\bar{F}_t(Q_t^*) = P_t^*$.

iv) The set of observed outputs $(Q_{1t}, ..., Q_{nt})$ is a Cournot equilibrium in pure strategies for each $t \in T$.

Let $R_i(t) = \{t' \in T : Q_{it'} \geq Q_t^*\}$.

That is, $R_i(t)$ is the set of observations when the output of firm i is greater than the aggregate output in observation t.

Let $S_i(t) = \{t' \in T : Q_{it'} < Q_{it}\}$.

The set $S_i(t)$ is those observations when the output of firm i is less than its own output in observation t.

We shall also want to compare firms' outputs with regards to the following quantity $\hat{Q}_t = Q_t^*/(|A_t| + 1)$.

Let $M_i(t) = \{t' \in T : Q_{it'} \geq \hat{Q}_t\}$ which is the set of observations when the output of firm i is greater than or equal to $\hat{Q}_t$.

Definition

A set of generic homogeneous-good oligopoly observations, $(P_{it}, Q_{it}, C_{it})_{i \in N, t \in T}$, satisfy the increasing cost condition (ICC) if, whenever $S_i(t) \neq \emptyset$, then $C_{it} - C_{it'} > 0$ for all $t' \in S_i(t)$.

Definition

A set of generic homogeneous-good oligopoly observations, $(P_{it}, Q_{it}, C_{it})_{i \in N, t \in T}$, satisfy the monopoly deviation condition (MDC) if, whenever $R_i(t) \neq \emptyset$, then $P_t^* Q_{it} - C_{it} \geq P_t^* Q_t^* - C_{it'}$ for all $t' \in R_i(t)$ with the inequality holding strictly whenever $Q_{it'} > Q_t^*$.

Definition

A set of generic homogeneous-good oligopoly observations, $(P_{it}, Q_{it}, C_{it})_{i \in N, t \in T}$, satisfy the tie deviation condition (TDC) if, whenever $M_i(t) \neq \emptyset$, then $P_t^* \hat{Q}_t \leq C_{it'}$ for all $t' \in M_i(t)$ and $i \in N \backslash A_t$ with the inequality holding strictly whenever $Q_{it'} > \hat{Q}_t$.

Definition

A single-price data set, $(P_t^*, Q_{it}, C_{it})_{i \in N, t \in T}$, satisfies the marginal condition (MC) if, whenever $S_i(t) \neq \emptyset$, then $P_t^* Q_{it'} - C_{it'} < P_t^* Q_{it} - C_{it}$ for all $t' \in S_i(t)$.

Results

Theorem

A set of generic homogeneous-good oligopoly observations, $(P_{it}, Q_{it}, C_{it})_{i \in N, t \in T}$, is Bertrand rationalizable if and only if it satisfies ICC, MDC and TDC.

Theorem

A single-price data set, $(P_t^*, Q_{it}, C_{it})_{i \in N, t \in T}$, is Cournot rationalizable if and only if it satisfies ICC and MC (Carvajal and Quah, 2009).

Theorem

A single-price data set, $(P_t^*, Q_{it}, C_{it})_{i \in N, t \in T}$, is Bertrand and Cournot rationalizable if and only if it satisfies ICC, MC and MDC.

The result follows from combining the conditions in the first two theorems and noting that a single-price data set trivially satisfies TDC.

Observations of prices and quantities

Suppose cost information is not available so that our set of observations reduces to $(P_{it}, Q_{it})_{i \in N, t \in T}$. Can any set of generic homogeneous-good market observations be Bertrand or Cournot rationalized?

Consider the following example of a symmetric duopoly, $n = 2$, with two observations, $m = 2$.

	(P, Q)
t=1	$(1, 4)$
t=2	$(2, 2)$

First observation requires $0 < \bar{C}(4) \leq 4$. Monopoly deviation condition requires that we can choose the cost function so that $4 - \bar{C}(2) \geq 8 - \bar{C}(4)$ with $0 < \bar{C}(2) < \bar{C}(4)$.

Inspection reveals that these observations cannot be simultaneously satisfied. Hence, the observations cannot be Bertrand rationalized.

Definition

A generic homogeneous-good set of prices and outputs, $(P_{it}, Q_{it})_{i \in N, t \in T}$, satisfies the revenue condition (RC) if, whenever $R_i(t) \neq \emptyset$, then $P^*_{t'} Q_{it'} > P^*_t (Q^*_t - Q_{it})$ for all $t' \in R_i(t)$ and $i \in A_t$.

The revenue condition states that the observed revenue which a firm obtains when it supplies an output at least as large as the observed aggregate output must be strictly greater than the increase in the revenue which a firm would obtain from supplying the entire market at the existing price.

Definition

A generic homogeneous-good set of prices and outputs, $(P_{it}, Q_{it})_{i \in N, t \in T}$, satisfies the tied revenue condition (TRC) if $i \in A_t$ then we do not observe a $t' \in T$ such that $i \in N \setminus A_{t'}$ and $P^*_{t'} \hat{Q}_{t'} \geq P^*_t Q_{it}$ and $\hat{Q}_{t'} \leq Q_{it}$ with at least one of the inequalities holding strictly.

Partial characterizations of rationalizable price-quantity observations

Proposition

A generic homogeneous-good set of prices and outputs, $(P_{it}, Q_{it})_{i \in N, t \in T}$, is Bertrand rationalizable only if it satisfies RC and TRC.

Proof. Suppose we have a generic set of observations which violate RC.

Then there is an i, t and $t' \in R_i(t)$ such that $P^*_{t'} Q_{it'} \leq P^*_t(Q^*_t - Q_{it})$.

If the observations are rationalizable we should be able to find a cost function such that $\bar{C}_i(Q_{it'}) \leq P^*_{t'} Q_{it'}$ and $\bar{C}_i(Q_{it}) \leq P^*_t Q_{it}$ as firms do not make losses.

MDC also requires that $\bar{C}_i(Q_{it'}) - \bar{C}_i(Q_{it}) > P^*_t(Q^*_t - Q_{it})$.

However, if $\bar{C}_i(Q_{it'}) \leq P^*_{t'} Q_{it'}$ we must have that $\bar{C}_i(Q_{it'}) - \bar{C}_i(Q_{it}) < P^*_t(Q^*_t - Q_{it})$ and the observations are not Bertrand rationalizable.

Suppose that we have a set of generic observations which violate TRC. Then there is an i, t and t' such that $i \in A_t$, $i \in N \setminus A_{t'}$ and $P^*_{t'}\hat{Q}_{t'} > P^*_t Q_{it}$ and $\hat{Q}_{t'} \leq Q_{it}$.

If the observations are rationalizable we should be able to find a cost function so that $\bar{C}_i(Q_{it}) \leq P^*_t Q_{it}$.

However, as $\hat{Q}_{t'} \leq Q_{it}$ and $P^*_{t'}\hat{Q}_{t'} > P^*_t Q_{it}$ this implies that $P^*_{t'}\hat{Q}_{t'} - \bar{C}_i(\hat{Q}_{t'}) > 0$.

Firm i has a profitable deviation by posting the minimum price in observation t'. ■

If one returns to the example data set it is clear that the reason that set of observations could not be Bertrand rationalized is because it violates RC: in observation one each firm produces a output equal to the aggregate market output in the second observation and obtains a revenue of 4. The revenue condition then requires that $4 > 2(4-2)$ which is violated.

Definition

A single-price data set of prices and outputs, $(P_t^*, Q_{it})_{i \in N, t \in T}$, satisfies the strengthened revenue condition (SRC) if, whenever $R_i(t) \neq \emptyset$, then $P_{t'}^* Q_{it'} > P_t^* Q_t^*$ for all $t' \in R_i(t)$.

The following result shows that this condition alone is sufficient for a single-price set of prices and outputs to be Bertrand rationalizable.

Proposition

A single-price data set of prices and outputs, $(P_t^*, Q_{it})_{i \in N, t \in T}$*, is Bertrand rationalizable if it satisfies SRC.*

What about Cournot rationalizable observations when cost information is unobservable?

Proposition

Any single-price data set of prices and outputs, $(P_t^*, Q_{it})_{i \in N, t \in T}$, is Cournot rationalizable (Carvajal and Quah, 2009).

This represents a significant difference between the two models.

One of the advantages of the revealed preference method is that we can write down possible observations and test their compatibility with different equilibrium concepts.

Examples of revealed Nash equilibria in oligopoly games- Example 1

	(P, Q, C)
t=1	$(3, 1, 1)$
t=2	$(4, 2, 7)$

Consider the example observations given in the table of a symmetric duopoly, $n = 2$, with two observations, $m = 2$.

First, we can note that $S_i(2) = \{1\}$. The increasing cost condition requires $C_{i2} - C_{i1} = 7 - 1 > 0$ which is satisfied.

Second, note that $R_i(1) = \{2\}$. The monopoly deviation condition requires
$P^*_{i1} Q_{i1} - C_{i1} = (3)(1) - (1) = 2 \geq (3)(2) - 7 = -1 = P^*_1 Q^*_1 - C_{i2}$ which is satisfied. The observations are Bertrand rationalizable.

Finally, the marginal condition requires
$P^*_2 Q_{i2} - C_{i2} = (4)(2) - 7 = 1 > (4)(1) - 1 = 3 = P^*_2 Q_{i1} - C_{i1}$ which is violated. We can conclude that this set of observations *cannot* be Cournot rationalized.

Example 2

	(P, Q, C)
t=1	$(3, 1, 2)$
t=2	$(4, 2, 4)$

First, note that $S_i(2) = \{1\}$. The increasing cost condition requires $C_{i2} - C_{i1} = 4 - 2 > 0$ which is satisfied.

Second, note that $R_i(1) = \{2\}$. The monopoly deviation condition requires that
$P_{i1}^* Q_{i1} - C_{i1} = (3)(1) - 2 = 1 \geq (3)(2) - 4 = 2 = P_1^* Q_1^* - C_{i2}$ which is violated. Therefore this set of observations *cannot* be Bertrand rationalized.

The marginal condition requires that no firm can benefit by reducing their outputs. The condition requires that
$P_2^* Q_{i2} - C_{i2} = (4)(2) - 4 = 4 > (4)(1) - 2 = 2 = P_2^* Q_{i1} - C_{i1}$.

Therefore this set of observations can be Cournot rationalized.

Concluding remarks

The aim of this work has been to provide a revealed preference approach to the classical oligopoly games.

Established an elegant theoretical framework and intuitive characterizations of the rationalizable observations.

What about more applied models? In Vives (1999, Ch. 6) it is assumed that differentiated-good market demand is generated from a representative consumer with utility $U(\mathbf{q}) - \mathbf{p}.\mathbf{q}$ so partial equilibrium analysis is valid.

A revealed preference analysis of this model would have the advantage of generating applied work- this remains an interesting topic for future research.