On strategic behavior of non-expected utility players in games with payoff uncertainty*

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Abstract

This paper investigates the extent to which we can distinguish expected and non-expected utility players on the basis of their behavior. A model of incomplete information games is used in which players can choose mixed strategies. Expected and non-expected players sometimes cannot be distinguished by observing their equilibrium actions, since they behave observationally equivalent. It is shown that uncertainty-averse non-expected utility players can often be identified by looking at their best responses. They may behave differently in the use of mixed strategies, called hedging behavior, and the response to mixed strategy combinations, called reversal behavior. We find that these are the sole behavioral differences between expected and non-expected utility players. Furthermore, the absence of hedging behavior is sufficient for observational equivalence. The paper provides necessary and sufficient conditions for the existence of hedging and reversal behavior in terms of the payoff structure of a two-person two-strategies game. Finally, an equilibrium concept is introduced that allows for players who are not uncertainty-averse.

Keywords: Non-expected utility, Incomplete information games, Uncertainty aversion, Mixed strategies, Strategic behavior

JEL classifications: D81, C72

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1 Introduction

Most real-life strategic situations involve incomplete information. Harsanyi (1967-68) showed that incomplete information games can be transformed into game-theoretically equivalent games with complete, but imperfect information, commonly known as *Bayesian games*. One key assumption of Harsanyi's approach is that players are Bayesian expected utility maximizers and share a common prior distribution over the state space. However, experiments demonstrate that in some situations individuals consistently violate the expected utility (or, briefly, EU) hypothesis. In particular, Ellsberg (1961) exemplified that in situations under *ambiguity*, i.e., situations in which probabilities are imperfectly known, many individuals display behavior which is inconsistent with EU theory.

A number of models of incomplete information games with non-EU players have been proposed, see Section 1.2. However, to my knowledge, it has not been systematically investigated whether, and if so, under which conditions, these models predict behavior that differs from the behavior predicted by models with EU players. Such an investigation is useful not only for gaining theoretical insights, but also as a guide to design experiments testing one model against another.

This paper offers a first attempt at systematically examining the question of when one can distinguish EU and non-EU players in the context of an increasingly used model. The key assumption of this model is that players behave as expected utility maximizers with correct beliefs concerning mixed strategy combinations, but face ambiguity[1] about the environment. Comparable models were first introduced by Bade (2011a) and Azrieli and Teper (2011). Therefore, I shall occasionally refer to this model as Bade-Azrieli-Teper-model (or, briefly, BAT-model). For instance, the applications described in Section 1.2 are based on BAT-type models.

Unfortunately, ambiguity-neutral and ambiguity-averse players sometimes behave observationally equivalent, which means that it is impossible to identify non-EU players by observing equilibrium actions. I show that EU and non-EU players can be distinguished

[1]The main theorems also apply to non-EU players who are probabilistically sophisticated in the sense of Machina and Schmeidler (1992), but we will focus on players with non-probabilistic beliefs when we interpret the results.

by looking at their best-response correspondences. More precisely, the strategic behavior of uncertainty-averse non-EU players can differ substantially from the behavior of EU players in both: the use (*hedging behavior*) and the response (*reversal behavior*) to mixed strategies. From a decision theoretic perspective, both characteristics are due to the same cause, namely a preference for mixtures. However, from a game theoretic perspective, it does matter whether a player prefers to randomize over her strategies or whether she exhibits a preference for mixed strategy combinations of the opponents.

In the formal analysis, attention is restricted to games with payoff uncertainty, but without private information. However, the results can also be applied to incomplete information games with private information, see Remark 2. The first main theorem shows that EU and non-EU players behave observationally equivalent whenever the non-EU players do not exhibit hedging behavior. In other words, the absence of hedging behavior is sufficient for observational equivalence. That is, we need hedging behavior if we want to distinguish EU from non-EU players by observing equilibrium actions. The second main theorem shows that non-EU players behave as if they were EU players if and only if they do not exhibit hedging and reversal behavior. To put it differently, hedging and/or reversal behavior are necessary and sufficient for the existence of behavioral differences between EU and non-EU players, which means that there are no other behavioral differences. The last part of the paper provides necessary and sufficient conditions for the existence of hedging and reversal behavior in terms of the payoff structure of a game. For tractability, we consider two-person two-strategies games and players with maxmin expected utility preferences axiomatized by Gilboa and Schmeidler (1989). In laboratory experiments, both restrictions are frequently satisfied.

This paper is organized as follows. The following subsection gives two examples to illustrate the BAT-model and the notions of hedging and reversal behavior. Then, I review the related literature. Section 2 introduces the basic concepts. Section 3 provides the results. The subsequent section discusses the underlying model and introduces a generalized equilibrium concept. Finally, Section 5 concludes with a summary of the main results. Unless noted otherwise, the proofs of the results are given in the Appendix.

1.1 Two examples

In the following, we consider two examples.[2] These examples illustrate the BAT-model and our notions of hedging and reversal behavior. In addition, they show potential economic applications of the BAT-model.

Example 1 (Discrete Cournot duopoly with uncertain demand)**.** *There are two firms, $i \in \{1, 2\}$, which produce a homogeneous product. The firms compete in quantities, and decide simultaneously whether to produce a low quantity normalized to one, $q_l = 1$, or a high quantity, $q_h = 2$. Marginal costs of production are constant and normalized to one. The market price, p, depends on the total quantity in the industry, Q, and on an uncertain state of the world, $\omega \in \{\omega_1, \omega_2\}$: $p = A(\omega) - b(\omega) \cdot Q$, where $(A, b)(\omega_1) = (6, \frac{3}{2})$ and $(A, b)(\omega_2) = (2, 0)$. When choosing whether to produce q_l or q_h, both firms do not know the state of the world. Firms' state-dependent profits are:*

ω_1:

	q_l	q_h
q_l	$2, 2$	$\frac{1}{2}, 1$
q_h	$1, \frac{1}{2}$	$-2, -2$

ω_2:

	q_l	q_h
q_l	$1, 1$	$1, 2$
q_h	$2, 1$	$2, 2$

Since firms' profits depend on a state of nature, every pure strategy profile induces a state-contingent vector of profits for both firms. For instance, the strategy profile (q_l, q_l) induces the vector $f_1(q_l, q_l) = (f_1^{\omega_1}(q_l, q_l), f_1^{\omega_2}(q_l, q_l)) = (2, 1)$ for firm 1 (row). Every mixed strategy profile generates a probability distribution over pure strategy profiles. In a given state ω, each firm's payoff from a mixed profile corresponds to its expected profit with respect to this distribution. Hence, every mixed profile induces state-contingent vectors of expected profits.

Suppose each firm $i \in \{1, 2\}$ has the following non-EU preferences over state-contingent (expected) profits: $V_i(f_i) = \min\{f_i^{\omega_1}, f_i^{\omega_2}\}$. Then, firm i's best-response correspondence, R_i, takes the form illustrated in Figure 1, where $\sigma_j(q_l)$ denotes the

[2]The equilibria for the games in the examples and the formal derivation of players' best-response correspondences are given in the Appendix.

probability with which the other firm j produces q_l, and, at the same time, the mixed strategy of firm j. In this section, mixed strategies are denoted by their first component, $\sigma_j(q_l) = (\sigma_j(q_l), \sigma_j(q_h))$, since $\sigma_j(q_l) = 1 - \sigma_j(q_h)$.

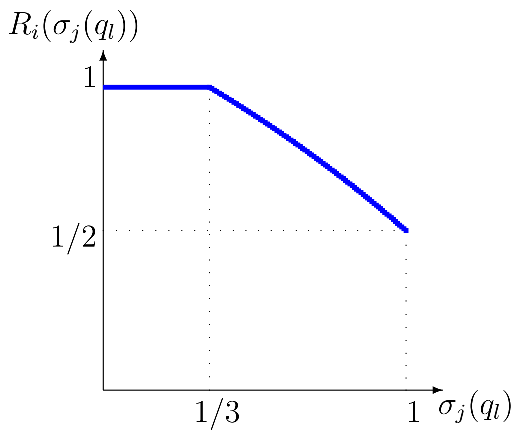

Figure 1: Best responses of a firm in Example 1

As Figure 1 shows, firm i has a unique best response to all strategies of firm j. Furthermore, its unique best response is a mixed strategy if j plays q_l with more than 1/3 probability. EU players would never show this type of strategic behavior. They use mixed strategies to make the other players indifferent between playing their pure strategies, for instance, like in matching pennies-type games, to avoid exploitation by their opponents. However, for an EU player, mixed strategies are always weakly optimal: if a mixed strategy is a best response to some strategy profile of the other players, then, at the same time, all pure strategies to which it assigns positive probability are best responses. Consequently, mixed strategies are never unique best responses.

Why are non-EU players able to behave differently? The reason is that they randomize over their pure strategies not only for strategic purposes, but also as a kind of "hedging" against environmental uncertainty. In Example 1, q_l is a strictly dominant strategy in ω_1 and q_h in ω_2 for both firms. If firm j chooses a strategy $\sigma_j(q_l) \leq 1/3$, then, firm i's expected profit in state ω_1 is lower than in ω_2, regardless of its strategy choice. Therefore, firm i will play its strictly dominant strategy in ω_1, $\sigma_i(q_l) = 1$. Otherwise, if $\sigma_j(q_l) > 1/3$, firm i seeks to smooth its expected profits across states by playing a mixed strategy. For instance, given $\sigma_j(q_l) = 1$, firm i will play q_l (and q_h) with 1/2 probability, which induces the vector $f_i\left(\frac{1}{2},1\right) = \left(\frac{3}{2},\frac{3}{2}\right)$.

This is not new from a decision theoretic perspective. In an early reply to Ellsberg (1961), Raiffa (1961) claimed that ambiguous uncertainty can be eliminated by randomizing. Furthermore, Schmeidler (1989) defines uncertainty aversion as a weak preference for randomization.[3] More recently, Battigalli et al. (2013) study a framework of mixed extensions of decision problems under uncertainty that involves preference for randomization as an expression of uncertainty aversion. They and other authors, e.g., Gilboa and Schmeidler (1989) and Saito (2013), use the term "hedging" to refer to situations in which decision-makers prefer randomized choices. Although some confusion may arise from the connotations of this traditional term[4], I will follow this terminology and refer to a preference for playing mixed strategies as "hedging behavior".[5]

In the game theory literature, only a few authors, e.g., Klibanoff (1996) and Lo (1996), explicitly discuss a preference for randomized strategies in the context of their models, which involve strategic ambiguity, but no environmental uncertainty.

Example 2 (Uncertain investment). *There is an investor, I, with initial wealth 1 and a fund manager, M. The investor decides whether to invest her money in the fund, In, or keep it at the bank, Bk, with a guaranteed payoff of 1. The fund manager chooses an investment strategy: He can either speculate on falling or rising share prices. For simplicity, suppose he can either buy one stock, S, or a put option on the stock, P. Initially, stock and put are worth 1. The future stock value $q^s(\omega)$ depends on an uncertain state of the world, $\omega \in \{\omega_1, \omega_2\}$, where $q^s(\omega_1) = 6$ and $q^s(\omega_2) = 0$. The strike price of the put is 6, hence, its future value is $q^p(\omega) = 6 - q^s(\omega)$. The fee for the fund manager is performance-based: He gets 1 if the investment is successful, otherwise 0. Players' state-dependent payoffs are:*

[3]Schmeidler's axiom states that a preference relation $\succsim$ reveals uncertainty aversion, if for any two acts f, g, and $\alpha \in [0,1]$: If $f \succsim g$, then $\alpha f + (1-\alpha)g \succsim g$.

[4]For instance, it could be associated with "hedging" in finance, which refers to activities that reduce portfolio risk.

[5]Klibanoff (2001) suggests the term "objectifying behavior". In my opinion, another suitable alternative is "Raiffa behavior", since he was was the first who pointed to this effect.

	S	P
Bk	$2, 0$	$2, 0$
In	$5, 1$	$0, 0$

ω_1

	S	P
Bk	$2, 0$	$2, 0$
In	$0, 0$	$5, 1$

ω_2

Again, suppose each player $i \in \{I, M\}$ has the following non-EU preferences over state-contingent (expected) payoffs: $V_i(f_i) = \min\{f_i^{\omega_1}, f_i^{\omega_2}\}$. Then, players' best-response correspondences, R_i, are:

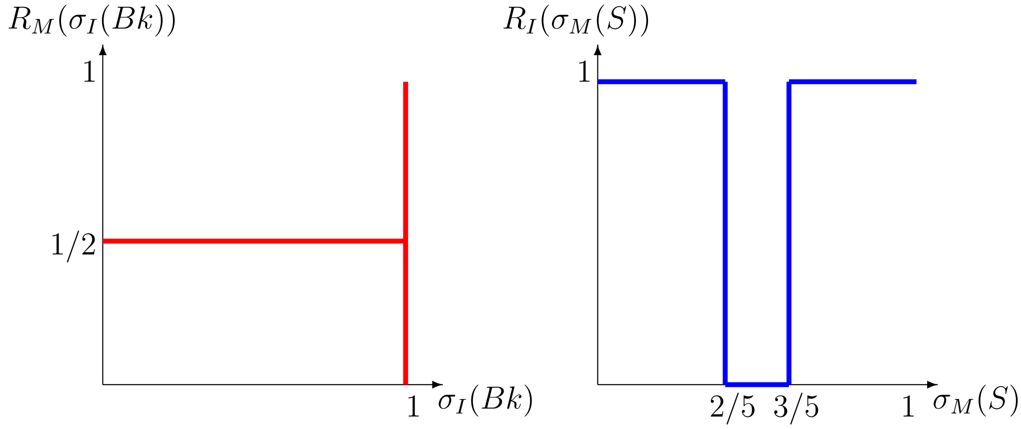

Figure 2: Players' best responses in Example 2

The fund manager (left graph) has a weakly dominant mixed strategy: Buying the stock and the put with 1/2 probability. The investor (right graph) has no preference for mixed strategies. However, she shows the second type of strategic behavior which differs from behavior of EU players: she prefers to keep her money at the bank if the investor buys the stock or the put with high probability. Otherwise, if his action is sufficiently uncertain for her, she will invest in the fund. In other words, her preference for strategy Bk over In, given S or P, reverses for some mixtures of S and P. Therefore, I will refer to this type of behavior as "reversal behavior". In contrast, if an EU player in a two-person game prefers to play a particular strategy in response to two strategies of her opponent, she will still prefer to play this strategy in response to any mixture of the two strategies. More formally, the preimage of each of her best responses is convex under her best-response correspondence.[6]

[6]Note that this holds only for two-person games. For the general case, see Definition 3.

In Example 2, the reason for reversal behavior is that, no matter what the investor chooses, her expected profit in ω_1 is lower than in ω_2 if $\sigma_M(S) > 1/2$ and higher if $\sigma_M(S) < 1/2$. According to her objective function $V_I = \min\{f_I^{\omega_1}, f_I^{\omega_2}\}$, she will maximize $f_I^{\omega_1}$ if $\sigma_M(S) > 1/2$, and, otherwise, $f_I^{\omega_2}$. Hence, given $\sigma_M(S) > 1/2$, the investor's best-response correspondence equals her best responses in ω_1, and, otherwise, her best responses in ω_2.

To summarize, non-EU players can behave differently to EU players in that they may prefer randomized strategies and/or change their preferences for strategies due to mixture operations of one of their opponents.[7]

1.2 Related Literature

The majority of the literature on games played by non-EU players has focused on games with complete information in which players face only strategic uncertainty. In an early paper, Crawford (1990) generalize Nash equilibrium to allow for probabilistically sophisticated non-EU players. The subsequent papers on strategic ambiguity can be roughly divided into two groups. To the first group belong Klibanoff (1996), Lo (1996), and Lehrer (2012), who assume that players explicitly randomize. They provide equilibrium concepts with weaker requirements regarding the consistency between beliefs and strategies than Nash equilibrium. In contrast, the approach of the second group, which includes Dow and Werlang (1994), Eichberger and Kelsey (2000, 2014), and Marinacci (2000), is based on the interpretation of a mixed strategy as a player's belief about the pure strategy choices of his opponents. The equilibrium definitions of these papers require consistency conditions between the beliefs that players hold.

The approach of the first group has the drawback that, typically, players' beliefs will not coincide with the strategies that are actually played. A criticism of the approach of the second group is that it has limited abilities to predict behavior, since it usually does not specify the strategies that are played. The model studied in this paper does not have these drawbacks.

[7]In both cases, the matrix-form is an unsatisfactory representation of the game.

There is relatively small, but growing, literature on incomplete information games played by non-EU players. Epstein and Wang (1996) offer a general framework that provides a foundation for a "type-space" approach à la Harsanyi with non-EU players. Eichberger and Kelsey (2004) generalize perfect Bayesian equilibrium for the case of two-player games with ambiguity. Players' beliefs are represented by *capacities*, i.e., normalized and monotone, but not necessarily additive set functions. In their Dempster-Shafer equilibrium, players maximize Choquet expected utility introduced by Schmeidler (1989). Kajii and Ui (2005) investigate a model in which all players have maxmin expected utility preferences. Their model differs from Bayesian games in that it does not assume a common prior over the states. Instead, there is a set of priors for each player, which may vary among players. Bade (2011a) and Azrieli and Teper (2011) consider more general preferences. Their models assume that players choose mixtures as their strategies, and there is no ambiguity about the probabilities of mixed strategies, i.e., no strategic ambiguity. However, players face ambiguity about the environment. The papers differ in that Bade (2011a) requires payoffs to be state-independent, and in that Azrieli and Teper (2011) do not rule out correlation devices and diverging beliefs.

There is an increasing number of papers on applications of incomplete information games that use BAT-type models. These papers examine games with payoff ambiguity, but without private information. For instance, Bade (2011b) studies electoral competition between two parties in a two-stage game by assuming that parties are uncertain about voters' marginal rates of substitution between issues. Król (2012) investigates ambiguous demand in the context of a two-stage product-type-then-price competition game. Aflaki (2013) examines the tragedy of the commons in which players face ambiguity concerning the size of the resource endowment. Bade (2011a) and Król (2012) use maxmin expected utility preferences. Aflaki (2013) additionally considers Choquet expected utility, and smooth ambiguity preferences introduced by Klibanoff et al. (2005).

2 Preliminaries

2.1 The model

A basic normal-form game with incomplete information (or basic game, for short) is an ordered set $G = \langle I, \Omega, \{A_i, u_i\}_{i\in I}\rangle$, where

(1) $I = \{1, \ldots, n\}$ is a *finite set of players*;

(2) $\Omega = \{\omega_1, \ldots, \omega_m\}$ is a *finite set of states of nature*;

(3) A_i is the *finite set of pure actions of player* i. Let $A = \bigtimes_{i\in I} A_i$;

(4) $u_i : A \times \Omega \to \mathbb{R}$ is *the payoff function of player* i.

Players' payoffs (4) depend not only on an action profile, $a \in A$, but also on an uncertain state of the world (2). Note that (4) is a commonly used simplification. Technically, there exists an outcome function $\gamma : A\times\Omega \to X$ which maps from action profiles and states into physical outcomes, or consequences, X.[8] For each $i \in I$, there exists a utility function $v_i : X \to \mathbb{R}$ which assigns a real number to each consequence. Player i's payoff function (4) can be considered as the composition $u_i := v_i \circ \gamma : A \times \Omega \to \mathbb{R}$.

Remark 1. A basic game is a game-form together with fixed state-dependent payoffs.

According to (4), an action profile $a \in A$ induces payoff $u_i(a, \omega)$ in state $\omega \in \Omega$ for each $i \in I$. Hence, every action profile induces a payoff vector or, an act, $f_i(a) = \big(u_i(a, \omega_1), \ldots, u_i(a, \omega_m)\big) \in \mathbb{R}^m$ for each $i \in I$. The basic game description does not include private information. I shall restrict attention to this case to avoid cumbersome notation.

Remark 2. Private information can be introduced by defining an information partition P_i of Ω for each $i \in I$ which specifies *players' strategy sets*. A *pure strategy* of player i is then a P_i-measurable function $s_i : \Omega \to A_i$, cf. Bade (2011a) and Azrieli and Teper (2011). If player i has no private information, the partition P_i is trivial. In this case, player i's strategies correspond to her actions. The results in this paper hold also for basic games with private information.

[8]The sets (1) to (3) together with the mapping $\gamma(\cdot)$ are called *normal game-form with incomplete information.*

Of particular interest in this paper are mixed actions. The *mixed extension of a basic game* involves, in addition to the elements of the description above, players' mixed action sets. A *mixed action* of player i is a function $\sigma_i : A_i \to [0,1]$ where $\sum_{a_i \in A_i} \sigma_i(s_i) = 1$. The set of all mixed actions of i (i.e., the set of all probability distributions over A_i) is denoted by Σ_i. Let $\Sigma = \bigtimes_{i \in I} \Sigma_i$. Henceforth, $\sigma_i(a_i)$ denotes the probability which $\sigma_i \in \Sigma_i$ assigns to action $a_i \in A_i$. The model does not involve strategic ambiguity, which means that the probabilities $\sigma_i(a_i)$ are "objective" or, at least, known among players. It is assumed that, in any given state $\omega \in \Omega$, players' preferences w.r.t. (with respect to) a mixed profile $\sigma \in \Sigma$ have an EU representation, formally,

Assumption 1. Fix a state $\omega \in \Omega$, then player i's payoff from a mixed profile $\sigma \in \Sigma$ is

$$f_i^{\omega}(\sigma) = \sum_{a \in A} \left(\prod_{j \in I} \sigma_j(a_j) \right) u_i(a, \omega) \text{ for each } i \in I.$$

According to Assumption 1, every mixed action profile $\sigma \in \Sigma$ induces a vector of expected payoffs, $f_i(\sigma) = \sum_{a \in A} \left(\prod_{j \in I} \sigma_j(a_j) f_i(a) \right)$, which is a convex combination of player i's payoff vectors induced by pure strategy profiles $f_i(a)$. Given the actions of the other players, any degenerate mixed action is payoff equivalent to a pure action. Therefore, we may associate the set of player i's pure actions, A_i, with the subset of Σ_i that contains i's degenerate mixed actions. Henceforth, depending on the context, the symbols a_i and A_i may also stand for (the set of) i's degenerate mixed actions. Furthermore, Γ denotes the set of all basic games and -i the set of all players, except player i.

2.2 Preferences over acts and equilibrium points

The basic game description is not sufficient to characterize the solution of a game. In order to obtain a solvable game from a basic game $G \in \Gamma$, we need to specify each player i's preferences, $\succsim_i$, over m-dimensional payoff vectors, as in the examples in Section 1.1. That is, for each $i \in I$, there exists a function $V_i : \mathbb{R}^m \to \mathbb{R}$ such that, for all $f, g \in \mathbb{R}^m$,

$$f \succsim_i g \;\Leftrightarrow\; V_i(f) \;\geq\; V_i(g).$$

The preference ordering $\succsim_i$ of each player i induces, through the associated payoff vectors, a preference ordering on action profiles and hence on actions for any given action combination of the other players. Let $\succsim = \{\succsim_i\}_{i\in I}$ denote players' preferences over acts. I shall refer to the set $\langle G, \succsim \rangle$ as G played by, or, with $\succsim$ players, or simply as game. The analysis in this paper focuses on the representation function $V_i(\cdot)$ of player i's preferences. Throughout Section 3, we will assume,[9]

Assumption 2. For each $i \in I$, function V_i is continuous and quasiconcave on $\mathbb{R}^m$, and monotonic, i.e., for all $f, g \in \mathbb{R}^m$, $f(\omega) \geq (>) g(\omega)$ for all $\omega \in \Omega$ implies $V_i(f) \geq (>) V_i(g)$.

According to Assumption 2, the underlying preference relation $\succsim_i$ of each player i is complete, transitive, and monotonic. Furthermore, it satisfies uncertainty aversion in the sense of Schmeidler (1989), which translates into quasiconcavity of the representation function. There is a huge variety of preferences which are consistent with Assumption 1. For instance, maxmin expected utility preferences, Choquet expected utility preferences if the capacity is convex, and smooth ambiguity-averse preferences. For more details, compare Cerreira-Vioglio et al. (2011) who identify the representation of preferences that satisfy the properties mentioned above.

The following examples illustrate two possible representation functions. Let $\Delta(\Omega)$ be the set of all probability measures over Ω, and $\mathscr{C}$ be the collection of all nonempty, closed and convex subsets of $\Delta(\Omega)$. An element of $\Delta(\Omega)$ (i.e., a probability vector or prior) is denoted by $\pi = (\pi(\omega_1), \ldots, \pi(\omega_m))$ where $\pi(\omega)$ is the probability of state $\omega \in \Omega$.

Example 3 (Expected utility)**.** *The belief of an EU player i is represented by a unique prior $\pi_i \in \Delta(\Omega)$. She evaluates a state-contingent vector $f \in \mathbb{R}^m$ by the expected utility with respect to her prior:*

$$EU_{\pi_i}(f) = f \cdot \pi_i^\top \text{ where } f \text{ is a row vector and } \pi_i^\top \text{ a column vector.}$$

Hence, for all $f, g \in \mathbb{R}^m$, it holds that, $f \succsim_i^{EU} g \;\Leftrightarrow\; f \cdot \pi_i^\top \;\geq\; g \cdot \pi_i^\top$.

Consequently, a game played by EU players is a game $\langle G, \succsim^{EU} \rangle$ in which each player i has EU preferences, $\succsim_i^{EU}$, i.e., i's preferences are represented by an EU function, $V_i = EU_{\pi_i}$.

[9]In Section 4, we discuss an equilibrium concept that allows for preferences, which are not represented by quasiconcave functions.

Example 4 (Maxmin expected utility)**.** *The key idea of the maxmin expected utility approach is that, in case of ambiguous uncertainty, an individual i has too little information to form a unique prior probability distribution $\pi_i \in \Delta(\Omega)$. For this reason, she considers a set of priors $C_i \in \mathscr{C}$ as possible. A maxmin expected utility player i evaluates an act $f \in \mathbb{R}^m$ by the minimal expected utility over all priors in her prior set:*

$$MEU_{C_i}(f) = \min_{\pi \in C_i} \big\{ EU_\pi(f) \big\}.$$

Hence, for all $f, g \in \mathbb{R}^m$, it holds that, $f \succsim_i^{MEU} g \;\Leftrightarrow\; \min_{\pi \in C_i} \big\{ EU_\pi(f) \big\} \geq \min_{\pi \in C_i} \big\{ EU_\pi(g) \big\}$.

Finally, we turn to the solution of a game $\langle G, \succsim \rangle$. From now on, occasionally, I abuse notation and write $V_i(\sigma)$ instead of $V_i(f_i(\sigma))$. An (ex-ante) equilibrium for (the mixed extension of) a normal-form game with incomplete information is defined as follows:

Definition 1. An *equilibrium point in a game* $\langle G, \succsim \rangle$ is a profile $(\sigma_i^*, \sigma_{-i}^*) \in \Sigma$ such that

$$\sigma_i^* \in \arg\max_{\sigma_i \in \Sigma_i} V_i(\sigma_i, \sigma_{-i}^*) \text{ for each player } i.$$

Azrieli and Teper (2011) show that, under Assumption 1, uncertainty aversion, i.e., quasiconcavity of players' objective functions, is necessary and sufficient for equilibrium existence, provided that players' preferences are continuous and monotonic. Their theorem is essentially similar to the existence theorem of Debreu (1952), which shows that in a finite game Nash equilibrium is guaranteed to exist if players preferences are nonlinear but quasiconcave in their own strategies.

2.3 Hedging behavior and reversal behavior

The *best-response correspondence* of player i is a multivalued mapping $R_i : \Sigma_{-i} \rightrightarrows \Sigma_i$ defined by $R_i(\sigma_{-i}) = \{\sigma_i \mid \sigma_i \in \arg\max_{\sigma_i \in \Sigma_i} V_i(\sigma_i, \sigma_{-i})\}$. Furthermore, a pure action $a_i \in A_i$ is said to be contained in the *support of a mixed action* $\sigma_i \in \Sigma_i$ if σ_i assigns a strictly positive probability to a_i, formally, $\text{supp}(\sigma_i) = \{a_i \in A_i \mid \sigma_i(a_i) > 0\}$.

Definition 2. Player i with preferences $\succsim_i$ represented by function V_i exhibits *hedging behavior* in $G \in \Gamma$ if there exists a mixed action $\sigma_i' \in \Sigma_i$ which satisfies

(i) $\sigma_i' \in R_i(\sigma_{-i}')$ for some $\sigma_{-i}' \in \Sigma_{-i}$ and

(ii) $V_i(\sigma'_i, \sigma'_{-i}) > V_i(a_i, \sigma'_{-i})$ for some $a_i \in \text{supp}(\sigma'_i)$.

Property (i) in Definition 2 restricts the notion of hedging behavior to actions that are contained in a player's best-response correspondence. This is necessary, since if a mixed action is not a best response, an EU player can also prefer it over particular pure actions from its support. This is, however, not possible if the mixed action is a best response due to the linearity of the EU functional.[10] Furthermore, non-EU players may strictly prefer mixed actions. This is the case when property (ii) holds for all $a_i \in \text{supp}(\sigma'_i)$. As a consequence, mixed actions can be unique best responses.

The second type of strategic behavior refers to players' behavior concerning randomizing operations of the other players.

Definition 3. Let $(\sigma'_j, \overline{\sigma}_{-j}), (\sigma''_j, \overline{\sigma}_{-j}) \in \Sigma_{-i}$, where $\overline{\sigma}_{-j}$ denotes a fixed strategy profile of all players except player i and player j. Player i with preferences $\succsim_i$ exhibits *reversal behavior* in $G \in \Gamma$ if there exist actions $a'_i, a''_i \in A_i$ such that

(i) $a'_i \in R_i(\sigma'_j, \overline{\sigma}_{-j})$, $a'_i \in R_i(\sigma''_j, \overline{\sigma}_{-j})$, and $a'_i \notin R_i(\alpha\sigma'_j + (1-\alpha)\sigma''_j, \sigma'_{-j})$ for some $\alpha \in (0,1)$ and/or

(ii) $a'_i \in R_i(\sigma'_j, \overline{\sigma}_{-j})$, $a'_i \in R_i(\sigma''_j, \overline{\sigma}_{-j})$, and $a''_i \notin R_i(\sigma'_j, \overline{\sigma}_{-j})$ and/or $a''_i \notin R_i(\sigma''_j, \overline{\sigma}_{-j})$, and $a''_i \in R_i(\alpha\sigma'_j + (1-\alpha)\sigma''_j, \overline{\sigma}_{-j})$ for some $\alpha \in (0,1)$.

Definition 3 is more technical in nature. Condition (i) refers to a situation like in Example 2 where an action is a best response to some action profiles, but not to all convex combinations of the profiles. Condition (ii) describes a situation in which a pure action is a best response to a convex combination of two action profiles, but not to both profiles, and, at the same time, there exists another action which is a best response to both profiles. Due to the linearity of the EU function, (ii) is also not possible if player i is an EU player.

[10] This property can be easily shown, see, for instance, Dekel et al. (1991, p. 236).

3 Results

3.1 Main theorems

This section analyzes the extent to which the strategic behavior of EU players can be distinguished from the behavior non-EU players. It turns out that if non-EU players do neither exhibit hedging nor reversal behavior in a game, they are *quasi-expected utility players*. That is, they behave as if if they were EU players. In this case, it is impossible to distinguish the players on the basis of their strategic behavior.

In general, it is difficult to infer players' preferences from their equilibrium actions. Bade (2011a) shows that the sets of equilibria of a two-player game and its ambiguous act extension are "observationally equivalent" in the sense that their supports coincide. My first main theorem is similar in nature, but holds also for n-player games with state-dependent payoffs. More precisely, it shows that one cannot identify non-EU players by observing equilibrium actions whenever the players do not show hedging behavior.

Theorem 1. *Fix a basic game $\bar{G} \in \Gamma$. Consider players with preferences $\succsim'$. If none of the players shows hedging behavior in $\bar{G}$, then, under Assumption 1 and 2, for any equilibrium point $\sigma^* \in \Sigma$ in $\langle \bar{G}, \succsim' \rangle$, there exist priors $\{\pi_i\}_{i \in I}$ such that σ^* is an equilibrium point in game $\langle \bar{G}, \succsim^{EU} \rangle$ where each player i is an expected utility maximizer and her beliefs about nature are represented by π_i.*

Theorem 1 shows that if players do not show hedging behavior, we need to consider their beliefs about nature or their best-response correspondences to distinguish EU from non-EU players. However, on the basis of beliefs, we are only able to identify non-EU players who are not probabilistically sophisticated in the sense of Machina and Schmeidler (1992). Moreover, from an experimental point of view, it might be difficult to measure players' beliefs. In complete information games, eliciting players' ex-ante beliefs about their opponents' strategy choice may affect their decisions in the game. In addition, there is evidence that players' ex-post beliefs are biased, see Rubinstein and Salant (2016). These difficulties could also limit the ability to measure players' beliefs about nature.

To state the second main theorem, we need to introduce the notion of best response equivalence. Roughly, two games are said to be best response equivalent if player i's best responses coincide both games for each $i \in I$. The precise definition is as follows.

Definition 4. Let R_i^G be the best-response correspondence of player i in $G \in \Gamma$. Two games $\langle G, \succsim \rangle$ and $\langle G', \succsim' \rangle$ are said to be *best response equivalent* if they consist of the same number of players and the same set of pure actions for each player, and if

$$R_i^G = R_i^{G'} \text{ for each player } i \in I.$$

The second theorem says that in any given two-person game players do neither exhibit hedging nor reversal behavior if and only if there exists a game with EU players which is best response equivalent to that game. That is, non-EU players behave strategically as if they were EU players if and only if they do not exhibit hedging and reversal behavior. To put it differently, hedging and reversal behavior are the sole behavioral differences between EU and non-EU players.

Theorem 2. *Consider a two-person game $\langle G, \succsim \rangle$, then, under Assumption 1 and 2, the following statements are equivalent:*

(i) Each player $i \in I$ exhibits neither hedging behavior nor reversal behavior in G.

(ii) There exists a game $\langle G', \succsim^{EU} \rangle$ which is best response equivalent to $\langle G, \succsim \rangle$.

Taken together, non-EU players who do not exhibit hedging and reversal behavior in a two-person basic game G cannot be distinguished from EU players by observing their equilibrium actions due to Theorem 1, and behave structurally as if they were EU players by Theorem 2. Therefore, these players can be termed quasi-expected utility players.

According to Definition 4, two games, which are best response equivalent, have the same number of players and, for each player, the same set of pure actions. However, the games may still have different state spaces and state-dependent payoffs. Consequently, one may ask under which conditions the game with EU players in statement (ii) of Theorem 2 has the same basic game structure as the game with non-EU players, i.e., under which conditions it holds that $G = G'$. In this case, it is impossible to identify non-EU players on the basis of their behavior. Proposition 1 gives sufficient conditions for this to be true in the context of two-person two-action games, which will be treated in Section 3.2. The proposition says that, given that players do not exhibit hedging and reversal behavior, it suffices that each player has a strictly dominant strategy and/or her state-dependent payoffs satisfy a particular condition. This condition says that there exists two states of the world such that in one of the two states, player i's first pure strategy is strictly worse than the second if the opponent plays her first pure strategy and strictly better than the second if the opponent plays her second pure strategy, and vice versa in the other state of the world. This condition can also be viewed as the requirement that player i's state-dependent utility function has strictly decreasing differences in one state of the world and strictly increasing differences in another state.[11] However, note that the requirement (ii) in Proposition 1 below is slightly stronger, since it requires increasing differences with respect to the reference point 0.

Proposition 1. *Fix a two-person two-actions game $\bar{G} \in \Gamma$. Let $A_i = \{a_i', a_i''\}$ be player i's action set. Consider players with preferences $\succsim$ who do not exhibit hedging and reversal behavior in $\bar{G}$. If, for each player i, at least one of the following conditions is met*

(i) player i has a strictly dominant strategy (strategic dominance),

(ii) there exist $\omega', \omega'' \in \Omega$ such that (strictly increasing and decreasing differences)

$$u_i(a_i', a_{-i}', \omega') - u_i(a_i'', a_{-i}', \omega') < 0 < u_i(a_i', a_{-i}'', \omega') - u_i(a_i'', a_{-i}'', \omega') \text{ and}$$

$$u_i(a_i', a_{-i}', \omega'') - u_i(a_i'', a_{-i}', \omega'') > 0 > u_i(a_i', a_{-i}'', \omega'') - u_i(a_i'', a_{-i}'', \omega''),$$

then, there exist priors $\{\pi_i\}_{i \in I}$ such that $\langle \bar{G}, \succsim^{EU} \rangle$ is best response equivalent to $\langle \bar{G}, \succsim \rangle$.

The results of this section show that we are only able to behaviorally distinguish non-EU and EU players if the non-EU players exhibit hedging and/or reversal behavior. In the next section, we will provide conditions under which players show hedging or reversal behavior, especially in terms of the payoff structure of a game.

[11] A function $f : X \times Y \to \mathbb{R}$ has *strictly increasing differences* if for $x', x'' \in X$, $x'' > x'$, and for $y', y'' \in Y$, $y'' > y'$, it holds that $f(x'', y'') - f(x', y'') > f(x'', y') - f(x', y')$.

3.2 Existence of hedging behavior and reversal behavior

General preferences

In this section, we consider games in which players are strictly uncertainty-averse. A player i is said to be *strictly uncertainty-averse in* $G \in \Gamma$ if her objective function V_i is strictly quasiconcave on the convex hull of her payoff vectors induced by the pure action profiles of the game, formally, $conv\{f_i(a) \mid a \in A\}$. In this case, the existence of hedging behavior is closely tied to the existence of strictly dominant strategies, as the following proposition demonstrates.

Proposition 2. *Let* $G \in \Gamma$ *be a basic game which has the following property, for each* $i \in I$ *and all* $a_i', a_i'' \in A_i$, $a_i' \neq a_i''$ *and any given* $\sigma_{-i} \in \Sigma_{-i}$, *it holds that*

$$f_i(a_i', \sigma_{-i}) \neq f_i(a_i'', \sigma_{-i}). \tag{P}$$

Consider a set of players who have strictly uncertainty-averse preferences, $\succsim^{UA}$, *in* G. *The following statements are equivalent:*

(i) Some players have no strictly dominant pure strategies.

(ii) Some players exhibit hedging behavior in G.

Proof. The proof of the proposition is straightforward. □

Although Proposition 2 is simple from a mathematical point of view, it has two interesting implications. Firstly, if we observe a mixed equilibrium in a basic game like the one in the proposition and we know that the players are strictly uncertainty-averse, then we can conclude that some players show hedging behavior.

Corollary 1. *If we observe a mixed equilibrium in a game* $\langle \bar{G}, \succsim^{UA} \rangle$ *where* G *meets property* (P), *then some players exhibit hedging behavior.*

Secondly, suppose that it is known that a player is strictly uncertainty-averse, but his particular objective function V_i is unknown. Then, we can exclude that the player exhibits hedging behavior if and only if she has a pure action that gives a strictly higher payoff in each state of the world than any other action for every fixed strategy combination of the opponents.

Corollary 2. *A strictly uncertainty-averse player i shows no hedging behavior in a basic game G which satisfies property* (P) *if and only if she has a pure action $a_i' \in A_i$ such that $u_i(a_i', a_{-i}, \omega) > u_i(a_i, a_{-i}, \omega)$ for all $\omega \in \Omega$ and all $a_i \in A_i$, $a_i \neq a_i'$ and any $a_{-i} \in A_{-i}$.*

Maxmin expected utility

This section studies two-person two-strategies games played by players whose preferences are represented by maxmin expected utility (or MEU players, for short). Experiments on game theory are often based on two-player two-strategies games and experiments on ambiguity frequently assume MEU subjects.

It is worth noting that the results of this section hold also for uncertainty-averse players with Choquet expected utility (or, briefly, CEU) preferences. This follows from the fact that uncertainty-averse CEU preferences, i.e., CEU with a convex capacity, correspond to MEU preferences where the prior set equals the set of probabilities in the core of the capacity, see Schmeidler (1986). Hence, preferences that can be represented by CEU can also be represented by MEU.[12]

The focus of this section lies on hedging behavior due to Theorem 1. All results, except Proposition 3, provide conditions under which we can exclude hedging and reversal behavior, respectively, for all possible prior sets. The negations of the results give existence conditions. One may think of a situation similar to that of Corollary 2: suppose that we know that a player i has MEU preferences, but his particular prior set C_i is unknown.

Ghirardato et al. (1998) and Klibanoff (2001) provide useful results concerning hedging behavior. They examine additivity and preference for mixtures, respectively, in the context of single-person decision problems and MEU preferences. A natural starting point for the question of additivity of the MEU functional is comonotonicity.[13] However, comonotonicity does not ensure additivity as an example in Klibanoff (1996) illustrates.

[12]Under uncertainty aversion, MEU is even a strict generalization of CEU, see Klibanoff (2001).

[13]Two vectors $f, g \in \mathbb{R}^m$ are comonotonic if $(f^\omega - f^{\omega'})(g^\omega - g^{\omega'}) \geq 0$ for all $\omega, \omega' \in \Omega$.

Ghirardato et al. (1998) show that we need a stronger condition for additivity of the MEU functional, called affine-relatedness.

Definition 5. Two vectors $f, g \in \mathbb{R}^m$ are *affinely related* if there exist $a \geq 0$ and $b \in \mathbb{R}$ such that $f^\omega = ag^\omega + b$ and/or $g^\omega = af^\omega + b$ for all $\omega \in \Omega$.

Definition 5 says that f and g are affinely related if either f or g is constant or there exist $a > 0$ and $b \in \mathbb{R}$ such that $f^\omega = ag^\omega + b$. We say that two vectors $f, g \in \mathbb{R}^m$ are *negatively affinely related* if f is affinely related to $-g$. Affine-relatedness implies comonotonicity, but the converse is not true. For the special case of two states of nature, affine-relatedness is equivalent to comonotonicity. According to Theorem 1 in Ghirardato et al. (1998), affine-relatedness guarantees additivity: let $f, g \in \mathbb{R}^m$, then $MEU_{C_i}(f+g) = MEU_{C_i}(f) + MEU_{C_i}(g)$ for all $C_i \in \mathscr{C}$ if and only if f and g are affinely related.

Another property, which we use in this section, is dominance-relatedness. This property refers to a situation in which one payoff vector (weakly) dominates another w.r.t. the state-dependent payoffs.

Definition 6. Two vectors $f, g \in \mathbb{R}^m$ are *dominance related* if $f^\omega \geq g^\omega$ and/or $g^\omega \geq f^\omega$ for all $\omega \in \Omega$.

Two vectors $f, g \in \mathbb{R}^m$ are said to be *strictly dominance related* if $f^\omega > g^\omega$ or $g^\omega > f^\omega$ for all $\omega \in \Omega$. Furthermore, a vector f is *constant* if $f^\omega = f^{\omega'}$ for all $\omega, \omega' \in \Omega$.

In the sequel, as in Proposition 1, $A_i = \{a_i', a_i''\}$ is player i's pure action set. By using negative affine-relatedness, we obtain a strong existence result for hedging behavior. Fix an action of the other player, if player i's pure actions induce negatively affinely related payoff vectors, then player i will show hedging behavior for all prior sets contained in a particular subset of $\mathscr{C}$. The following proposition makes this precise.

Proposition 3. *Fix a* $\overline{\sigma}_{-i} \in \Sigma_{-i}$. *Let* $\mathscr{C}^*$ *be the collection of all closed, convex and nonempty subsets of* $\Delta(\Omega)$ *which contain some* π', π'' *such that*

(i) $f_i(a_i', \overline{\sigma}_{-i})\pi' > f_i(a_i'', \overline{\sigma}_{-i})\pi'$ *and* $f_i(a_i', \overline{\sigma}_{-i})\pi'' < f_i(a_i'', \overline{\sigma}_{-i})\pi''$ *and*

(ii) $f_i(a_i', \overline{\sigma}_{-i})\pi' \neq f_i(a_i', \overline{\sigma}_{-i})\pi''$ *and* $f_i(a_i'', \overline{\sigma}_{-i})\pi' \neq f_i(a_i'', \overline{\sigma}_{-i})\pi''$.

If $f_i(a_i', \overline{\sigma}_{-i})$ and $f_i(a_i'', \overline{\sigma}_{-i})$ are negatively affinely related, then, a MEU_{C_i} player shows hedging behavior for all $C_i \in \mathscr{C}^*$.

In general the set $\mathscr{C}^*$ can vary strongly across different payoff vectors. Apparently, the set does not contain singletons, but it can be empty. For instance, $\mathscr{C}^*$ is empty when $f_i(a_i', \overline{\sigma}_{-i})$ and $f_i(a_i'', \overline{\sigma}_{-i})$ are dominance related. For the special case of two states of nature, there are only two possibilities: either $\mathscr{C}^*$ is empty or it contains all prior sets $C_i \in \mathscr{C}$ that are not singletons.

The first lemma illustrates that, given an action of the opponent, player i does not show hedging behavior for all prior sets if and only if i's pure actions induce payoff vectors that are strictly dominance related and/or affinely related.

Lemma 1. *Fix a $\overline{\sigma}_{-i} \in \Sigma_{-i}$. The following statements are equivalent:*

(i) $f_i(a_i', \overline{\sigma}_{-i})$ and $f_i(a_i'', \overline{\sigma}_{-i})$ are (a) strictly dominance related or (b) affinely related.

(ii) Given $\overline{\sigma}_{-i}$, a MEU_{C_i} player i shows no hedging behavior for all $C_i \in \mathscr{C}$.

Proof. The lemma is a variant of Theorem 2 in Klibanoff (2001). Therefore, the proof is omitted. □

The next proposition gives the most important result in this section. It states that, in many games, player i shows neither hedging nor reversal behavior for all prior sets if and only if player i's payoff vectors, which are induced by pure action profiles, are pairwise affinely related. That is, for any prior set $C_i \in \mathscr{C}$, the function MEU_{C_i} is additive on the convex hull of player i's payoff vectors. Hence, for every prior set C_i, there exist a prior $\pi_i' \in \Delta(\Omega)$ such that a MEU_{C_i} player and a $EU_{\pi_i'}$ player behave identically.

Proposition 4. *Consider a two-player two-strategies basic game $G \in \Gamma$ in which $f_i(a_i', \sigma_{-i})$ and $f_i(a_i'', \sigma_{-i})$ are not strictly dominance related for any $\sigma_{-i} \in \Sigma_{-i}$ and $f_i(a_i', a_{-i}) \neq f_i(a_i'', a_{-i})$ for any $a_{-i} \in A_{-i}$. If at most one of the vectors from the set $\{f_i(a) \mid a \in A\}$ is constant, the following statements are equivalent:*

(i) Player i's payoff vectors induced by pure action profiles are pairwise affinely related.

(ii) A MEU_{C_i} player i shows neither hedging nor reversal behavior in G for all $C_i \in \mathscr{C}$.

The statement in Proposition 4 is restricted to basic games where, given any action of the other player, the induced vectors of player i's pure actions are not strictly dominance related. However, this is not a strong restriction. To see why, note that, in most games, there exists a closed and convex subset, $\tilde{\Sigma}_{-i} \subseteq \Sigma_{-i}$, that satisfies the property that the induced vectors of player i's pure actions are not strictly dominance related.[14] Proposition 4 can be analogously applied to these games: player i shows no hedging and reversal behavior for all prior sets only if MEU_{C_i} is additive on the set of all vectors which are induced by the profiles which involve elements of $\tilde{\Sigma}_{-i}$.

Furthermore, Proposition 4 is restricted to basic games in which at most one of player i's payoff vectors induced by pure action profiles is constant, and in which any two pure action profiles induce different payoff vectors. The last two propositions discuss the existence of hedging behavior for basic games which do not satisfy these properties.

At first, we consider the case where more than one of player i's payoff vectors induced by pure action profiles is constant. In this case, a MEU_{C_i} player i exhibits no hedging behavior if and only if the MEU_{C_i} functional is additive for all induced vectors of the game and/or the vectors of one of player i's actions are constant, given any action of the other player.

Proposition 5. *Consider a two-player two-strategies basic game $G \in \Gamma$ in which there exists a $\sigma_{-i} \in \Sigma_{-i}$ such that $f_i(a_i', \sigma_{-i})$ and $f_i(a_i'', \sigma_{-i})$ are not strictly dominance related. If at least two of the vectors from $\{f_i(a) \mid a \in A\}$ are constant, the following statements are equivalent:*

(i) (a) Player i's payoff vectors induced by pure action profiles are pairwise affinely related and/or (b) the vectors $f_i(a_i', a_{-i}')$ and $f_i(a_i', a_{-i}'')$ are constant.

(ii) A MEU_{C_i} player i shows no hedging behavior in G for all $C_i \in \mathscr{C}$.

Finally, we turn to games in which player i's pure actions can induce equal payoff vectors, given an action of the other player. In these games, a MEU_{C_i} player i shows no hedging behavior if and only if the MEU_{C_i} functional is additive and/or player i's pure

[14] Exceptions are games where, for any given action of the opponent, one action induces a payoff vector which strictly dominates the vector induced by the other action.

actions induce equal vectors, given any pure action of the opponent.[15]

Proposition 6. *Consider a two-player two-strategies basic game $G \in \Gamma$ where $f_i(a'_i, \sigma_{-i})$ and $f_i(a''_i, \sigma_{-i})$ are not strictly dominance related for any $a_{-i} \in \Sigma_{-i}$ and some non-degenerate $\sigma_{-i} \in \Sigma_{-i}$. Furthermore, it holds that $f_i(a'_i, a_{-i}) = f_i(a''_i, a_{-i})$ for some $a_{-i} \in A_{-i}$. If at most one of the vectors from $\{f_i(a) \mid a \in A\}$ is constant, the following statements are equivalent:*

(i) (a) Player i's payoff vectors induced by pure action profiles are pairwise affinely related and/or (b) $f_i(a'_i, a_{-i}) = f_i(a''_i, a_{-i})$ for any given $a_{-i} \in A_{-i}$.

(ii) A MEU_{C_i} player i shows no hedging behavior in $\bar{G}$ for all $C_i \in \mathscr{C}$.

4 Discussion

4.1 Preference for randomization

A preference for randomization plays a central role in this paper. One may ask whether there exists evidence for such a preference. There is little experimental literature on this topic. One study by Dominiak and Schnedler (2011) finds no evidence for a preference for mixtures. However, the study is about single-person decisions and does not explicitly test for ex-ante and ex-post randomization attitudes, which I will elaborate on in the next subsection.

A further question is whether a preference for randomization leads to an infinite sequence of randomization operations: suppose a player strictly prefers a 1/2-mixture of two pure actions a_1 and a_2 over either alone, say, he prefers to flip a coin to determine his strategy choice. After flipping the coin, it turns out to be a_1. Due to his preferences before the coin flip, one may think that he would strictly prefer to flip the coin again and again...ad infinitum. An argument against this view is dynamic consistency, as Machina (1989) eloquently argues. In addition, an infinite sequence of randomization operations is impossible when mixed actions are generated by some kind of exogenous random device

[15] Note that the latter is not necessarily equivalent to the case where all payoff vectors induced by pure action profiles are equal, since it is still possible that $f_i(a'_i, a'_{-i}) \neq f_i(a'_i, a''_{-i})$.

and players accept binding commitments to play a pure action based on the outcome of this device.

4.2 The model

Assumption 1 of the model is crucial for the existence of hedging and reversal behavior. Recall that Assumption 1 states that a mixed action profile induces an expected utility value in each state of the world. There is no compelling reason for this assumption. Alternatively, we could have assumed that players' payoffs from a mixed profile equals the expectation of the representation function values of pure action profiles taken with respect to the distribution given by the mixed profile, formally,

Assumption 1′. Player i's payoff from a mixed profile $\sigma \in \Sigma$ is

$$U_i(\sigma) = \sum_{a \in A} \left(\prod_{j \in I} \sigma_j(a_j) V_i\big(f_i(a)\big) \right).$$

To see the difference between Assumption 1 and 1′, consider a player i with MEU preferences. According to Assumption 1, we need to take two expectations to determine player i's payoff from a mixed profile σ. At first, we take the expectation of the payoffs of pure action profiles w.r.t. the probability measure given by the mixed profile. This generates a vector of expected payoffs. Afterwards, expectations of this vector are taken w.r.t. each prior in a given prior set C_i. The minimum of this set of expectations corresponds to player i's minimum expected utility of σ. Let $EU_\pi(\sigma)$ be the expectation of the expected payoff vector w.r.t. π, then:

$$MEU_{C_i}(\sigma) = \min_{\pi \in C_i} \big\{ EU_\pi(\sigma) \big\},$$

In contrast, under Assumption 1′, player i's payoff from σ equals the expectation of player i's minimum expected utility from pure action profiles taken w.r.t. the probability measure given by σ:

$$\mathcal{MEU}_{C_i}(\sigma) = \sum_{a \in A} \left(\prod_{j \in I} \sigma_j(a_j) MEU_{C_i}(a) \right).$$

This implies that player i's objective function is linear in both her strategies and the strategies of the other players. Hence, under Assumption 1′, players do neither exhibit hedging nor reversal behavior. That is, they are quasi-expected utility players.

Now, which assumption is the "correct" one? In the class of games, we consider in this paper, there are two sources of uncertainty: strategic risk and ambiguous uncertainty about the environment. From a decision-theoretic perspective, this situation can be viewed as a two-stage lottery which involves

1. An ambiguous lottery which represents Nature's move.
2. A risky lottery which is given by players' mixed strategies.

In my view, the underlying assumption of the model depends on how players evaluate the two-stage lottery above. This is closely tied to the distinction between *ex-ante* and *ex-post randomization*. That is, how do players perceive the sequence of lottery 1. and 2., i.e., whether Nature's move takes place before or after the randomization by mixed strategies. In a recent paper, Eichberger et al. (2014) show that dynamically consistent individuals will be indifferent to ex-ante randomizations, but may exhibit a strict preference for ex-post randomizations. Following this result, we can associate Assumption 1 with ex-post randomization and Assumption $1'$ with ex-ante randomization.

Finally, it is worth mentioning that the model avoids the drawbacks of strategic ambiguity models described in Section 1.2 by not allowing for strategic ambiguity. From my point of view, it would be desirable to get an appropriate generalization of the model with a richer state space that incorporates strategic ambiguity.

4.3 Equilibrium without uncertainty-aversion

Azrieli and Teper (2011) show that the equilibrium according to Definition 1 may fail to exist when players are not ambiguity-averse (or ambiguity-neutral). In the following, I define an equilibrium for the BAT-model that allows for more general preferences. This equilibrium concept is based on the notion of equilibrium in beliefs introduced by Crawford (1990) and its n-player version defined in Zimper (2007).

Let $\Delta(\Sigma_i)$ be the set of all probability distributions with finite support on player i's mixed strategy set. An element $\beta_i \in \Delta(\Sigma_i)$ is a *belief about player i's mixed strategy choice.* Consequently, an element of the product space $\beta_{-i} \in \bigtimes_{j \in I \setminus \{i\}} \Delta(\Sigma_j)$ is *a belief of player i about the mixed strategy choices of the other players.* Let $\beta_{-i}(\sigma_{-i})$ be the probability with which player i believes that her opponents will play the strategy combination σ_{-i}. In analogy to Assumption 1, we assume that, in any given state $\omega \in \Omega$, each player's payoff from a mixed strategy $\sigma_i \in \Sigma_i$ and a belief $\beta_{-i} \in \bigtimes_{j\in I\setminus\{i\}} \Delta(\Sigma_j)$ corresponds to her expected utility. The probability which is assigned to a particular strategy combination of the other players equals the expectation taken with respect to the belief β_{-i}.

Assumption 3. Fix a state $\omega \in \Omega$, then player i's payoff from a mixed strategy $\sigma_i \in \Sigma_i$ and a belief $\beta_{-i} \in \bigtimes_{j\in I\setminus\{i\}} \Delta(\Sigma_j)$ is

$$f_i^\omega(\sigma_i, \beta_{-i}) = \sum_{a\in A} \Bigg(\sigma_i(a_i) \cdot \big[\sum_{\sigma_{-i}\in \text{supp}(\beta_{-i})} \beta_{-i}(\sigma_{-i}) \cdot \sigma_{-i}(a_{-i})\big]\Bigg) u_i(a,\omega) \text{ for each } i \in I.$$

According to Assumption 3, every mixed action σ_i together with a belief β_{-i} induces a vector of expected payoffs:

$$f_i(\sigma_i, \beta_{-i}) = \big(f_i^{\omega_1}(\sigma_i, \beta_{-i}), \ldots, f_i^{\omega_m}(\sigma_i, \beta_{-i})\big).$$

For each player i, let $V_i : \mathbb{R}^m \to \mathbb{R}$ be a continuous function that represents her preferences $\succsim_i$ over over m-dimensional payoff vectors. Then, players best responses to their beliefs can be defined in the usual way.

Definition 7. Consider a player i with preferences $\succsim_i$ represented by function V_i. *Player i's best responses to her belief* β_{-i} are

$$R_i(\beta_{-i}) = \{\sigma_i \mid \sigma_i \in \arg\max_{\sigma_i\in\Sigma_i} \; V_i\left(f_i(\sigma_i, \beta_{-i})\right)\}.$$

Now, we can introduce the solution concept for a game played by players whose preferences are not necessarily represented by quasiconcave functions.

Definition 8. An *equilibrium in beliefs* in a game $\langle G, \succsim \rangle$ is a beliefs combination $(\beta_i^*, \beta_{-i}^*)$ such that, for each player $i \in I$,

(i) $\beta_i^* \in \Delta(\Sigma_i)$ is the identical belief of all players $j \neq i$ about player i's mixed strategy choice and

(ii) $\sigma_i \in R_i(\beta_{-i}^*)$ for all $\sigma_i \in \text{supp}(\beta_i^*)$, i.e., player i's mixed strategies in the support of β_i^* are best responses to her belief β_{-i}^*.

An equilibrium in beliefs exists in any game $\langle G, \succsim \rangle$ in which the preferences of each player i are represented by a continuous functions V_i. This follows from the existence result in Zimper (2007, p. 69). Finally, observe that the equilibrium according to Definition 8 coincides with the equilibrium in the sense of Definition 1 whenever players' preferences are represented by quasiconcave functions.

5 Conclusion

This paper examines the extent to which we can distinguish expected and non-expected utility players on the basis of their behavior. Both types of players sometimes behave observationally equivalent, which means that we cannot infer players' preferences by observing their equilibrium actions. It is shown that expected and uncertainty-averse non-expected utility players can be distinguished on the basis of their best responses. Non-expected utility players may use mixed strategies differently, called hedging behavior, and may respond differently to mixed strategy combinations, called reversal behavior.

The first main theorem shows that if non-expected utility players do not show hedging behavior, then they behave observationally equivalent to expected utility players. The second main theorem states that hedging and/or reversal behavior are necessary and sufficient for distinguishing expected and non-expected utility players by looking at their best responses, i.e., these are the sole behavioral differences between the players. The last part of the paper provides necessary and sufficient conditions for the existence of hedging and reversal behavior in terms of the payoff structure of a two-person two-strategies game. Furthermore, I discuss the underlying model and introduce an equilibrium concept that allows for players who are not uncertainty-averse.

Our analysis provides insights into the model studied in this paper. It is useful for economic applications of this model and can serve as a guide to design experiments testing the model. Furthermore, this paper can be a starting point for further experimental and theoretical research. Besides the points mentioned in Section 4, one interesting question is, for instance, whether hedging or reversal behavior can be strategically exploited.

Appendix

Example 1 and 2 (Best response correspondences and equilibria).

Example 1. Given a strategy profile $(\sigma_1, \sigma_2) = \big(\sigma_1(q_l), \sigma_2(q_l)\big)$, firm 1's state-dependent expected profits are

$f_1^{\omega_1}(\sigma_1, \sigma_2) = \big(\frac{1}{2}\sigma_1[5 - 3\sigma_2] + 3\sigma_2 - 2\big)$ and $f_1^{\omega_2}(\sigma_1, \sigma_2) = \big(2 - \sigma_1\big)$.

If $\sigma_2 \leq 1/3$, then $f_1^{\omega_1}(\sigma_1, \sigma_2) \leq f_1^{\omega_2}(\sigma_1, \sigma_2)$ for all $\sigma_1 \in [0, 1]$. In this case, firm 1 will maximize $f_1^{\omega_1}(\sigma_1, \sigma_2)$ by playing $\sigma_1 = 1$. Otherwise, for any given $\sigma_2 > 1/3$, there exists a mixed strategy σ_1' such that $f_1^{\omega_1}(\sigma_1', \sigma_2) = f_1^{\omega_2}(\sigma_1', \sigma_2)$, which maximizes $V_1\big(f(\sigma_1, \sigma_2)\big) = \min\big\{f_1^{\omega_1}(\sigma_1, \sigma_2),\ f_1^{\omega_2}(\sigma_1, \sigma_2)\big\}$. By setting $f_1^{\omega_1}(\sigma_1, \sigma_2) = f_1^{\omega_2}(\sigma_1, \sigma_2)$, we obtain $\sigma_1' = \big(8 - 6\sigma_2\big)/\big(7 - 3\sigma_2\big)$. Due to the symmetry of the game, the same argumentation applies to firm 2. Consequently, firm i's best response correspondence is:

$$BR_i(\sigma_j(q_l)) = \begin{cases} 1, & \text{if } \sigma_j(q_l) \leq 1/3 \\ \big(8 - 6\sigma_j(q_l)\big)/\big(7 - 3\sigma_j(q_l)\big), & \text{if } \sigma_j(q_l) > 1/3 \end{cases}$$

The game has only one equilibrium: $\big(\sigma_1^*(q_l), \sigma_2^*(q_l)\big) \approx (0.74, 0.74)$.

Example 2. Given a strategy profile $(\sigma_M, \sigma_I) = \big(\sigma_M(S), \sigma_I(Bk)\big)$, players' state-dependent expected profits are

$f_M^{\omega_1}(\sigma_M, \sigma_I) = \big(\sigma_M[1 - \sigma_I]\big)$ and $f_M^{\omega_2}(\sigma_M, \sigma_I) = \big(\sigma_M[\sigma_I - 1] + 1 - \sigma_I\big)$, and

$f_I^{\omega_1}(\sigma_M, \sigma_I) = \big(\sigma_I[2 - 5\sigma_M] + 5\sigma_M\big)$ and $f_I^{\omega_2}(\sigma_M, \sigma_I) = \big(\sigma_I[5\sigma_M - 3] + 5 - 5\sigma_M\big)$.

If $\sigma_I = 1$, M is indifferent between all of his strategies, since $f_1^{\omega_1}(\sigma_M, 1) = 0 = f_1^{\omega_2}(\sigma_M, 1)$ for all $\sigma_M \in [0, 1]$. Otherwise, M's unique best response is $\sigma_M = 1/2$, where $f_1^{\omega_1}(1/2, \sigma_I) = f_1^{\omega_2}(1/2, \sigma_I)$ for all $\sigma_I \in [0, 1]$. Hence,

$$BR_M(\sigma_I(Bk)) = \begin{cases} 1/2, & \text{if } \sigma_I(Bk) \in [0, 1) \\ [0, 1], & \text{if } \sigma_I(BK) = 1 \end{cases}$$

Let $BR_I(\sigma_M(S)|\,\omega)$ be the investor's best response correspondence in state $\omega \in \{\omega_1, \omega_2\}$. Since $f_I^{\omega_1}(\sigma_M, \sigma_I) \leq (\geq) f_I^{\omega_2}(\sigma_M, \sigma_I)$ for $\sigma_M \leq (\geq) 1/2$ and all $\sigma_I \in [0,1]$, I's best response correspondence is

$$BR_I(\sigma_M(S)) = \begin{cases} BR_I(\sigma_M(S)|\,\omega_1), & \text{if } \sigma_M(S) \leq 1/2 \\ BR_I(\sigma_M(S)|\,\omega_2), & \text{if } \sigma_M(S) \geq 1/2 \end{cases}$$

The game has one equilibrium where the investor buys the stock: $\big(\sigma_M^*(S), \sigma_I^*(Bk)\big) = (0.5, 0)$, and infinitely many equilibria where she keeps her money: $\{\big(\sigma_M^*(S), \sigma_I^*(Bk)\big) \mid \sigma_M^*(S) \in [0, \frac{2}{5}] \cup [\frac{3}{5}, 1]$ and $\sigma_I^*(Bk) = 1\}$.

Notation 1. From now on, $f, g, h, k \in \mathbb{R}^m$ denote row payoff vectors and $\pi \in \Delta(\Omega)$ column probability vectors. A zero vector of proper dimension is denoted by $\mathbf{0}$. The following convention for ordering relations will be used. For real numbers, the relations $=, >, \geq$ are defined as usual. If $x, y \in \mathbb{R}^n$, $n > 1$, then

$x = y \Leftrightarrow x_i = y_i$ for $i = 1, \ldots, n$.

$x \geqq y \Leftrightarrow x_i \geqq y_i$ for $i = 1, \ldots, n$.

$x \geq y \Leftrightarrow x \geqq y$ and $x \neq y$.

$x > y \Leftrightarrow x_i > y_i$ for $i = 1, \ldots, n$.

Furthermore, for any set S, ∂S denotes the boundary of S, $int(S)$ the interior of S, and $cl(S)$ the closure of S. Matrix operations, e.g. matrix multiplication, inner product, and scalar multiplication, et cetera, are defined as usual. The same holds true for set operations such as intersection, union, set difference, et cetera.

Proof of Theorem 1. Fix a basic game $\bar{G} \in \Gamma$ and consider players with preferences $\succsim'$. Suppose $(\sigma_i^*, \sigma_{-i}^*) \in \Sigma$ is an equilibrium for the game $\langle \bar{G}, \succsim' \rangle$. Consider an arbitrary player i. Let V_i' be a function which represents i's preferences $\succsim_i'$ and satisfies Assumption 2. We prove the theorem by showing that if $\sigma_i^* \in \underset{\sigma_i \in \Sigma_i}{\arg\max}\ V_i'(\sigma_i, \sigma_{-i}^*)$, then there exists a $\pi_i \in \Delta(\Omega)$ such that $\sigma_i^* \in \underset{\sigma_i \in \Sigma_i}{\arg\max}\ EU_{\pi_i}(\sigma_i, \sigma_{-i}^*)$, whenever player i with preferences $\succsim_i'$ shows no hedging behavior in $\bar{G}$. In other words, if player i's best response to σ_{-i}^* is σ_i^*, given that her preferences are $\succsim_i'$, then there exists a prior such that σ_i^* is also a best response to σ^*_{-i} if i's preferences are $\succsim_i^{EU}$. This proves the theorem, since we consider an arbitrary player i. The proof for general finite strategy spaces is a bit tedious and confusing. For this reason, the proof is given for four actions, $A_i = \{a_1, a_2, a_3, a_4\}$, the generalization is straightforward. Given σ^*_{-i}, let $f, g, h, k \in \mathbb{R}^m$ be the payoff vectors induced by i's pure actions, i.e. $f = f_i(a_1, \sigma^*_{-i})$, $g = f_i(a_2, \sigma^*_{-i})$, et cetera. Hence, i's payoffs are

	σ^*_{-i}
a_1	f
a_2	g
a_3	h
a_4	k

We distinguish two cases: Player i's equilibrium strategy σ^*_i in $\langle \bar{G}, \succsim' \rangle$ is 1. a degenerate mixed action (resp. a pure action) or 2. a proper mixed action.

Case 1. W.l.o.g. (without loss of generality), we may assume that $\sigma^*_i = a_1$ is i's equilibrium action in $\langle \bar{G}, \succsim' \rangle$. Given that i exhibits no hedging behavior, we need to show that there exists a prior $\pi_i \in \Delta(\Omega)$ such that $EU_{\pi_i}(a_1, \sigma^*_{-i}) \geq EU_{\pi_i}(a_i, \sigma^*_{-i})$ for $a_i \in \{a_1, a_2, a_3, a_4\}$. Note that this is equivalent to

$$\exists \pi_i \in \Delta(\Omega): \ (f-g)\pi_i \geq 0, (f-h)\pi_i \geq 0, \text{ and } (f-k)\pi_i \geq 0 \tag{1}$$

Let I be a $m \times m$ identity matrix and define

$$x = \begin{pmatrix} \pi_i \\ \gamma \end{pmatrix} \in \mathbb{R}^{(m+1)},\ B = \begin{bmatrix} & 0 \\ I & \vdots \\ & 0 \end{bmatrix} \in \mathbb{R}^{m\times(m+1)},\ C = \begin{bmatrix} f-g & 0 \\ f-h & 0 \\ f-k & 0 \end{bmatrix} \in \mathbb{R}^{3\times(m+1)}, \text{ and}$$

$$D = \begin{pmatrix} 1 & \dots & 1 & -1 \end{pmatrix} \in \mathbb{R}^{1\times(m+1)}.$$

Then, condition (1) is equivalent to the system:

$$Bx \geq \mathbf{0},\ Cx \geqq \mathbf{0}, \text{ and } Dx = 0 \tag{2}$$

$Bx \geq \mathbf{0}$ ensures nonnegativity of the probabilities and $Dx = 0$ translates into $\sum_{\omega\in\Omega} \pi_i(\omega) = \gamma$, which can be normalized to $\sum_{\omega\in\Omega} \pi_i(\omega) = 1$. $Cx \geqq \mathbf{0}$ is the condition that a_1 is a best response to σ^*_{-i}.

Claim. System (2) *has a solution* $x \in \mathbb{R}^{(m+1)}$.

Proof. By Tucker's theorem of the alternative, cf. Mangasarian (1969, p. 29), either (2) has a solution $x \in \mathbb{R}^{(m+1)}$ or the equation $B^\top y^2 + C^\top y^3 + D^\top y^4 = \mathbf{0}$ has a solution $(y^2, y^3, y^4) \in \mathbb{R}^m \times \mathbb{R}^3 \times \mathbb{R}$ with $y^2 > \mathbf{0}$ and $y^3 \geqq \mathbf{0}$, which equals

$$\begin{bmatrix} \begin{pmatrix} y_1^2 \\ \vdots \\ y_m^2 \end{pmatrix} + (f-g)y_1^3 + (f-h)y_2^3 + (f-k)y_3^3 + \begin{pmatrix} y^4 \\ \vdots \\ y^4 \end{pmatrix} \\ -y^4 \end{bmatrix} = \mathbf{0} \tag{3}$$

Since $y^4 = 0$ and $y^2 > \mathbf{0}$, (3) has a solution iff (if and only if) there exists $y_1^3, y_2^3, y_3^3 \geq 0$ such that $(f-g)y_1^3 + (f-h)y_2^3 + (f-k)y_3^3 < \mathbf{0}$. This condition is equivalent to the existence of $\alpha, \beta \in [0,1]$ such that $f < \alpha g + \beta h + (1-\alpha-\beta)k$. Given σ^*_{-i}, the right-hand side of this inequality corresponds to the induced payoff vector of the following mixed action of player i: $\sigma'_i = \big(\sigma'_i(a_1), \sigma'_i(a_2), \sigma'_i(a_3), \sigma'_i(a_4)\big) = \big(0, \alpha, \beta, 1-\alpha-\beta\big)$. Hence, $f^\omega(a_1, \sigma^*_{-i}) < f^\omega(\sigma'_i, \sigma^*_{-i})$ for all $\omega \in \Omega$. Then, by Assumption 2 (monotonicity), $V_i(a_1, \sigma^*_{-i}) < V_i(\sigma'_i, \sigma^*_{-i})$ - a contradiction to the starting assumption that a_1 is the equilibrium strategy σ^*_i of player i in $\langle \bar{G}, \succsim' \rangle$. Consequently, (3) has no solution, which proves that (2) has a solution. □

Case 2. The proof of the second case follows the same line as the proof of the first case. W.l.o.g. assume that player i's equilibrium strategy, σ^*_i, is a proper mixed action with $\text{supp}(\sigma^*_i) = \{a_1, a_2\}$. We need to show that there exists a prior $\pi_i \in \Delta(\Omega)$ such that such that $EU_{\pi_i}(\sigma^*_i, \sigma^*_{-i}) \geq EU_{\pi_i}(a_i, \sigma^*_{-i})$ for $a_i \in \{a_1, a_2, a_3, a_4\}$. This is equivalent to the condition $\exists \pi_i \in \Delta(\Omega): \ (f-g)\pi_i = 0, (f-h)\pi_i \geq 0, (f-k)\pi_i \geq$

$0\,, (g-h)\pi_i \geq 0, (g-k)\pi_i \geq 0$ which can be expressed as

$$Bx \geq \mathbf{0},\ Cx \geqq \mathbf{0},\ \text{and } Dx = \mathbf{0}, \tag{4}$$

where $x = \begin{pmatrix} \pi_i \\ \gamma \end{pmatrix} \in \mathbb{R}^{(m+1)}$, $B = \begin{bmatrix} & 0 \\ I & \vdots \\ & 0 \end{bmatrix} \in \mathbb{R}^{m\times(m+1)}$, $C = \begin{bmatrix} f-h & 0 \\ f-k & 0 \end{bmatrix} \in \mathbb{R}^{2\times(m+1)}$,

and

$$D = \begin{bmatrix} 1 & \dots & 1 & -1 \\ & f-g & & 0 \end{bmatrix} \in \mathbb{R}^{2\times(m+1)}.$$

Claim. System (4) *has a solution* $x \in \mathbb{R}^{(m+1)}$.

Proof. According to Tucker's theorem, the alternative to the claim is that

$$\begin{bmatrix} \begin{pmatrix} y_1^2 \\ \vdots \\ y_m^2 \end{pmatrix} + (f-h)y_1^3 + (f-k)y_2^3 + (f-g)y_2^4 + \begin{pmatrix} y_1^4 \\ \vdots \\ y_1^4 \end{pmatrix} \\ -y_1^4 \end{bmatrix} = \mathbf{0} \tag{5}$$

has a solution $(y^2, y^3, y^4) \in \mathbb{R}^m \times \mathbb{R}^2 \times \mathbb{R}^2$ with $y^2 > \mathbf{0}$ and $y^3 \geqq \mathbf{0}$.

Equation (5) has a solution iff $(f-h)y_1^3+(f-k)y_2^3+(f-g)y_2^4 < 0$ for some $y_1^3, y_2^3 \geq 0$ and $y_2^4 \in \mathbb{R}$. For $y_2^4 \geq 0$, one obtains the same contradiction as before. If $y_2^4 < 0$, then there are $\alpha, \beta \in [0,1]$ such that $(\alpha+\beta)f+(1-\alpha-\beta)g < \alpha h+\beta k+(1-\alpha-\beta)f$. Given σ_{-i}^*, let σ_i' be the mixed action of i that induces the vector on the left-hand side of the inequality and σ_i'' the action that induces the vector on the right-hand side. By Assumption 2 (monotonicity), $V_i(\sigma_i', \sigma_{-i}^*) < V_i(\sigma_i'', \sigma_{-i}^*)$. Furthermore, since i shows no hedging behavior, it holds that $V_i(\sigma_i', \sigma_{-i}^*) = (\alpha+\beta)V_i(a_1, \sigma_{-i}^*) + (1-\alpha-\beta)(a_2, \sigma_{-i}^*) = V_i(\sigma_i^*, \sigma_{-i}^*)$. Consequently, $V_i(\sigma_i^*, \sigma_{-i}^*) < V_i(\sigma_i'', \sigma_{-i}^*)$, a contradiction to the starting assumption that σ_i^* is i's equilibrium strategy. This proves that (5) has no solution which implies that (4) has a solution. $\square$

Since player i was chosen arbitrarily, for any given equilibrium point σ^* of $\langle \bar{G}, \succsim' \rangle$, there exists a prior π_i for each $i \in I$, such that σ^* is an equilibrium point of $\langle \bar{G}, \succsim^{EU} \rangle$, which proves the theorem. □

In order to prove Theorem 2, we need the following lemma:

Lemma 2. *Let Δ^d be the d-dimensional unit simplex, $d < \infty$, and let $\mathscr{B}$ be a finite collection of closed, convex and nonempty sets . If*

(i) $\bigcup_{B \in \mathscr{B}} B = \Delta^d$ *and*

(ii) $int(B') \cap int(B'') = \varnothing$ *for all* $B', B'' \in \mathscr{B}$,

then each B in $\mathscr{B}$ is a polyhedron.

Proof. If $\mathscr{B}$ is a singleton, then the statement is trivial by (i). Assume that $\mathscr{B}$ is not a singleton. Since each B in $\mathscr{B}$ is closed (i.e. $\partial B \subseteq B$), (ii) implies that $B' \cap B'' = \partial B' \cap \partial B''$ for all $B', B'' \in \mathscr{B}$. Furthermore, by (i), if $x \in \partial B'$, then $x \in \partial B''$ for some $B'' \in \mathscr{B}$ and/or $x \in \partial \Delta^d$, formally $\partial B' = \left[\bigcup_{B'' \in \mathscr{B} \setminus B'} (B' \cap B'') \right] \cup (\partial B' \cap \Delta^d)$.

It holds that $\partial B' = cl(\partial B')$, because $\partial B'$ is closed. Hence, $\partial B' = int(\partial B') \,\dot{\cup}\, \partial\partial B'$. Due to $\partial\partial B' = \partial B'$, it follows that $int(\partial B') = \varnothing$. Therefore, $int(B' \cap B'') = \varnothing$. Furthermore, $(\partial B' \cap \partial B'')$ is closed and convex, since it is an intersection of closed and convex sets (recall that $B' \cap B'' = \partial B' \cap \partial B''$). Taken together, $(\partial B' \cap \partial B'')$ is a closed and convex set with empty interior, which implies that $(\partial B' \cap \partial B'')$ is contained in a hyperplane. In addition, $(\partial B' \cap \partial \Delta^d)$ is contained in a hyperplane, since $\partial \Delta^d$ is contained in a hyperplane. Thus, $(\partial B' \cap \partial B'')$ is contained in a hyperplane for all $B'' \in \mathscr{B} \setminus B'$ and $(\partial B' \cap \partial \Delta^d)$ is contained in a hyperplane. Therefore, $\partial B'$ is contained in the union of finitely many hyperplanes, formally $\partial B' \subseteq \bigcup_{n \in N} H_n$, where H_n is a hyperplane and N an index set. Let $\mathscr{H}_n$ be a half-space, which is associated with hyperplane n. Then, there exists n half-spaces such that $B' \subseteq \bigcap_{n \in N} \mathscr{H}_n$, since B' is a convex set. Furthermore, it holds that $B' \supseteq \bigcap_{n \in N} \mathscr{H}_n$, since the boundary of B' is contained in the hyperplanes associated with the half-spaces. Consequently, B' equals the intersection of finitely many half-spaces. That is, B' is a polyhedron, which proves the claim, since B' was chosen arbitrarily. □

Proof of Theorem 2. "(i) $\implies$ (ii)". The proof relies on the following fact: Consider a finite two-player normal-form game with complete information or with incomplete information and EU players. Let $i \in \{1, 2\}$ and $j = 3 - i$ denote the players. Then, for each player i, it holds that the preimages of i's pure actions under her best response correspondence are either empty or polyhedral subsets of the set of j's mixed strategies, Σ_j, which corresponds to the $|A_j|$-dimensional unit simplex. For instance, the preimages of player i's pure strategies in the well-known Rock-paper-scissors-game are:[16]

Proof. By statement (i) of the theorem, i exhibits no hedging behavior in G. This implies that for every $\sigma_j \in \Sigma_j$, there exists a pure action $a_i \in A_i$ which is a best response to σ_j, i.e. $\bigcup_{i=1}^{K} B_k \supseteq \Sigma_j$. Furthermore, by the definition of a best response correspondence, $\bigcup_{i=1}^{K} B_k \subseteq \Sigma_j$. Hence,

$$\bigcup_{i=1}^{K} B_k = \Sigma_j, \tag{6}$$

which means that (i) implies (*). □

Claim. (i) implies (**).

Proof. Due to (i), i shows no reversal behavior in G. The negation of condition (ii) in Definition 3 implies:

$$int(B_k) \cap int(B'_k) = \varnothing \tag{7}$$

for all $k, k' \in \{1, \ldots, K\}$, $k \neq k'$.

W.l.o.g., we may assume that $B_k \neq \varnothing$ and $B_k \neq B_{k'}$ for all $k, k' \in \{1, \ldots, K\}$, $k \neq k'$. According to Assumption 2, the function $V_i(\cdot)$, which represents i's preferences, is continuous. Therefore, each B_k is closed. Furthermore, the negation of condition (i) in Definition 3 implies that each B_k is convex. Considering these properties together with equation (6) and (7) and using Lemma 2, we see that each B_k is a polyhedron. □

To sum up, $(i) \Rightarrow$ (*) and (**) $\Rightarrow (ii)$.

"(i) $\Longleftarrow$ (ii)". The examples in Section 1.1 illustrate that $\neg(i) \Rightarrow \neg(ii)$, which is logically equivalent to (i) $\Leftarrow$ (ii). □

Notation 2. From now on, $f, g, h, k \in \mathbb{R}^m$ denote player i's payoff vectors which are induced by pure actions profiles in a given two-player two strategies basic game, i.e. i's payoff matrix is:

	a'_{-i}	a''_{-i}
a'_i	f	g
a''_i	h	k

Proof of Proposition 1. In some parts of the proof, the argumentation is based on theorems of the alternative like in the proof of Theorem 1. These parts of the proof will be only sketched.

(i) Consider player i and suppose she has a strictly dominant strategy in $\langle \bar{G}, \succsim \rangle$. W.l.o.g. assume that a_i' is strictly dominant. If

$$\exists \pi_i \in \Delta(\Omega) : (f - h)\pi_i > 0 \text{ and } (g - k)\pi_i > 0, \tag{8}$$

then a_i' is also a strictly dominant strategy for i in case she has prior π_i and EU preferences. By applying Motzkin's theorem, cf. Mangasarian (1969, p. 28-29), we obtain an alternative to the condition (8). This alternative has a solution iff $\alpha f + (1 - \alpha)g \leqq \alpha h + (1 - \alpha)k$ for some $\alpha \in [0, 1]$. Then, by Assumption 2 (monotonicity), there exists a $\sigma_{-i} \in \Sigma_{-i}$ such that $V_i(a_i', \sigma_{-i}) \leq V_i(a_i'', \sigma_{-i})$. This contradicts the assumption that a_i' is a strictly dominant strategy. Consequently, (8) has a solution.

(ii) Suppose i has no strictly dominant strategy. By Theorem 2, $\langle \bar{G}, \succsim \rangle$ is best response equivalent to some $\langle G', \succsim^{EU} \rangle$. Let $f', g', h', k' \in \mathbb{R}^m$ be i's payoff vectors induced by pure action profiles in G'. Note that $\langle G', \succsim^{EU} \rangle$ is best response equivalent to a two-player complete information game with identical action sets, where player i's payoffs equal the expected utility values: $U_{f'} = EU_{\pi_i}(f')$, $U_{g'} = EU_{\pi_i}(g')$, et cetera, see matrix (a) below. Furthermore, it is well-known that player i's best response sets are unaffected if we transform her payoff matrix (a) into matrix (b) where $z > 0$ and $\varepsilon, \delta \in \mathbb{R}$, see e.g. Weibull (1995).

(a)

	a_{-i}'	a_{-i}''
a_i'	$U_{f'}$	$U_{g'}$
a_i''	$U_{h'}$	$U_{k'}$

(b)

	a_{-i}'	a_{-i}''
a_i'	$zU_{f'} + \varepsilon$	$zU_{g'} + \delta$
a_i''	$zU_{h'} + \varepsilon$	$zU_{k'} + \delta$

Since $\langle \bar{G}, \succsim \rangle$ is best response equivalent to $\langle G', \succsim^{EU} \rangle$ and $\langle G', \succsim^{EU} \rangle$ is best response equivalent to a complete information game where player i's payoff matrix is matrix

(b) above. Therefore, the second part of the proposition is proven if

$$\begin{aligned}&\exists \pi_i \in \Delta(\Omega), z > 0, \varepsilon, \delta \in \mathbb{R} : \\ &f\pi_i = zU_{f'} + \varepsilon, h\pi_i = zU_{h'} + \varepsilon, g\pi_i = zU_{g'} + \delta \text{ and } k\pi_i = zU_{k'} + \delta.\end{aligned} \tag{9}$$

By using Motzkin's theorem again, we obtain an alternative to (9) which has a solution iff $(f-h)y_1^4 + (g-k)y_3^4 \leqq \mathbf{0}$ and $(U_{f'} - U_{h'})y_1^4 + (U_{g'} - U_{k'})y_3^4 > 0$ for some $y_1^4, y_3^4 \in \mathbb{R}$. For $y_1^4 = 0$, $y_3^4 = 0$, $y_1^4, y_3^4 > 0$ and $y_1^4, y_3^4 < 0$, we get a similar contradiction as in case of a strictly dominant strategy. If $y_1^4 > 0$ and $y_3^4 < 0$, the first part of the alternative condition equals $(f-h)a \leqq (g-k)$ for some $a > 0$. However, by (ii) of the proposition, there exists a $\omega'' \in \Omega$ such that $(f^{\omega''} - h^{\omega''}) > 0$ and $(g^{\omega''} - k^{\omega''}) < 0$ which contradicts this condition. Similarly, (ii) contradicts the first part of the alternative if $y_1^4 < 0$ and $y_3^4 > 0$. Therefore, (9) has a solution, which completes the proof.

□

Proof of Proposition 3. Consider a two-players two-strategies game and fix a $\bar{\sigma}_{-i} \in \Sigma_{-i}$. Let $f(a_i', \bar{\sigma}_{-i}) = f$ and $f(a_i'', \bar{\sigma}_{-i}) = g$ denote player i's payoff vectors induced by her pure actions. Note that every vector induces, through expectation, an ordering on probabilities. The proof is based on the fact that affine-relatedness implies that the induced orderings of two vectors are identical, see Ghirardato et al. (1998). That is, if f and $-g$ are affinely related, then f and g induce opposite orderings on the probabilities. Assume that the set $\mathscr{C}^*$ is nonempty, which implies that f and g are not dominance related and non-constant. Take an arbitrary $C_i \in \mathscr{C}^*$. Since there are $\pi', \pi'' \in C_i$ such that $f\pi' \neq f\pi''$ and $g\pi' \neq g\pi''$, it holds that $\arg\min_{\pi \in C_i} \{EU_\pi(f)\} \cap \arg\max_{\pi \in C_i} \{EU_\pi(f)\} = \varnothing$ and $\arg\min_{\pi \in C_i} \{EU_\pi(g)\} \cap \arg\max_{\pi \in C_i} \{EU_\pi(g)\} = \varnothing$. Furthermore, if f and g are negatively affinely related, it holds that $\arg\min_{\pi \in C_i} \{EU_\pi(f)\} \cap \arg\min_{\pi \in C_i} \{EU_\pi(g)\} = \varnothing$. Then, by Lemma 1 in Ghirardato et al. (1998), $MEU_{C_i}(f+g) \neq MEU_{C_i}(f) + MEU_{C_i}(g)$. Hence, we are done if $MEU_{C_i}(f) = MEU_{C_i}(g)$. W.l.o.g. assume that $MEU_{C_i}(f) > MEU_{C_i}(g)$. Let $MEU_{C_i}(f) = f\tilde{\pi}$. Since there is a $\pi'' \in C_i$ such that $f\pi'' < g\pi''$, we have that

$f\tilde{\pi} \leq f\pi'' < g\pi''$. Moreover, it holds that $g\pi'' \leq g\tilde{\pi}$ because f is negatively affinely related to g. Hence, $f\tilde{\pi} < g\tilde{\pi}$. Then, for sufficiently high $\alpha \in [0,1] : MEU_{C_i}(\alpha f + (1-\alpha)g) = \alpha f\tilde{\pi} + (1-\alpha)g\tilde{\pi} > f\tilde{\pi} = MEU_{C_i}(f)$. This means that there exists a mixed action, $(\sigma_i(a_i'), \sigma_i(a_i'')) = (\alpha, 1-\alpha)$, which is a strictly better response to $\bar{\sigma}_{-i}$ than a_i'. Since we assumed that a_i' is a strictly better response to $\bar{\sigma}_{-i}$ than a_i'', player i exhibits hedging behavior, which proves the proposition. □

Before proving Proposition 4, we need a couple of lemmas.

Lemma 3. *Fix a two-player two-strategies basic game $\bar{G} \in \Gamma$ where at most one of $f, g, h, k \in \mathbb{R}^m$ is constant and f, h and g, k are not strictly dominance related. If a MEU_{C_i} player i exhibits no hedging behavior in $\bar{G}$ for all $C_i \in \mathscr{C}$, then one of the following statements is true:*

(i) *f, h and g, k are affinely related, there is no $\omega' \in \Omega$ such that $f^{\omega'} = h^{\omega'}$ and $g^{\omega'} = k^{\omega'}$, and h is weakly dominated by f and k by g or vice versa.*

(ii) *f, h and g, k are affinely related and $f = h$ and/or $g = k$.*

(iii) *f, g, h, k are pairwise affinely related.*

(iv) *$f, -g, h, -k$ are pairwise affinely related.*

Proof. Since f, h and g, k are not strictly dominance related, by Lemma 1, if a MEU_{C_i} player i shows no hedging behavior in $\bar{G}$ for all $C_i \in \mathscr{C}$, then f, h and g, k are affinely related. Hence, for all $\omega \in \Omega$, it holds that,

(*) $h^\omega = a'f^\omega + b'$ for some $a' \geq 0$, $b' \in \mathbb{R}$ and

(**) $k^\omega = a''g^\omega + b''$ for some $a'' \geq 0$, $b'' \in \mathbb{R}$.

Furthermore, either

(***) $\alpha f + (1-\alpha)g$ and $\alpha h + (1-\alpha)k$ are strictly dominance related for all $\alpha \in (0,1)$ or not. In the latter case, i shows no hedging behavior in $\bar{G}$ for all $C_i \in \mathscr{C}$ only if $\alpha' f + (1-\alpha')$ is affinely related to $\alpha' h + (1-\alpha')k$, whenever $\alpha' f + (1-\alpha')$ and $\alpha' h + (1-\alpha')k$ are not strictly dominance related. Hence, there exist $\alpha' \in (0,1)$ such that, for all $\omega \in \Omega$,

(****) $\alpha' f^\omega + (1-\alpha')g^\omega = a'''[\alpha' h^\omega + (1-\alpha')k^\omega] + b'''$ for some $a''' \geq 0$, $b''' \in \mathbb{R}$.

Suppose (***) is true.

(i) Since f, h and g, k are not strictly dominance related (***) is true iff there is no $\omega' \in \Omega$ such that $f^{\omega'} = h^{\omega'}$ and $g^{\omega'} = k^{\omega'}$, and h is weakly dominated by f and k by g or vice versa.

Now, suppose (***) is not true. Then, there exist $\alpha' \in (0,1)$ such that $\alpha' f + (1-\alpha')g$ and $\alpha' h + (1-\alpha')k$ are not dominance related and non-constant.

(ii) If $f = h$, then (**) implies (****). Similarly, (*) implies (****), whenever $g = k$.

W.l.o.g. assume that f and g are non-constant and let $f \neq h$ and $g \neq k$.

(iii) If f is affinely related to g, then (*) and (**) imply that h and k are affinely related, either because one of the vectors is constant or by transitivity. Hence, all vectors are pairwise affinely related.

(iv) If f and g are not affinely related, then (*),(**), (****) imply that $f^\omega = \tilde{b} g^\omega + \hat{b}$ for all $\omega \in \Omega$ and some $\tilde{b}, \hat{b} \in \mathbb{R}$ where $\tilde{b} \neq 0$, otherwise f is constant. Since f and g are not affinely related, it holds that $\tilde{b} < 0$, which means that f is affinely related to $-g$. Then, h and $-k$ are affinely related by transitivity or because one of the vectors is constant.

□

Lemma 4. *Let at most one of the payoff vectors f, g, h, k be constant and $f, -g, h, -k$ be pairwise affinely related. If there exist $\pi', \pi'' \in \Delta(\Omega)$ such that $\frac{f\pi' - f\pi''}{g\pi'' - g\pi'} \neq \frac{h\pi' - h\pi''}{k\pi'' - k\pi'}$, then $\alpha f + (1-\alpha)g$ is not affinely related to $\alpha h + (1-\alpha)k$ for some $\alpha \in (0,1)$.*

Proof. W.l.o.g. assume that $\frac{f\pi' - f\pi''}{g\pi'' - g\pi'} > \frac{h\pi' - h\pi''}{k\pi'' - k\pi'}$ and $f\pi' > f\pi''$. The latter implies that $h\pi' > h\pi''$, $g\pi' < g\pi''$, and $k\pi' < k\pi''$, since $f, -g, h, -k$ are pairwise affinely related. Therefore, it holds that $\alpha f\pi' + (1-\alpha)g\pi' \geq \alpha f\pi'' + (1-\alpha)g\pi''$ for all $\alpha \geq \frac{g\pi'' - g\pi'}{g\pi'' - g\pi' + f\pi' - f\pi''}$ and $\alpha h\pi' + (1-\alpha)k\pi' \geq \alpha h\pi'' + (1-\alpha)k\pi''$ for all $\alpha \geq \frac{k\pi'' - k\pi'}{k\pi'' - k\pi' + h\pi' - h\pi''}$. Furthermore, $\frac{f\pi' - f\pi''}{g\pi'' - g\pi'} > \frac{h\pi' - h\pi''}{k\pi'' - k\pi'}$ implies that $\frac{g\pi'' - g\pi'}{g\pi'' - g\pi' + f\pi' - f\pi''} < \frac{k\pi'' - k\pi'}{k\pi'' - k\pi' + h\pi' - h\pi''}$. Consequently, $\alpha f\pi' + (1-\alpha)g\pi' > \alpha f\pi'' + (1-\alpha)g\pi''$ and $\alpha h\pi' + (1-\alpha)k\pi' < \alpha h\pi'' + (1-\alpha)k\pi''$ for all $\alpha \in \left(\frac{g\pi'' - g\pi'}{g\pi'' - g\pi' + f\pi' - f\pi''}, \frac{k\pi'' - k\pi'}{k\pi'' - k\pi' + h\pi' - h\pi''}\right)$. That is, there exist $\alpha \in (0,1)$ such that $\alpha f + (1-\alpha)g$ and $\alpha h + (1-\alpha)k$ induce different orderings on probabilities, which means that these payoff vectors are not affinely related. □

Lemma 5. *Let* $e(\omega) = \frac{(k^\omega - g^\omega)}{(k^\omega - g^\omega + f^\omega - h^\omega)}$ *for* $\omega \in \Omega$ *and define the sets:*
$E_- = \{e(\omega) \mid (k^\omega - g^\omega + f^\omega - h^\omega) < 0\}$ *and* $E_+ = \{e(\omega) \mid (k^\omega - g^\omega + f^\omega - h^\omega) > 0\}$.
The following statements are equivalent.

(i) $\alpha f + (1-\alpha)g$ *strictly dominates* $\alpha h + (1-\alpha)k$ *for some* $\alpha \in [0, 1]$.

(ii) (a) For each $\omega \in \Omega$*:* $f^\omega > h^\omega$ *and/or* $g^\omega > k^\omega$ *and (b)* $\max\{E_+\} < \min\{E_-\}$.

Proof. Statement (i) says that there exist $\alpha \in [0, 1]$ which solves the following system of linear inequalities:

$$\begin{gathered}\alpha f^{\omega_1} + (1-\alpha)g^{\omega_1} > \alpha h^{\omega_1} + (1-\alpha)k^{\omega_1}\\ \vdots \\ \alpha f^{\omega_m} + (1-\alpha)g^{\omega_m} > \alpha h^{\omega_m} + (1-\alpha)k^{\omega_m}\end{gathered}$$

This system is solvable iff each inequality has a nonempty solution set, which corresponds to condition (ii)(a), and the solutions sets of all inequalities have a nonempty intersection, which is equivalent to condition (ii)(b). □

Proof of Proposition 4. The proof of "(i) $\implies$ (ii)" is trivial. "(i) $\impliedby$ (ii)". Under the assumptions of the proposition, Lemma 3 shows that statement (ii) implies either (i) or $f, -g, h, -k$ are pairwise affinely related. Suppose that (ii) implies the latter. By the assumptions of the proposition, it holds that $\nexists\, \alpha \in [0, 1] : \alpha f + (1-\alpha)g > \alpha h + (1-\alpha)k$ or vice versa. The negation of Lemma 5 implies that

$$f^{\omega'} \leq h^{\omega'} \text{ and } g^{\omega'} \leq k^{\omega'} \text{ for some } \omega' \in \Omega \text{ and/or } \max\{E_+\} \geq \min\{E_-\} \text{ and} \tag{10}$$

$$h^{\omega''} \leq f^{\omega''} \text{ and } k^{\omega''} \leq g^{\omega''} \text{ for some } \omega'' \in \Omega \text{ and/or } \max\{E_-\} \geq \min\{E_+\}. \tag{11}$$

At first, consider the case where the first condition of (10) and/or (11) is violated. W.l.o.g. assume that the first condition of (10) is violated. That is, for each $\omega \in \Omega$: $f^\omega > h^\omega$ and/or $g^\omega > k^\omega$. Furthermore, $\max\{E_+\} \geq \min\{E_-\}$, otherwise $\alpha f + (1-\alpha)g > \alpha h + (1-\alpha)k$ for some $\alpha \in [0, 1]$. If $f^\omega > h^\omega$ for all $\omega \in \Omega$ and/or $g^\omega > k^\omega$ for all $\omega \in \Omega$, then f strictly dominates h and/or g strictly dominates k, which contradicts the assumptions of the proposition. Therefore, suppose that there are $\omega', \omega'' \in \Omega$ such that

$f^{\omega'} \leq h^{\omega'}$ and $g^{\omega''} \leq k^{\omega''}$. Let $e(\omega_+) \in \max\{E_+\}$ and $e(\omega_-) \in \min\{E_-\}$. Due to $f^{\omega_-} > h^{\omega_-}$ and/or $g^{\omega_-} > k^{\omega_-}$, it holds that $e(\omega_-) > 0$. If $g^{\omega_+} \geq k^{\omega_+}$, then $e(\omega_+) \leq 0 < e(\omega_-)$ - a contradiction. Therefore, $g^{\omega_+} < k^{\omega_+}$ and $f^{\omega_+} > h^{\omega_+}$, which implies that $e(\omega_+) < 1$. If $f^{\omega_-} \geq h^{\omega_-}$, then $e(\omega_-) \geq 1 > e(\omega_+)$ - a contradiction. Therefore, $f^{\omega_-} < h^{\omega_-}$ and $g^{\omega_-} > k^{\omega_-}$. Taken together, we have that,

(*) $g^{\omega_+} < k^{\omega_+}$ and $f^{\omega_+} > h^{\omega_+}$; $f^{\omega_-} < h^{\omega_-}$ and $g^{\omega_-} > k^{\omega_-}$.

W.l.o.g. we may assume that $f^{\omega_+} \leq f^{\omega_-}$. Then, since f is affinely related to h and negatively affinely related to g and k,

(**) $h^{\omega_+} < h^{\omega_-}$, $g^{\omega_+} \geq g^{\omega_-}$ and $k^{\omega_+} > k^{\omega_-}$.

Now, consider the prior set $\bar{C}_i = \{\beta\delta_{\omega_+} + (1-\beta)\delta_{\omega_-} \mid \beta \in [0,1]\}$ where δ_ω denotes the measure concentrated on $\omega \in \Omega$. Then, by (*) and (**), $MEU_{\bar{C}_i}(f) = f^{\omega_+} > h^{\omega_+} = MEU_{\bar{C}_i}(h)$ and $MEU_{\bar{C}_i}(g) = g^{\omega_-} > k^{\omega_-} = MEU_{\bar{C}_i}(k)$. This means that action a_i' is the unique best response of a $MEU_{\bar{C}_i}$ player i to a_{-i}' and a_{-i}''. Consequently, it needs to hold that a_{-i}' is the unique best response to $\alpha a_{-i}' + (1-\alpha)a_{-i}''$ for all $\alpha \in [0,1]$. Otherwise, player i exhibits reversal behavior, which contradicts statement (ii). Let $\underline{\alpha} = \frac{k^{\omega_+} - g^{\omega_+}}{k^{\omega_+} - g^{\omega_+} + f^{\omega_+} - h^{\omega_+}} \in (0,1)$ and $\overline{\alpha} = \frac{g^{\omega_-} - k^{\omega_-}}{g^{\omega_-} - k^{\omega_-} + h^{\omega_-} - f^{\omega_-}} \in (0,1)$. Then, $\alpha f^{\omega_+} + (1-\alpha) g^{\omega_+} \leq \alpha h^{\omega_+} + (1-\alpha) k^{\omega_+}$ for all $\alpha \in [0, \underline{\alpha}]$ and $\alpha f^{\omega_-} + (1-\alpha) g^{\omega_-} \leq \alpha h^{\omega_-} + (1-\alpha) k^{\omega_-}$ for all $\alpha \in [\overline{\alpha}, 1]$. Player i exhibits no reversal behavior only if $\underline{\alpha} < \frac{g^{\omega_+} - g^{\omega_-}}{g^{\omega_+} - g^{\omega_-} + f^{\omega_-} - f^{\omega_+}} < \overline{\alpha}$, which is equivalent to

(***) $\frac{k^{\omega_+} - g^{\omega_+}}{f^{\omega_+} - h^{\omega_+}} < \frac{g^{\omega_+} - g^{\omega_-}}{f^{\omega_-} - f^{\omega_+}}$ and $\frac{k^{\omega_-} - g^{\omega_-}}{f^{\omega_-} - h^{\omega_-}} > \frac{g^{\omega_+} - g^{\omega_-}}{f^{\omega_-} - f^{\omega_+}}$.

However, (*), (**), (***), and the affine-relatedness condition from Lemma 4, $\frac{f^{\omega_-} - f^{\omega_+}}{g^{\omega_+} - g^{\omega_-}} = \frac{h^{\omega_-} - h^{\omega_+}}{k^{\omega_+} - k^{\omega_-}}$, lead to a contradiction, see the Mathematica code at the end of this proof. That is, either a $MEU_{\bar{C}_i}$ player exhibits reversal behavior or there exists a $C_i \in \mathscr{C}$ such that a MEU_{C_i} player exhibits hedging behavior.

Consequently, the first condition of (10) and (11) need to be both fulfilled. This implies that there are $\omega', \omega'' \in \Omega$ such that

(****) $f^{\omega''} - f^{\omega'} \geq h^{\omega''} - h^{\omega'}$ and $g^{\omega'} - g^{\omega''} \leq k^{\omega'} - k^{\omega''}$.

Define the prior set $\tilde{C}_i = \{\beta\delta_{\omega'} + (1-\beta)\delta_{\omega''} \mid \beta \in [0,1]\}$. If the inequalities in (****) are strict, it holds that $\frac{f^{\omega''} - f^{\omega'}}{g^{\omega'} - g^{\omega''}} > \frac{h^{\omega''} - h^{\omega'}}{k^{\omega'} - k^{\omega''}}$, which means that a $MEU_{\tilde{C}_i}$ player i shows hedging behavior due to Lemma 4. At least one of the inequalities in (****) is not strict iff

(*****) ($f^{\omega'} = h^{\omega'}$ and $f^{\omega''} = h^{\omega''}$) and/or ($g^{\omega'} = k^{\omega'}$ and $g^{\omega''} = k^{\omega''}$).

Consider (*****) with "and". Then, $f^{\omega'} = h^{\omega'}$ and $g^{\omega'} = k^{\omega'}$. By the proposition, at most one of the acts is constant. Suppose that f is constant, which implies that g, h, k are not constant. Since $f \neq h$ and $g \neq k$, there exists a $\pi' \in \Delta(\Omega)$ such that $h^{\omega'} \neq h\pi'$, which implies that $f\pi' \neq h\pi'$, and $g^{\omega'} \neq g\pi'$, $k^{\omega'} \neq k\pi'$ and $g\pi' \neq h\pi'$. Define the prior set $\hat{C}_i = \{\beta\delta_{\omega'} + (1-\beta)\pi' \mid \beta \in [0,1]\}$. Since $f, -g, h, -k$ are pairwise affinely related either $MEU_{\hat{C}_i}(f) = f^{\omega'}$ and $MEU_{\hat{C}_i}(h) = h^{\omega'}$ or $MEU_{\hat{C}_i}(g) = g^{\omega'}$ and $MEU_{\hat{C}_i}(k) = k^{\omega'}$, but not both. W.l.o.g. assume that $MEU_{\hat{C}_i}(f) = f^{\omega'}$ and $MEU_{\hat{C}_i}(h) = h^{\omega'}$. Given $\alpha a'_{-i} + (1-\alpha)a''_{-i}$, a $MEU_{\hat{C}_i}$ player is indifferent between her actions for all $\alpha \in [0, \alpha']$ and strictly prefer one of her pure actions for $\alpha \in [\alpha'', 1]$, where α' is sufficiently low and α'' is sufficiently large. That is, a $MEU_{\hat{C}_i}$ shows reversal behavior - a contradiction. Similarly, one can show that (*****) with "or" yields a contradiction.

To sum up, if (ii) implies that $f, -g, h, -k$ are pairwise affinely related, we obtain a contradiction to the assumptions of the proposition, which proves that (ii) implies (i). □

```
Define
f⁺ = f ' ,  f⁻ = f ' ' etc.
f, h and g, k are affinely related :
h = a * f + b
k = a ' * g + b '
Affine - relatedness condition :
(f ' ' - f ') / (g ' - g ' ') = (h ' ' - h ') / (k ' - k ' ') = (a * (f ' ' - f ')) / (a ' * (g⁺ - g⁻))
implies that a = a '.
Condition (*)
g ' < a * g ' + b ',  f ' > a * f ' + b,  f ' ' < a * f ' ' + b,  g ' ' > a * g ' ' + b '
Condition (**)
a * f ' + b < a * f ' ' + b,  g ' ≥ g ' ',  f ' ≤ f ' ',  a * g ' + b ' > a * g ' ' + b '
Condition (***)
((a - 1) * g ' + b ') / ((1 - a) * f ' - b) < (g ' - g ' ') / (f ' ' - f ') and
((1 - a) * g ' ' - b ') / ((a - 1) * f ' ' + b) > (g ' - g ' ') / (f ' ' - f ')
implies that
((a - 1) * g ' + b ') / ((1 - a) * f ' - b) < ((1 - a) * g ' ' - b ') / ((a - 1) * f ' ' + b).
These conditions together yield a contradiction :
In[38]:= Reduce[((1 - a) * g ' ' - b ') / ((a - 1) * f ' ' + b) > ((a - 1) * g ' + b ') / ((1 - a) * f ' - b) &&
   f ' < f ' ' && g ' > g ' ' && ((1 - a) * g ' ' - b ') > 0 &&
   ((a - 1) * f ' ' + b) > 0 && ((a - 1) * g ' + b ') > 0 && ((1 - a) * f ' - b) > 0 &&
   (g ' - g ' ') / (f ' ' - f ') > ((a - 1) * g ' + b ') / ((1 - a) * f ' + b) && f ' < f ' ' &&
   g ' > g ' ' && ((1 - a) * f ' + b) > 0 && ((a - 1) * g ' + b ') > 0 && a > 0 &&
   (g ' - g ' ') / (f ' ' - f ') < ((a - 1) * g ' ' + b ') / ((1 - a) * f ' ' + b) && f ' < f ' ' &&
   g ' > g ' ' && ((1 - a) * f ' ' + b) > 0 && ((a - 1) * g ' ' + b ') > 0 && a > 0]
Out[38]= False
```

Proof of Proposition 5. The proof of "(i)(a) $\implies$ (ii)" and of "(i)(b) $\implies$ (ii)" is straightforward. We prove "(i) $\impliedby$ (ii)" by its contrapositive "$\neg(i) \Rightarrow \neg(ii)$". Suppose that $\neg(i)(a)$ and $\neg(i)(b)$ is true. Then, it holds that f, g, h, k are not pairwise affinely related and if f (resp. h) is constant, then g (resp. k) is not constant and vice versa. There are two cases to consider:

Case 1. Let f and h be constant and g and k be non-constant. Since f, g, h, k are not pairwise affinely related, it needs to hold that g is not affinely related to k. By the proposition, there exists $\alpha' \in [0,1]$ such that $\alpha' f + (1-\alpha')g$ and $\alpha' h + (1-\alpha')k$ are not strictly dominance related. By Lemma 1, a MEU_{C_i} player i exhibits no hedging behavior for all $C_i \in \mathscr{C}$ only if $\alpha' f + (1-\alpha')g$ is affinely related to $\alpha' h + (1-\alpha')k$, i.e. (*) $\alpha' f^\omega + (1-\alpha')g^\omega = a[\alpha' h^\omega + (1-\alpha')k^\omega] + b$ for all $\omega \in \Omega$ and some $a > 0, b \in \mathbb{R}$. Since f and h are constant, (*) is equivalent to $g^\omega = ak^\omega + \tilde{b}$ for all $\omega \in \Omega$ and some $a > 0, \tilde{b} \in \mathbb{R}$, which means that g is affinely related to k - a contradiction. Therefore, a MEU_{C_i} player i shows hedging behavior, whenever $\neg(i)(a)$ is true.

Case 2. Let f and k be constant and g and h be non-constant. This case can be proven similarly to the previous one.

Therefore, "$\neg(i) \Rightarrow \neg(ii)$" $\Leftrightarrow$ "(i) $\Leftarrow$ (ii)". □

Proof of Proposition 6. The proof of "(i)(a) $\implies$ (ii)" and of "(i)(b) $\implies$ (ii)" is straightforward. As in the previous proof, we prove "(i) $\impliedby$ (ii)" by its contrapositive. Let $\neg(i)$ be true. Then, f, g, h, k are not pairwise affinely related and if $f = h$ (resp. $g = k$), then $g \neq k$ (resp. $f \neq h$). W.l.o.g. assume that $f = h$ and $g \neq k$. Since g and k are not strictly dominance related, $\alpha f + (1-\alpha)g$ and $\alpha h + (1-\alpha)k$ are not strictly dominance related for all $\alpha \in [0,1]$. Due to Lemma 1, if there exists a $\alpha' \in [0,1]$ such that $\alpha' f + (1-\alpha')g$ is not affinely related $\alpha' h + (1-\alpha')k$, then a MEU_{C_i} player i shows hedging behavior for some $C_i \in \mathscr{C}$, i.e. $\neg(ii)$ is true. If g is affinely related to k, then (*) $g^\omega = a'k^\omega + b'$ for all $\omega \in \Omega$ and some $a' > 0, b' \in \mathbb{R}$. Let $\alpha' \in (0,1)$. If $\alpha' f + (1-\alpha')g$ is affinely related to $\alpha' h + (1-\alpha')k$, then (**) $\alpha' f^\omega + (1-\alpha')g^\omega = a[\alpha' h^\omega + (1-\alpha')k^\omega] + b$ for all $\omega \in \Omega$ and some $a > 0, b \in \mathbb{R}$. If f, g, h, k are not pairwise affinely related, (*) and (**) cannot be true at the same time. Hence, "$\neg(i) \Rightarrow \neg(ii)$" $\Leftrightarrow$ "(i) $\Leftarrow$ (ii)". □

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