# Price Discovery and Interventions in Frozen Markets\*

Braz Camargo<sup>†</sup>

GV University of Iowa

Kyungmin Kim

Sao Paulo School of Economics - FGV

Benjamin Lester<sup>‡</sup> Federal Reserve Bank of Philadelphia

February 19, 2014

#### Abstract

We study how government intervention in frozen markets affects the process of information aggregation or *price discovery*. We find a fundamental trade-off between ensuring that buyers participate in the market—so that trade actually occurs—and ensuring that prices are informative about the quality of the asset being traded. The policy that balances this trade-off, and hence maximizes price discovery, is quite different than the policy that maximizes gains from trade, which has been the sole objective in most previous studies of interventions in frozen markets. We then study how the policy maximizing price discovery depends on features of the environment, such as the severity of the initial market freeze and the opacity of the assets for sale.

<sup>\*</sup>This paper previously circulated under the title "Subsidizing Price Discovery." We thank Philip Bond, Ken Burdett, Vincent Glode, and Pablo Kurlat for excellent discussions of this paper, along with Viral Acharya, George Alessandria, Roc Armenter, Gadi Barlevy, Mitchell Berlin, Hal Cole, Hanming Fang, Douglas Gale, Itay Goldstein, Boyan Jovanovic, Andrew Postlewaite, and Xianwen Shi for helpful comments. All errors are our own.

<sup>&</sup>lt;sup>†</sup>Braz Camargo gratefully acknowledges financial support from CNPq.

<sup>&</sup>lt;sup>‡</sup>The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/.

"The complete evaporation of liquidity in certain market segments of the US securitisation market has made it impossible to value certain assets fairly." — Official Statement of BNP Paribas, after stopping withdrawals from three investment funds on August 9, 2007.

"For some of the securities there are just no prices [...] As there are no prices, we can't calculate the value of the funds." — Alain Papiasse, Head of Asset Management, BNP Paribas.

## **1** Introduction

Information aggregation is one of the fundamental functions of a market.<sup>1</sup> Hence, when markets fail or "freeze," not only are gains from trade left unrealized, but the process of information aggregation or *price discovery* is disrupted as well. Given the potential ramifications of frozen markets, a natural question is what role a government can play in "unfreezing" a market. Answering this question has become particularly important in light of the financial crisis that occurred in 2007-2008, when the collapse of trade in several key financial markets had deleterious effects on the economy as a whole. However, nearly all of the literature that has emerged to study policy interventions in frozen markets has focused exclusively on the ability of various government programs to restore gains from trade, while ignoring the effect of these programs on price discovery.

This failure to consider the effects of government intervention on the process of price discovery is a serious omission. After all, the information contained in asset prices often plays a crucial role in the decision-making processes of many agents in the economy. For example, information produced about a particular type of asset can reduce information asymmetries in markets for similar (or even identical) assets, thereby helping other agents to realize gains from trade.<sup>2</sup> The information produced about a particular class of assets could also allow for a more accurate assessment of the balance sheet of a bank that owns these assets. This could be valuable to depositors who have to

<sup>&</sup>lt;sup>1</sup>See, e.g., Hayek [1945], Grossman [1976], and Hellwig [1980].

<sup>&</sup>lt;sup>2</sup>For recent examples of papers that study information spillovers in financial markets, see Benveniste et al. [2003], who provide evidence from the IPO market, or Cespa and Foucault [2013], who document the effects of informational spillovers after a "flash crash."

decide whether to withdraw their funds from such a bank or to invest additional funds.<sup>3</sup> The same information could also be valuable to regulators who have to decide whether or not to "bail out" such a bank if it faces financial distress.<sup>4</sup> In either case, more accurate asset prices could reduce the incidence of liquidating banks that would ultimately be solvent and bailing out banks that would ultimately find themselves insolvent. More generally, there are myriad of channels through which information contained in asset prices guides real economic decisions.<sup>5</sup>

Once we recognize that the informational content of asset prices is important for a variety of markets, agents, and real economic decisions, a crucial question arises: *what is the effect of government interventions in frozen markets on price discovery?* The goal of this paper is to provide some answers to this question.

We begin with a standard model of trade: an auction with one seller and N buyers. An auction is a natural starting point because it provides rigorous micro-foundations for price formation and, hence, has served as a workhorse model in the literature on information aggregation (see, e.g., [Wilson, 1977] and [Milgrom, 1979]). Then we add three simple ingredients that allow us to address the question posed above.

First, we introduce a friction that serves as an impediment to trade: asymmetric information about the quality of the seller's asset. In particular, we assume that the seller's asset is either of high or low quality, and that this is the seller's private information. This friction is not only a classic explanation for market failures in general, but it is also one of the most commonly cited reasons for the specific interruptions that occurred during the recent financial crisis.<sup>6</sup>

Second, we allow buyers to acquire information about the quality of the asset before they bid. More specifically, each buyer can acquire a noisy signal about the quality of the asset at a cost, which is drawn independently for each buyer from a given distribution. This ingredient

<sup>&</sup>lt;sup>3</sup>See, e.g., Goldstein and Pauzner [2005], who provide a model that describes how information about fundamentals can change the probability of a bank run.

<sup>&</sup>lt;sup>4</sup>During the recent financial crisis, Hart and Zingales [2011], McDonald [2013], and Flannery [2010] all offered policy prescriptions that utilize the information contained in current market prices. See Bond and Goldstein [2012] for a recent theoretical contribution.

<sup>&</sup>lt;sup>5</sup>See, e.g., Dow and Gorton [1997], Chen et al. [2007], Foucault and Gehrig [2008], Bakke and Whited [2010], and Foucault and Frésard [2012] for specific examples, and Bond et al. [2012] for a broad overview of the literature that studies the interaction between price discovery and real investment decisions.

<sup>&</sup>lt;sup>6</sup>See, e.g., Gorton [2009].

ultimately allows us to analyze buyers' incentives to produce information and how those incentives are influenced by features of the economic environment.

Last, we introduce a simple policy to "unfreeze" the market, whereby the government provides partial insurance to the winning bidder against the event of acquiring a low quality asset or "lemon." In particular, we assume that a buyer who pays a high price and discovers that the asset is of low quality only suffers a fraction  $\gamma$  of the loss, and the government bears the remainder of the loss. This is a natural policy to study for two reasons. First, it directly addresses the underlying friction in the market; since the market freeze occurs because of buyers' concerns for over-paying for a "lemon," providing a sufficient level of insurance to buyers can unambiguously restore trade in the environment we consider. Second, this form of intervention captures the essential features of several policies that have actually been implemented in response to market interruptions. In fact, as we describe in detail later in the text, the program we study is almost identical to the Public Private Investment Program for Legacy Assets, or PPIP, which was introduced in March of 2009 in order to "support market functioning and facilitate price discovery, mostly in the mortgage-backed securities market[.]"<sup>7</sup>

We present our model of trade with these key ingredients in Section 2, and provide a complete characterization of the model's equilibrium in Section 3. The equilibrium strategies are composed of a cut-off cost  $k^*$  such that buyers acquire a signal about the quality of the asset if, and only if, their cost of acquiring the signal is not greater than  $k^*$ , along with optimal bidding strategies for each buyer conditional on their information set at the time of bidding. Then, in Section 4, we use the equilibrium characterization to achieve our primary goal—namely, to gain a better understanding of how government policy affects the amount of information that is produced in the auction. We do this in a number of steps.

First, we analyze the relationship between each buyer's incentive to acquire a signal about

<sup>&</sup>lt;sup>7</sup>This quote is taken from the Quarterly Report of the Department of the Treasury, January 30, 2013. The idea of curing a frozen market by sharing in participants' potential losses was not exclusive to PPIP, though. For example, Swagel [2009] describes an FDIC proposal for foreclosure avoidance that included "a loss-sharing insurance plan, under which the federal government would make good on half of the loss suffered by a lender that modified a loan according to the IndyMac protocol but later saw the loan go into default and foreclosure." A similar philosophy underlies the "ring fence insurance schemes" he describes, whereby money from the Troubled Asset Relief Program was used to share losses on a large pool of assets owned by Citi.

the asset's quality (captured by the equilibrium cut-off  $k^*$ ) and the amount of insurance provided by the intervention (captured by the policy parameter  $\gamma$ ). We show that there exists a unique, interior policy choice,  $\tilde{\gamma} \in (0, 1)$ , that maximizes buyers' incentives to become informed (i.e., acquire a signal). Intuitively, when the amount of insurance is too low, the problem of adverse selection remains severe, the expected gains from trade are small, and thus buyers are hesitant to acquire information and bid for the asset. However, when the program provides too much insurance against losses, a moral hazard problem emerges: buyers become more willing to "gamble" and bid aggressively without first becoming informed.

Next, we examine how the buyers' equilibrium behavior affects the amount of information produced in the auction. Note that this is not a straightforward task: as we will show, each choice of  $\gamma$  induces a distribution of winning bids, and thus a distribution of signals about the quality of the asset, and it is notoriously difficult to compare such distributions or *information structures* in a way that does not depend crucially on the specific details of the decision problem to which they are applied (see, e.g., [Athey and Levin, 1998]). We overcome this challenge by studying the expected reduction in *entropy* that results from an agent observing the winning bid. This "entropy informativeness" metric has been popular—and provides a natural benchmark here—both because it allows for a complete ordering of information structures and because it is model-free, i.e., it does not depend on the endowments or preferences of the agents in the model, nor on the decision-making process to which the information is applied.<sup>8</sup> In fact, since our results do not depend on a specific model of information spillovers, they are informative both for environments in which price discovery is beneficial for welfare and for environments in which information production is harmful (such as [Dang et al., 2012]).

Using entropy informativeness as a metric, we show that there is an inherent trade-off between ensuring that gains from trade are realized and promoting price discovery. We establish that providing full insurance for the buyers (i.e., setting  $\gamma = 0$ ) maximizes gains from trade, but minimizes the informational content of the winning bid. The quantity of information produced is maximized at a value of  $\gamma \in (0, \tilde{\gamma})$ , at which buyers obtain enough insurance so that they are willing to bid for

<sup>&</sup>lt;sup>8</sup>The notion of entropy was first introduced in this context by Shannon [1948]. For early discussions of this measure's use in economics, see Marschak [1959] and Arrow [1972]. For a more recent treatment, see Sims [2003], Veldkamp [2011], and Cabrales et al. [2013].

the asset, but not so much insurance that they place these bids without first acquiring information. This trade-off has not been identified in the existing literature. In particular, previous studies have focused on two potential costs of government intervention: the direct cost of raising funds for an intervention, and the indirect cost of encouraging risky behavior in the future. Hence, one important contribution of our paper is to identify a third cost of government intervention: distorting prices and disrupting the process of information production.

In Section 4, we also examine how the policy that maximizes entropy informativeness depends on certain features of the economic environment, such as the (initial) severity of the lemons problem (i.e., the ex ante probability that the asset is a lemon) and the opacity of the market (i.e., the informativeness of signals about the quality of the asset). We show that reducing adverse selection and reducing market opacity have different, opposing implications for the policy that maximizes information production: this policy requires providing less insurance when adverse selection is less severe, while it requires providing more insurance when the asset is less opaque. This difference stresses the subtleties of how policy interacts with price discovery.

In Section 5, we discuss some of our assumptions and possible extensions to our framework. First, since the number N of buyers in each auction is potentially a policy choice, a natural question is how the informational content of prices varies with N. We show that, for each  $\gamma$ , there is an optimal number of participants for maximizing information production. Second, we discuss the robustness of our results to alternative signal structures. Finally, we illustrate how our framework can accommodate an alternative theory of market crashes, the so-called "cash-in-the-market pricing." Section 6 concludes.

**Related Literature.** This paper primarily contributes to the literature on optimal interventions in frozen markets, which is young but growing fast; a non-exhaustive list includes Tirole [2012], Philippon and Skreta [2012], Chari et al. [2010], Camargo and Lester [2011], Guerrieri and Shimer [2011], Chiu and Koeppl [2011], Philippon and Schnabl [2011], House and Masatlioglu [2010], Diamond and Rajan [2012], and Farhi and Tirole [2012]. As we noted above, the majority of this literature focuses on how government interventions can improve allocations, while ignoring the effects of these interventions on the process of information production. To the best of our

knowledge, the only other paper that explicitly studies the effects of government interventions on information production is Bond and Goldstein [2012]. The focus of their analysis is very different from ours, though; most notably, they are not interested in inefficiencies due to adverse selection, and thus the interventions in their model play an entirely different role than in our environment.<sup>9</sup>

From a technical point of view, our paper is related to two strands of the auction literature. The first strand studies the incentives of bidders to acquire information under various auction formats, and whether these incentives align with the socially optimal level; see, for example, Matthews [1984], Persico [2000], Bergemann and Valimaki [2002], and Bergemann et al. [2009]. However, these papers are interested in neither the informational content of the winning bid, nor the effects of any form of intervention on information acquisition.<sup>10</sup> The second strand examines the extent to which the winning bid(s) of an auction reflects the underlying value of the good(s) for sale; see, for example, Wilson [1977], Milgrom [1979], Milgrom [1981], Pesendorfer and Swinkels [1997], Pesendorfer and Swinkels [2000], Kremer [2002], and Lauermann and Wolinsky [2013]. In contrast to our paper, this literature typically treats the information set of each bidder as exogenous and focuses on conditions for the winning bid to completely reveal the underlying value of the object.<sup>11</sup>

## 2 The Model

**Environment.** There is a single seller who possesses one indivisible asset, and there are N buyers who are interested in purchasing the asset. The asset is either of high (H) or low (L) quality. If the asset is of high quality, then the seller receives payoff c from retaining the asset, while a buyer receives payoff v from acquiring it, where v > c > 0. If the asset is of low quality, then it is of no value to either the seller or the buyers; that is, it yields all of them zero payoff.

The quality of the asset is the seller's private information. The buyers have a common prior

<sup>&</sup>lt;sup>9</sup>They highlight an interesting feedback effect that is absent from our analysis: the government decides how much to use market prices in formulating a policy, which affects the incentives of speculators to trade and hence changes the informational content of these prices.

<sup>&</sup>lt;sup>10</sup>Perhaps closest to our framework is Cao and Shi [2001], who study how the number of bidders affects information acquisition and bidding behavior in the market for loans.

<sup>&</sup>lt;sup>11</sup>A notable exception is Jackson [2003], who allows for endogenous information acquisition.

belief that the asset is of high quality with probability  $\pi \in (0, 1)$ . Should a buyer acquire an asset of quality  $j \in \{L, H\}$  at some price b, this buyer receives payoff v - b if j = H and -b if j = L, the seller receives payoff b, and all other buyers receive payoff zero. Should no trade occur, the seller receives payoff c if j = H and zero if j = L, and all buyers receive payoff zero.

**Trading.** The game proceeds as follows. First, each buyer  $i \in \{1, ..., N\}$  has the opportunity to inspect the asset at a cost  $k_i$ , where each  $k_i$  is independently and identically drawn from the interval  $[0, \infty)$  according to a continuous and strictly increasing cumulative distribution function G. If buyer i incurs the cost  $k_i$ , then he receives a private and independently drawn signal  $s_i \in \{\ell, h\}$  about the quality of the asset. In order to deliver our results most clearly, we focus on a simple signal generating process summarized by the matrix

	H	L	
h	1	$1-\rho$	,
$\ell$	0	ρ	

where  $\rho \in (0, 1)$ . In words, a buyer who inspects the asset always receives the "good" signal h if the asset is of high quality. However, if the asset is of low quality, then the buyer receives the "bad" signal  $\ell$  only with probability  $\rho$ .<sup>12</sup> We refer to a buyer who has decided to receive a costly signal as "informed" and a buyer who has chosen not to receive a signal as "uninformed." For ease of exposition, we say that an uninformed buyer observes the signal  $s_i = u$ . A buyer cannot observe the other buyers' costs, nor can he observe whether the other buyers are informed or uninformed.

After the information acquisition stage, the buyers simultaneously submit a non-negative bid for the asset; we denote buyer *i*'s bid by  $b_i$ . The seller then decides whether to accept the highest bid or to reject all bids and retain the asset. If the highest bid is offered by two or more buyers (and the seller accepts), the asset is awarded to each of those buyers with equal probability.

<sup>&</sup>lt;sup>12</sup>Although the informational structure is rather stylized, it has a natural interpretation. One can imagine that there are certain "red flags" associated with low quality assets, corresponding to signal  $\ell$  in our environment. A buyer who studies a seller's asset will never uncover such a red flag if the asset is of high quality, while he may (with probability  $\rho$ ) find one if the asset is of low quality. Many of our results are robust to other specifications, including the case in which the bad signal occurs with positive probability when the asset is of high quality; see Section 5 for a discussion.

**Assumptions.** A buyer who receives the bad signal knows with certainty that the asset is of low quality. Alternatively, a buyer who receives the good signal is still uncertain about the quality of the asset, but updates his belief that the asset is of high quality to

$$\tilde{\pi} = \frac{\pi}{\pi + (1-\pi)(1-\rho)} > \pi$$

In order to focus on the most relevant case, we make the following two assumptions: ASSUMPTION 1. (*Initial Lemons Problem*)

$$\pi(v-c) - (1-\pi)c < 0 \iff \pi < \frac{c}{v}; \tag{1}$$

ASSUMPTION 2. (Positive value of inspection)

$$\tilde{\pi}(v-c) - (1-\tilde{\pi})c > 0 \iff \tilde{\pi} > \frac{c}{v} \iff \rho > \frac{c-\pi v}{(1-\pi)c}.$$
(2)

Assumption 1 implies that buyers are not willing to place a "serious" bid  $b \ge c$  without inspecting the asset. Assumption 2 implies that inspection is sufficiently informative about the quality of the asset to generate the potential for trade between a buyer who receives the signal h and a seller with a high-quality asset; that is, a buyer who receives the good signal is willing to bid  $b \ge c$ .

**Policy.** We consider a simple form of intervention that directly addresses the fundamental friction, adverse selection. In particular, we assume that the government chooses to provide some level of insurance to buyers against the possibility of acquiring a low quality asset.

Motivated by the PPIP program that was introduced during the recent financial crisis, we assume that this policy is implemented as follows. A buyer who purchases the asset at price b is required to put up an amount  $\gamma b$  of his own equity, and is issued a nonrecourse loan from the government for the remaining portion of his bid,  $(1 - \gamma)b$ . Should the buyer choose not to repay the loan, the government can seize the asset, but the buyer is not liable for any additional payments. Therefore, the buyer's loss is limited to  $\gamma b$  under this program. We assume that a buyer who purchases the asset observes its quality before deciding whether to repay the loan.<sup>13</sup> Hence, a buyer

<sup>&</sup>lt;sup>13</sup>Perfect observability is not a crucial assumption. All of our results go through if the purchaser receives additional

who acquires the asset at price b repays the loan if the asset is of high quality, earning a payoff of v - b, and defaults otherwise, suffering a loss of  $\gamma b$ . Thus, it is easy to see that the government policy is tantamount to insurance: it provides a rebate of size  $(1 - \gamma)b$  to an "unlucky" buyer who pays price  $b \ge c$  and receives a low-quality asset.<sup>14</sup>

Strategies and Equilibrium. The seller's behavior is straightforward in our model: a seller with a low-quality asset accepts any positive bid, while a seller with a high-quality asset accepts the highest bid b if  $b \ge c$  and rejects all bids otherwise. It simplifies our analysis to assume that a bid of zero is rejected by both types of sellers, even though sellers with low-quality assets are indifferent between accepting and rejecting such a bid; one could imagine, for example, an arbitrarily small transaction cost associated with trading an asset of low quality. In what follows, we take the behavior of the sellers as given and focus on the behavior of the buyers.

A strategy for buyer  $i \in \{1, ..., N\}$  has two components. First, he must decide whether or not to inspect the asset given the cost  $k_i$ . The optimal inspection strategy for a buyer is obviously a cutoff rule: inspect the asset if, and only if, the cost  $k_i$  is not greater than the value of doing so. Therefore, we represent a buyer's inspection strategy by his cutoff cost k. Second, a buyer must formulate an optimal bidding strategy as a function of his private information: namely, the signal  $s_i \in \{u, \ell, h\}$  he receives about the quality of the asset.<sup>15</sup> We let a cumulative distribution function  $F_s$  represent the mixed bidding strategy of a buyer with signal s;  $F_s(b)$  is the probability that a buyer with signal s bids b or less.

A symmetric equilibrium is a strategy profile  $(k, F_u, F_\ell, F_h)$  in which: (i) a buyer inspects the asset if, and only if, the cost of doing so is not greater than the benefit; and (ii) each buyer's bid is

information about the quality of the asset before he makes the repayment decision.

<sup>&</sup>lt;sup>14</sup>The actual PPIP was only slightly different. Like our model, an auction would be organized for a pool of assetbacked securities with a fixed (known) number of bidders. The winning bidder was required to finance a fraction  $\frac{1}{12}$  of the purchase price with his own equity. The treasury would match the investor's equity investment in exchange for a 50% share in profits. The remaining  $\frac{5}{6}$  of the purchase price would be financed by a nonrecourse loan from the FDIC. Hence, the main features of PPIP—an auction in which downside insurance was offered to the winning bidder by way of a nonrecourse loan—is identical to our formulation. We abstract from the profit-sharing feature, though including this would not substantially change our results.

<sup>&</sup>lt;sup>15</sup>In principle, a buyer can also condition his bidding strategy on his cost of inspection,  $k_i$ . However, at the time of bidding, the costs of inspection are already sunk and, therefore, intrinsically irrelevant to the buyers' bidding problems. Hence, we assume that a buyer's bidding strategy depends only on his signal. In addition, it is possible to show that allowing buyers to condition their bids on their inspection costs does not affect equilibrium payoffs and outcomes.

optimal given his private information and the strategies of the other buyers.

## **3** Equilibrium Characterization

In this section, we characterize the symmetric equilibrium of the trading game described in the previous section. The main challenge is to jointly characterize the buyers' equilibrium inspection and bidding strategies. After all, the value of inspection for buyers depends on their bidding behavior, which in turn depends on the inspection decisions of other buyers.

We overcome this challenge by proceeding in two steps. First, we consider the model in which the probability that each buyer is informed, which we denote by  $\lambda$ , is taken as exogenous and characterize the equilibrium for all possible values of  $\lambda$  and  $\gamma$ . Then, we use the equilibrium characterization of the model with exogenous inspection probabilities to determine the value of inspection for a buyer, conditional on the other buyers inspecting the asset with probability  $\lambda$ . This allows us to characterize the equilibrium of the original game, where the probability that each buyer is informed is determined endogenously, for each choice of  $\gamma$ . In particular, we show that, for each  $\gamma \in [0, 1]$ , there exists a unique cutoff  $k^*$  such that the value of being informed is exactly  $k^*$  when the probability that each buyer is informed is  $\lambda^* = G(k^*)$ .

We restrict the analysis in the text to the case of  $\gamma > 0$ , as this is the most relevant case. However, we also report several properties of the equilibrium as  $\gamma$  converges to zero, as this captures an important benchmark—the case in which buyers are fully insured against losses. As we will show, as the buyers' "skin in the game" vanishes, the incentive to acquire information converges to its minimum value, while gains from trade are maximized.<sup>16</sup>

<sup>&</sup>lt;sup>16</sup>The analysis at  $\gamma = 0$  itself is slightly different, as multiplicity of equilibria arises, with different bidding strategies across these equilibria (though ex ante payoffs are the same). For example, besides the (natural) equilibrium in which agents who observe the signal  $\ell$  bid zero, there exists an equilibrium in which these agents bids b = v even if they know that the asset is worthless: after all, buyers are fully insured. The second equilibrium is neither interesting nor robust (it is not the limit of equilibria as  $\gamma$  converges to zero). This is why we analyze only the case of  $\gamma > 0$  and consider the full insurance case as the limiting case when  $\gamma$  converges to zero. However, for completeness, in the Appendix we establish the key property of *all* equilibria at  $\gamma = 0$ ; namely, that the equilibrium cutoff cost for inspecting the asset is zero (see Proposition 7).

### 3.1 Equilibrium with Exogenous Information Acquisition

Suppose each buyer inspects the asset with probability  $\lambda \in (0, 1)$ .<sup>17</sup> The behavior of a buyer who observes the signal  $\ell$  is trivial: since this signal reveals that the asset is of low quality, a weakly dominant strategy for the buyer is to bid b = 0, yielding payoff  $V_{\ell} = 0$ . Therefore, in what follows, we take as given the behavior of buyers who receive the signal  $\ell$ , and concentrate on the behavior of uninformed buyers and informed buyers who receive the signal h. In a slight abuse of notation, we refer to the former as "type u" buyers and the latter as "type h" buyers. Note that bidding  $b \in (0, c)$  is suboptimal for both types of buyers, as such an offer is accepted only when the asset is of low quality, in which case the buyer surely suffers a payoff loss of  $\gamma b$ .

For each  $s \in \{u, h\}$ , denote the minimum and maximum of the support of  $F_s$  by  $\underline{b}_s$  and  $\overline{b}_s$ , respectively. In addition, denote by  $V_s(b)$  the expected payoff to a type s buyer who bids b, and let  $V_s$  be the equilibrium payoff of a type s buyer. We first establish several basic properties of the equilibrium bidding strategies.

**Lemma 1.** The following holds in equilibrium for all  $\gamma > 0$ : (i)  $\overline{b}_u \leq \underline{b}_h$ ; (ii)  $\overline{b}_h > c$ ; (iii)  $F_s(b)$ is continuous and strictly increasing in b when  $b \in [\max\{c, \underline{b}_s\}, \overline{b}_s]$  for each  $s \in \{u, h\}$ ; and (iv)  $\underline{b}_u = 0$ .

We relegate the proof of Lemma 1 to the Appendix and sketch the intuition here. The first property is a typical single crossing property: a buyer who is more optimistic about the quality of the asset has a higher willingness to pay and therefore bids a higher price. As a result, the supports of  $F_h$  and  $F_u$  overlap at most at a single point. The second property states that type h buyers place serious bids with positive probability. Indeed, if type h buyers always bid zero, then a profitable deviation for such a buyer would be to bid b = c and obtain a payoff  $\tilde{\pi}(v - c) - (1 - \tilde{\pi})\gamma c$ , which is positive by Assumption 2. The third property states that the mixed bidding strategies  $F_u$  and  $F_h$ have neither atoms nor gaps on  $[c, \bar{b}_h]$ . If there were an atom, then a buyer who bids slightly above the atom obtains a strictly higher payoff than a buyer who bids at the atom. Similarly, if there were a gap in the support of the distribution of bids, then a bid at the lower end of the gap would be strictly preferable to a bid at the upper end of the gap, as both bids have the same probability

<sup>&</sup>lt;sup>17</sup>Below, we give conditions under which  $\lambda \in (0, 1)$  for all  $\gamma > 0$  when information acquisition is endogenous.

of winning. The last property states that  $\underline{b}_u = 0$ , which implies that  $V_u = 0$ . Indeed, if  $\underline{b}_u \ge c$ , then a type u bidder who bids  $b = \underline{b}_u$  wins only when the asset is of low quality, in which case his expected payoff is strictly negative.

Taken together, the properties in Lemma 1 imply that any equilibrium takes one of the following three forms. First, it could be that  $c < \overline{b}_u \leq \underline{b}_h$ , so that even type u buyers make serious bids. Second, it could be that  $\overline{b}_u = 0$  and  $\underline{b}_h \geq c$ , so that type u bidders never make serious bids, while type h buyers always place a serious bid. Finally, it could be that  $\overline{b}_u = \underline{b}_h = 0$ , so that even type h buyers make an offer of zero with positive probability. The equilibrium set can be characterized by analyzing each case separately. Since the analysis is similar for all three cases, we focus on the second case in detail here and relegate the other cases to the Appendix.

In the second case, in which uninformed buyers always bid zero, the expected payoff to a type  $s \in \{u, h\}$  buyer who bids  $b \ge c$  is

$$V_s(b) = \pi_s \left[ 1 - \lambda + \lambda F_h(b) \right]^{N-1} (v - b) - (1 - \pi_s) \left\{ 1 - \lambda + \lambda \left[ \rho + (1 - \rho) F_h(b) \right] \right\}^{N-1} \gamma b, \quad (3)$$

where  $\pi_u = \pi$  and  $\pi_h = \tilde{\pi}$ . Indeed, an uninformed buyer bids less than b for sure, while an informed buyer bids less than b with probability  $F_h(b)$  if the asset is of high quality and with probability  $\rho + (1 - \rho)F_h(b)$  if the asset is of low quality. Hence, if the asset is of high quality, then the buyer wins the asset with probability  $[1 - \lambda + \lambda F_h(b)]^{N-1}$  and obtains a payoff of v - b. On the other hand, if the asset is of low quality, then the buyer wins the asset with probability  $\{1 - \lambda + \lambda [\rho + (1 - \rho)F_h(b)]\}^{N-1}$  and suffers a loss of  $\gamma b$ .

Given (3), an equilibrium in which  $\overline{b}_u = 0$  and  $\underline{b}_h \ge c$  can be constructed as follows. First, it must be that  $\underline{b}_h = c$ ; if  $\underline{b}_h > c$ , then a type h buyer strictly prefers bidding c to  $\underline{b}_h$ , as this decreases his payment without changing his probability of winning. Second, the expected payoff to a type h buyer can be found by considering a type h buyer who bids  $\underline{b}_h = c$ . From (3), it follows that

$$V_{h} = \tilde{\pi} (1 - \lambda)^{N-1} (v - c) - (1 - \tilde{\pi}) (1 - \lambda + \lambda \rho)^{N-1} \gamma c.$$

Finally, for each  $b \in [c, \overline{b}_h]$ ,  $F_h(b)$  can be derived from (3) and the fact that the buyer must be indifferent between all bids in the support of  $F_h$ .

The equilibrium under consideration exists if, and only if, a type h buyer has no incentive to bid 0 and a type u buyer has no incentive to bid more than c. The first condition is that  $V_h \ge 0$ , which is equivalent to

$$\left(1 + \frac{\lambda\rho}{1-\lambda}\right)^{N-1} \le \frac{\tilde{\pi}(v-c)}{(1-\tilde{\pi})\gamma c}.$$
(4)

The second condition is that  $V_u(b) \leq 0$  for all  $b \geq c$ . Since  $\pi < \tilde{\pi}$  implies that a type u bidder strictly prefers b to b' > b whenever a type h buyer is indifferent between b and b', a necessary and sufficient condition for  $V_u(b) \leq 0$  for all  $b \geq c$  is that  $V_u(c) \leq 0$ , which is equivalent to

$$\left(1 + \frac{\lambda\rho}{1-\lambda}\right)^{N-1} \ge \frac{\pi(v-c)}{(1-\pi)\gamma c}.$$
(5)

Combining (4) and (5), an equilibrium in which  $\overline{b}_u = 0$  and  $\underline{b}_h \ge c$  exists if, and only if,

$$\frac{\pi(v-c)}{(1-\pi)\gamma c} \le \left(1 + \frac{\lambda\rho}{1-\lambda}\right)^{N-1} \le \frac{\tilde{\pi}(v-c)}{(1-\tilde{\pi})\gamma c}.$$

It is clear from the explicit construction above that the equilibrium is unique for each pair  $(\lambda, \gamma) \in (0, 1) \times (0, 1]$  satisfying (4) and (5).

Proposition 1 summarizes the characterization of the equilibria with  $\bar{b}_u = 0$  and  $\underline{b}_h = c$  described above and provides a characterization of the other two types of equilibria, namely, the equilibria with  $c < \bar{b}_u \leq \underline{b}_h$  and the equilibria with  $\bar{b}_u = \underline{b}_h = 0$ . Loosely speaking, for an equilibrium with  $c < \bar{b}_u \leq \underline{b}_h$  to exist, it must be that a type u buyer obtains a non-negative payoff from bidding c when all type u buyers bid zero; a necessary and sufficient condition for this is found by reversing the sign in the inequality in (5). Similarly, for an equilibrium with  $\bar{b}_u = \underline{b}_h = 0$  to exist, it must be that a type h buyer obtains a negative payoff from bidding c when all other type h buyer obtains a negative payoff from bidding c when all other type h buyer obtains a negative payoff from bidding c when all other type h buyers bid zero; a necessary and sufficient condition for this is found by reversing the sign in the inequality in (5).

**Proposition 1.** For each  $(\lambda, \gamma) \in (0, 1) \times (0, 1]$ , there exists a unique symmetric equilibrium. Let  $\underline{\lambda}(\gamma) \geq 0$  be the smallest value of  $\lambda$  that satisfies

$$\left[1 + \frac{\lambda \rho}{1 - \lambda}\right]^{N-1} \ge \frac{\pi (v - c)}{(1 - \pi)\gamma c}$$



and  $\overline{\lambda}(\gamma) \in (\underline{\lambda}(\gamma), 1)$  be the only value of  $\lambda$  that satisfies

$$\left[1 + \frac{\lambda \rho}{1 - \lambda}\right]^{N-1} = \frac{\tilde{\pi}(v - c)}{(1 - \tilde{\pi})\gamma c}$$

(1) If  $\lambda \in (0, \underline{\lambda}(\gamma))$ , then  $\underline{b}_h = \overline{b}_u > c$ . The expected payoff of a type h buyer is

$$V_h = \frac{\rho v}{\pi + (1 - \pi)(1 - \rho)} \left\{ \frac{1}{(1 - \pi)(1 - \lambda + \lambda \rho)^{N - 1} \gamma} + \frac{1}{\pi (1 - \lambda)^{N - 1}} \right\}^{-1}.$$
 (6)

(2) If  $\lambda \in [\underline{\lambda}(\gamma), \overline{\lambda}(\gamma)]$ , then  $\overline{b}_u = 0$  and  $\underline{b}_h = c$ . The expected payoff of a type h buyer is

$$V_{h} = \tilde{\pi} (1 - \lambda)^{N-1} (v - c) - (1 - \tilde{\pi}) (1 - \lambda + \lambda \rho)^{N-1} \gamma c.$$
(7)

(3) If  $\lambda > \overline{\lambda}(\gamma)$ , then  $\underline{b}_h = \overline{b}_u = 0$ . In this case,  $V_h = 0$ .

Figure 1 plots the values of  $\lambda$  and  $\gamma$  where each of the three types of equilibria exist.<sup>18</sup> To understand each type of equilibrium, it is helpful to note that, holding  $\gamma$  constant, an increase in  $\lambda$  worsens the winner's curse for both types of buyers and thus weakens their incentives to place serious bids. This effect, however, is stronger for type u buyers, as type h buyers are better informed about the quality of the asset. Similarly, holding  $\lambda$  constant, an increase in  $\gamma$  implies that buyers are less insured against acquiring a lemon, which also leads to less aggressive bidding.

<sup>&</sup>lt;sup>18</sup>One can easily show that  $\overline{\lambda}(\gamma)$  is strictly decreasing in  $\gamma$  and that  $\underline{\lambda}(\gamma)$  is strictly decreasing in  $\gamma$  as long as  $\pi(v-c) > (1-\pi)\gamma c$ , with  $\underline{\lambda}(\gamma) = 0$  otherwise.

Therefore, when both  $\lambda$  and  $\gamma$  are small, both types of buyers bid aggressively; in the first case of Proposition 1, even uninformed buyers place serious bids. As  $\lambda$  and  $\gamma$  increase, the winner's curse for uninformed buyers becomes sufficiently strong that they stop placing serious bids; in the second case, only informed buyers place bids greater than c. Finally, when  $\lambda$  and  $\gamma$  are sufficiently close to one, the winner's curse becomes sufficiently strong for informed buyers, too—so much so, in fact, that they bid b = 0 with positive probability in the last case.

### 3.2 Equilibrium with Endogenous Information Acquisition

We now complete the description of equilibria by endogenizing the information acquisition decision of buyers. In equilibrium, the cutoff inspection cost coincides with the value of conducting an inspection. We first derive the ex ante value of inspection as a function of the policy  $\gamma$  and the probability  $\lambda$  that other buyers inspect the asset, which we denote by  $V_I = V_I(\lambda, \gamma)$ . Then, for each  $\gamma > 0$ , we identify the cutoff inspection cost  $k^*$  that equates the value of inspection to the cost itself; that is, since G(k) is the probability that each buyer inspects the asset given a cutoff strategy k, we find the value of  $k^*$  by solving  $k^* = V_I(G(k^*), \gamma)$ .

As noted above, the expected payoffs of type  $\ell$  and u buyers are always zero. Therefore,

$$V_I(\lambda,\gamma) = \left[\pi + (1-\pi)(1-\rho)\right] V_h(\lambda,\gamma),\tag{8}$$

where  $V_h(\lambda, \gamma)$  denotes the expected payoff of a type h buyer given  $\lambda$  and  $\gamma$ . The following result is then immediate from Proposition 1.

**Lemma 2.** For each  $\gamma > 0$ ,  $V_I(\lambda, \gamma)$  is continuous in  $\lambda$ , strictly decreasing in  $\lambda$  if  $\lambda < \overline{\lambda}(\gamma)$ , and equal to zero if  $\lambda \ge \overline{\lambda}(\gamma)$ .

Given these properties of  $V_I$ , it is straightforward to characterize the equilibrium of the original game, where information acquisition is endogenous. For simplicity, we assume that G(0) > 0. Together with the fact that G(v - c) < 1, the assumption that G(0) > 0 ensures that in equilibrium the probability that each buyer inspects the asset lies in the open interval (0, 1).<sup>19</sup>

<sup>&</sup>lt;sup>19</sup>While the assumption that G(0) > 0 helps ensure an interior probability of inspection, this assumption could easily be relaxed without changing the substance of our results.

To summarize the analysis so far, for each  $\gamma > 0$ , a symmetric equilibrium is a strategy profile  $(k^*, F_\ell^*, F_u^*, F_h^*)$  such that: (i) a type  $\ell$  buyer bids zero; (ii)  $F_u^*$  and  $F_h^*$  are the unique bidding strategies for type u and type h buyers, respectively, of the game with exogenous information acquisition when  $\lambda = G(k^*)$ ; and (iii)  $k^* = V_I(G(k^*), \gamma)$ . The existence of an equilibrium cutoff cost follows from the fact that  $V_I(\lambda, \gamma)$  is continuous in  $\lambda$  for all  $\gamma > 0$ . The cutoff cost is unique given that  $V_I(\lambda, \gamma)$  is nonincreasing in  $\lambda$ . Note that  $G(k^*) < \overline{\lambda}(\gamma)$  for all  $\gamma > 0$  since  $V_I(\lambda, \gamma) = 0$  if  $\lambda \ge \overline{\lambda}(\gamma)$ . Thus, the payoff to type h buyers is positive in equilibrium when  $\gamma > 0$ .

**Proposition 2.** For each  $\gamma > 0$ , there exists a unique symmetric equilibrium. The cutoff inspection cost  $k^*$  is positive if, and only if,  $G(0) < \overline{\lambda}(\gamma)$ .

In what follows, we assume that  $G(0) < \overline{\lambda}(1)$ , which is a sufficient condition for  $k^* > 0$  for all  $\gamma > 0$ ; this assumption simply rules out the possibility that the mass of agents who become informed at no cost is so large that the value of becoming informed is zero.<sup>20</sup>

## 4 Policy Analysis

We now examine how policy affects buyers' equilibrium behavior, and the consequences for both information production and trade. We begin, in Section 4.1, by characterizing the relationship between the policy parameter  $\gamma$ , the equilibrium cutoff  $k^*$ , and the expected gains from trade that are realized. We highlight that there exists a unique policy choice  $\tilde{\gamma} > 0$  that maximizes buyers' incentives to acquire information, while gains from trade are maximized as  $\gamma$  converges to zero.

Then, in Section 4.2, we use the results from 4.1 to address our main question; namely, understanding the implications of policy on the amount of information that is revealed to an outsider who observes the winning bid of the auction. We first define a metric for information production by deriving the outsider's expected reduction in entropy after observing the winning bid. Then we establish that the quantity of information produced is maximized at a value of  $\gamma \in (0, \tilde{\gamma})$ , which provides enough insurance to encourage buyers to participate in the auction by bidding  $b \ge c$ , but not so much insurance that they place these bids without first acquiring information. Since setting

<sup>&</sup>lt;sup>20</sup>For a given  $\gamma > 0$ , if  $G(0) > \overline{\lambda}(\gamma)$ , then  $k^* = 0$  and the equilibrium is simply described by Proposition 1 with  $\lambda = G(0)$ . In this case, a marginal change in the policy  $\gamma$  obviously has no effect on  $k^*$ .

 $\gamma = 0$  maximizes gains from trade, this result shows that policymakers face an inherent trade-off between ensuring that gains from trade are realized and promoting price discovery. This trade-off has not been identified in the existing literature, but it highlights an important, indirect cost of government interventions: distorting prices and disrupting the process of information production.

Last, in section 4.3, we examine how the policy that maximizes price discovery depends on several features of the environment. In particular, we show that maximizing information production typically requires that the policy provides less insurance as the lemons problem becomes less severe (i.e., as  $\pi$  increases) and when the asset becomes more opaque (i.e., as  $\rho$  decreases).

### 4.1 Policy, Inspection, and Trade

Let  $k^*(\gamma)$  denote the equilibrium cutoff cost when the government policy is  $\gamma$ , and suppose first that  $\lambda^*(\gamma) \equiv G(k^*(\gamma)) < \underline{\lambda}(\gamma)$ , so that type u buyers make serious bids in equilibrium. It follows from (6) in Proposition 1 that  $V_I(\lambda, \gamma)$  is increasing in  $\gamma$ ; that is, a policy that provides the buyers with *less* insurance leads to an *increase* in the value of inspection. To understand the intuition behind this result, recall that the expected payoff of a type h buyer who bids  $\underline{b}_h = \overline{b}_u$  is

$$V_h(\underline{b}_h) = \tilde{\pi} \left(1 - \lambda\right)^{N-1} \left(v - \underline{b}_h\right) - \left(1 - \tilde{\pi}\right) \left(1 - \lambda + \lambda\rho\right)^{N-1} \gamma \underline{b}_h.$$

An increase in  $\gamma$  has two opposing effects on  $V_h(\underline{b}_h)$  and thus on  $V_I$ . First, less insurance against acquiring a lemon directly decreases the expected payoff of any buyer who bids  $b \ge c$ . This decreases  $V_h$  and hence the value of inspection. Second, less insurance also causes buyers to bid less aggressively. In particular, as  $\gamma$  increases the bids placed by type u buyers fall (i.e.,  $\overline{b}_u$ decreases), which makes it cheaper for type h buyers to outbid uninformed buyers. This increases  $V_h$  and hence the value of inspection. In equilibrium, the latter, indirect effect dominates the former, direct effect, and  $V_I$  is increasing in  $\gamma$ .<sup>21</sup>

Now suppose that  $\lambda^*(\gamma) > \underline{\lambda}(\gamma)$ . From (7) in Proposition 1, it follows that an increase in  $\gamma$ 

<sup>&</sup>lt;sup>21</sup>To see why this is true, recall that the expected payoff of a type u bidder is always zero, so that  $0 = \pi (1-\lambda)^{N-1} (v - \underline{b}_h) - (1-\pi) (1-\lambda+\lambda\rho)^{N-1} \gamma \underline{b}_h$ . Therefore, the two effects discussed above cancel out each other for type u buyers. In addition, the first, direct effect is relevant only when the asset is of low quality, while the second, indirect effect is relevant whether the asset is of high or low quality. It follows that the indirect effect dominates the direct effect for a type h buyer, because he assigns a higher probability to the asset being of high quality than a type u buyer.

causes a decrease in  $V_I$ . Indeed, since uninformed buyers never bid  $b \ge c$  in this type of equilibrium, the direct effect discussed above is still present, but the indirect effect is absent. Therefore, an increase in  $\gamma$  always decreases  $V_I$ .

Figure 2 plots  $V_I(G(k), \gamma)$  against k and illustrates the effect of an increase in  $\gamma$ . Recall that the equilibrium cutoff,  $k^*$ , lies at the intersection of  $V_I(G(k), \gamma)$  and the forty-five degree line. Also recall, from Proposition 1 and (8), that  $V_I$  has a kink at  $G(k) = \underline{\lambda}(\gamma)$ . Hence, Figure 2 corresponds to the case of  $G(k^*) < \underline{\lambda}(\gamma)$ , where the intersection occurs to the left of the kink. The other case, with  $G(k^*) > \underline{\lambda}(\gamma)$ , corresponds to a similar graph, only the intersection with the forty-five degree line occurs to the right of the kink in  $V_I$ .



Figure 2: The effect of an increase in  $\gamma$  when  $\lambda^* < \underline{\lambda}(\gamma)$ 

Given the response of  $V_I$  to an increase in  $\gamma$ , it is easy to see that  $k^*$  is increasing in  $\gamma$  if  $G(k^*(\gamma)) = \lambda^*(\gamma) < \underline{\lambda}(\gamma)$  and decreasing in  $\gamma$  otherwise. To formalize this result, for each  $\gamma > 0$ , let  $\underline{k}(\gamma)$  denote the smallest value of k such that  $G(k) \ge \underline{\lambda}(\gamma)$ .<sup>22</sup>

**Proposition 3.** There exists a unique and interior  $\tilde{\gamma} \in (0, 1)$  such that  $k^*(\tilde{\gamma}) = \underline{k}(\tilde{\gamma})$ . The cutoff cost  $k^*(\gamma)$  is strictly decreasing in  $\gamma$  when  $\gamma \in (\tilde{\gamma}, 1]$ , strictly increasing in  $\gamma$  when  $\gamma \in (0, \tilde{\gamma})$ , and converges to zero as  $\gamma$  decreases to zero. Hence,  $k^*(\gamma)$  is maximized when  $\gamma = \tilde{\gamma}$ .

Intuitively, when  $\gamma$  is relatively large, the policy provides little insurance and hence the losses associated with acquiring a low-quality asset are large. This risk depresses the expected value of

<sup>&</sup>lt;sup>22</sup>Note that  $\underline{k}(\gamma) = 0$  if  $\gamma \ge \pi (v - c)/(1 - \pi)c$  since G(0) > 0.

acquiring the asset—even after observing the signal h—and hence the value of becoming informed is small. On the other hand, when  $\gamma$  is relatively small, the policy insures a large portion of a buyer's downside risk and, as a result, a moral hazard problem emerges: even uninformed buyers bid aggressively. This price competition drives down the expected return from acquiring the asset and hence the value of acquiring information.

We now discuss how policy affects gains from trade. Notice that gains from trade are not realized if, and only if, the asset is of high quality but no buyer places a serious bid. We know that in equilibrium, the type h buyers always bid seriously ( $\lambda^*(\gamma) < \overline{\lambda}(\gamma)$  for all  $\gamma > 0$ ). Hence, gains from trade are maximized if the type u buyers also always bid seriously. Now observe that a consequence of the proof of Proposition 1 is that the probability that type u buyers bid seriously converges to one as  $\gamma$  decreases to zero. We then have the following result.

#### **Proposition 4.** Gains from trade are maximized as $\gamma$ converges to zero.

Propositions 3 and 4 show that the policy that maximizes information acquisition is different from the policy that maximizes gains from trade. Thus, policymakers face a trade-off between maximizing gains from trade and maximizing information acquisition. As we are going to see next, this trade-off implies that policymakers also face a trade-off between maximizing gains from trade and maximizing price discovery.

### 4.2 Information Production

In the previous section, we studied the relationship between the amount of insurance provided by the policy, each buyer's decision of whether to become informed about the quality of the asset, and their ensuing bidding behavior. We now turn our attention to understanding the implications of buyers' behavior on the informational content of the winning bid. Specifically, we first derive a metric for measuring the quantity of information contained in a winning bid and then, using this metric, show that information production is maximized at a value of  $\gamma \in (0, \tilde{\gamma})$  and is minimized as  $\gamma$  converges to zero. Entropy and the Quantity of Information Produced. We assume that the number of bidders, the policy rule, and the winning bid of an auction are observed by the public, and study the extent to which these observations reduce uncertainty about the quality of the asset.<sup>23</sup> To do so, consider an agent who does not participate in the auction but can observe both the policy choice,  $\gamma$ , and the winning bid, p. We adopt the convention that p = 0 when no trade takes place. Given the prior belief  $\pi$  that the asset is of high quality, and knowing both the signal structure and equilibrium strategies, the agent can use p to update his belief about the quality of the asset. Denote the agent's posterior belief by  $\pi^+$ . Since p is a random variable whose distribution depends on  $\gamma$ , each choice of  $\gamma$  induces a distribution  $\Omega^*(\cdot; \gamma)$  of posterior beliefs;  $\Omega^*(\pi^+; \gamma)$  is the (unconditional) probability that the agent's posterior belief is  $\pi^+$  or less when the policy is  $\gamma$ .

Following Sims [2003], we measure the quantity of information produced in a single auction with policy  $\gamma$  as the expected reduction in uncertainty that results from observing p, where uncertainty is measured by the entropy of the agent's beliefs. The entropy of a probability distribution q on a finite set  $\mathcal{J}$  of events is  $H(q) = -\sum_{j \in \mathcal{J}} q_j \log(q_j)$ , where  $q_j$  is the probability of  $j \in \mathcal{J}$ . Thus, the quantity of information or *entropy informativeness* of the auction is

$$I(\gamma) = H(\pi) - \mathbb{E}\left[H\left(\pi^{+}\right)\right],$$

where  $H(\phi) = -\phi \log(\phi) - (1 - \phi) \log(1 - \phi)$  is the entropy of a belief  $\phi$  that the asset is of high quality and the expectation is taken with respect to  $\Omega^*(\cdot; \gamma)$ . In order to evaluate how the choice of  $\gamma$  affects information production, we first deduce the distribution  $\Omega^*(\cdot; \gamma)$ . Afterwards, we study how  $I(\gamma)$  depends on  $\gamma$ .

The Distribution of Posterior Beliefs. For each  $\gamma \in (0, 1]$  and  $\lambda \in (0, \overline{\lambda}(\gamma))$ , let  $\phi(p; \lambda, \gamma)$  denote the agent's posterior belief that the asset is of high quality after observing a winning bid

<sup>&</sup>lt;sup>23</sup>The assumption that only the winning bid is observed is motivated in part by realism; losing bids are rarely observed in practice. This assumption is also consistent with the actual implementation of PPIP. For example, on September 19, 2009 the FDIC issued a press release providing details of an auction that occurred on August 31, 2009. The information in the press release included the assets for sale (a pool of residential mortgage loans with an unpaid principal balance of approximately \$1.3 billion), the name of the winning bidder (Residential Credit Solutions), the number of total bidders who participated in the auction (twelve), the winning bid (approximately \$885 million), and the leverage ratio used to finance the purchase (6-to-1). More details of this auction, or others like it, are available at http://www.fdic.gov.

p when the probability that buyers become informed is  $\lambda$  and the choice of policy is  $\gamma$ .<sup>24</sup> The following result reports basic properties of  $\phi(p; \lambda, \gamma)$ ; the proof and derivation are in the Appendix.

**Lemma 3.** The posterior belief  $\phi(p; \lambda, \gamma)$  satisfies the following properties: (i)  $\phi(0; \lambda, \gamma) < \phi(c; \lambda, \gamma)$ ; (ii)  $\phi(p; \lambda, \gamma)$  is strictly increasing in p when  $p \in [c, \overline{b}_h]$ ; and (iii)  $\phi(\overline{b}_h; \lambda, \gamma) = \tilde{\pi}$ .

The first two facts in Lemma 3 are intuitive. Indeed, since type  $\ell$  buyers only bid zero, while type u and type h buyers sometimes bid  $b \ge c$ , observing trade at some price  $p \ge c$  is more indicative that the asset is of high quality than observing no trade. Moreover, as p increases, so too does the conditional probability that the other buyers received signal h or u (as opposed to  $\ell$ ) but bid  $b \le p$ . However, for any  $p < \overline{b}_h$ , bids less than p are more likely when the asset is of low quality. Hence,  $\phi(p; \lambda, \gamma) < \tilde{\pi}$  for all  $p \in [c, \overline{b}_h)$  because of what an observer infers about the losing bids. It is only when  $p = \overline{b}_h$  that observing the winning bid is equivalent to observing the high signal, for in this case bids less than p have the same probability regardless of the asset's type.

Given  $\phi(p; \lambda, \gamma)$ , along with the equilibrium characterization of Section 3, we can now construct  $\Omega^*(\cdot, \gamma)$  for each  $\gamma \in (0, 1]$ . Let  $\Omega_j^*(\pi^+; \gamma)$  be the probability that the agent's posterior belief is  $\pi^+$  or less when the policy is  $\gamma$  and the asset quality is  $j \in \{L, H\}$ . Moreover, in a slight abuse of notation, let  $\phi(p) = \phi(p; \lambda^*(\gamma), \gamma)$  and  $\phi^{-1}(\pi^+) = \phi^{-1}(\pi^+; \lambda^*(\gamma), \gamma)$  be the inverse of  $\phi(p)$ , which is well defined by Lemma 3. Then,

$$\Omega_{H}^{*}(\pi^{+};\gamma) = \begin{cases}
0 & \text{if } \pi^{+} \in [0,\phi(0)) \\
[(1-\lambda^{*}(\gamma))F_{u}^{*}(0)]^{N} & \text{if } \pi^{+} \in [\phi(0),\phi(c)) \\
[(1-\lambda^{*}(\gamma))F_{u}^{*}(\phi^{-1}(\pi^{+}))]^{N} & \text{if } \pi^{+} \in [\phi(c),\phi(\overline{b}_{u})) \\
[(1-\lambda^{*}(\gamma))]^{N} & \text{if } \pi^{+} \in [\phi(\overline{b}_{u}),\phi(\underline{b}_{h})) \\
[1-\lambda^{*}(\gamma)+\lambda^{*}(\gamma)F_{h}^{*}(\phi^{-1}(\pi^{+}))]^{N} & \text{if } \pi^{+} \in [\phi(\underline{b}_{h}),\phi(\overline{b}_{h})]
\end{cases}$$
(9)

<sup>24</sup>We restrict attention to  $\lambda < \overline{\lambda}(\gamma)$  since we know that in equilibrium the probability of information acquisition is smaller than  $\overline{\lambda}(\gamma)$  for all  $\gamma > 0$ .

if  $\gamma < \tilde{\gamma}$ , and

$$\Omega_{H}^{*}(\pi^{+};\gamma) = \begin{cases} 0 & \text{if } \pi^{+} \in [0,\phi(0)) \\ [(1-\lambda^{*}(\gamma))]^{N} & \text{if } \pi^{+} \in [\phi(0),\phi(c)) \\ [1-\lambda^{*}(\gamma)+\lambda^{*}(\gamma)F_{h}^{*}(\phi^{-1}(\pi^{+}))]^{N} & \text{if } \pi^{+} \in [\phi(c),\phi(\overline{b}_{h})] \end{cases}$$
(10)

if  $\gamma \geq \tilde{\gamma}$ . Similar calculations, relegated to the Appendix for the sake of brevity, can be used to derive  $\Omega_L^*(\cdot;\gamma)$ . Given  $\Omega_H^*$  and  $\Omega_L^*$ , the unconditional distribution of posterior beliefs is then

$$\Omega^*(\pi^+;\gamma) = \pi \Omega^*_H(\pi^+;\gamma) + (1-\pi)\Omega^*_L(\pi^+;\gamma)$$

**Maximizing Information Production.** We now establish that the quantity of information is maximized at a value of  $\gamma$  that lies strictly between 0 and  $\tilde{\gamma}$ , and that maximizing gains from trade minimizes the informational content of prices.

**Proposition 5.**  $I(\gamma)$  is maximized at a point that is strictly positive, but strictly smaller than  $\tilde{\gamma}$ . In addition,  $I(\gamma)$  decreases to zero as  $\gamma$  decreases to zero.

The intuition for why  $I(\gamma)$  is maximized at an interior value of  $\gamma$  is simple. From Proposition 3, a decrease in  $\gamma$  initially increases the incentive of buyers to inspect the asset, which promotes price discovery by encouraging buyers to actively participate in the auction (i.e., to bid  $b \ge c$ ). However, we also know from Proposition 3 that too much insurance deters information acquisition: if buyers face little risk when acquiring the asset, they have no incentive to pay the cost of inspection before placing a serious bid. As a result, if  $\gamma$  becomes too small, buyers still place serious bids, but the probability that they are informed begins to fall, so the informational content of the winning bid decreases. In fact, the informational content of the winning bid disappears as  $\gamma$  converges to zero, since in this case the distribution of posterior beliefs becomes concentrated at the prior belief. The value of  $\gamma$  that maximizes the quantity of information produced strikes a balance between these two effects and ultimately encourages both participation and information acquisition.

To understand why  $I(\gamma)$  is maximized at a value of  $\gamma$  that is strictly less than  $\tilde{\gamma}$ , it is helpful to note that a change in  $\gamma$  affects the distribution of posterior beliefs through two margins. First, a change in  $\gamma$  affects the probability that each buyer acquires information,  $G(k^*) = \lambda^*$ ; we refer to this as the "extensive" margin. Second, a change in  $\gamma$  affects the informational content of winning bids by its effect on the equilibrium bidding strategies; we refer to this as the "intensive" margin. Starting at  $\tilde{\gamma}$ , a marginal decrease in  $\gamma$  causes  $\lambda^*$  to fall, which diminishes information production. However, it also causes uninformed buyers to start placing serious bids, which distinguishes their bids from type  $\ell$  buyers. This makes the distribution of winning bids more informative. In a neighborhood of  $\tilde{\gamma}$ , the first effect is of second-order importance (by the usual envelope argument), so that  $I(\gamma)$  rises in response to marginal decrease in  $\gamma$ . However, as  $\gamma$  continues to fall, the extensive margin effect dominates and  $I(\gamma)$  eventually falls as well.

### 4.3 **Comparative Statics**

We conclude this section with a discussion of how the policy that maximizes buyers' incentives to acquire information—and, ultimately, information production—depend on certain features of the economic environment. We focus on two specific model parameters: the prior belief  $\pi$ , which determines the severity of the lemons problem; and the signal precision  $\rho$ , which captures how difficult it is for a buyer to learn the true quality of the asset, or the asset's *opacity*.

Proposition 6 below establishes that an increase in  $\pi$  and an increase in  $\rho$  have the same implications for buyers' incentives to acquire information, but very different implications for how these incentives interact with the policymaker's choice of  $\gamma$ .

**Proposition 6.** For any  $\pi$  and  $\rho$  satisfying (1) and (2), the maximum cutoff cost,  $k^*(\tilde{\gamma})$ , is increasing in both  $\pi$  and  $\rho$ . However, the policy maximizing information acquisition,  $\tilde{\gamma}$ , is increasing in  $\pi$  but decreasing in  $\rho$ .

Loosely speaking, the first statement in Proposition 6 follows from the observation that there is little value in a buyer's acquiring information when either  $\pi$  or  $\rho$  are small: in the former case the buyer is likely to learn that the asset is worthless, and in the latter case the signal itself is largely uninformative. Hence, as either of these parameters increase, the expected payoff from acquiring the signal rises and buyers are willing to spend more in order to become informed.<sup>25</sup>

<sup>&</sup>lt;sup>25</sup>Note that this logic only applies in the presence of an initial lemons problem, i.e., when (1) is satisfied. For example, when  $\pi$  is sufficiently large, the expected payoff from acquiring information shrinks, as the posterior belief  $\tilde{\pi}$  converges to the prior as  $\pi$  approaches 1.

To understand the second statement in Proposition 6, it is helpful to recall that  $\tilde{\gamma}$  offers precisely the level of insurance that makes type u buyers indifferent between placing a serious bid and not participating in the auction. Therefore, when  $\pi$  increases and the adverse selection problem abates, type u buyers strictly prefer to place serious bids with positive probability, which decreases the payoffs to type h buyers and discourages information acquisition. Hence, by increasing  $\gamma$  and providing *less* insurance, the policymaker can discourage bids from uninformed buyers and restore maximal information acquisition. Alternatively, when  $\rho$  increases, the winner's curse is exacerbated and type u buyers strictly prefer not to participate in the auction. In this case, decreasing  $\gamma$ and providing *more* insurance encourages additional information acquisition.

The relationships between  $\tilde{\gamma}$  and the parameters  $\pi$  and  $\rho$ , established above, help to explain how these parameters ultimately affect the policy that maximizes information production, which we denote  $\gamma_I$ . Though we are unable to derive analytical results, simple numerical examples provide some general guidelines. Figures 3 and 4, respectively, depict how  $\gamma_I$  varies with  $\pi$  and  $\rho$ for a typical example.<sup>26</sup>



From Figure 3, one can see that the results in Proposition 6 extend to the relationship between  $\gamma_I$  and  $\pi$ : as the severity of the lemons problem diminishes, so too does the amount of insurance

<sup>&</sup>lt;sup>26</sup>In this case, we set  $\pi = .45$ , v = 1, c = .5,  $\rho = .75$ , and assume that k is distributed uniformly over the unit interval. We stress that these parameter values are not intended to represent a serious calibration, but rather are chosen purely for illustrative purposes. Extensive experimentation with alternative parameter values revealed very similar results.

required to maximize price discovery. Hence, if the policy goal is to encourage information production, then the benefits of intervention wane as the adverse selection problem disappears, which is consistent with the observation that such policies are not useful when markets are not frozen. Figure 4 suggests that the relationship between  $\gamma_I$  and  $\rho$  is more subtle. For most parameter values, the relationship we document in Proposition 6 is dominant, and the value of  $\gamma$  maximizing information production is decreasing in  $\rho$ . However, it is possible for  $\gamma_I$  to be increasing in  $\rho$  as  $\rho$ gets close to 1, since changes in  $\gamma$  are affecting both the intensive and extensive margins discussed in Section 4.2.<sup>27</sup>

## **5** Assumptions and Extensions

The Number of Buyers. The analysis above treats the number of buyers in the auction, N, as exogenous. However, one could imagine that N is actually a second potential instrument available to policymakers. Since the number of buyers participating in the auction clearly affects the equilibrium outcome, a natural question is how the informational content of the winning bid is affected by a change in N.

The answer is that there are two, opposing effects. On the one hand, an increase in the number of participants decreases the buyers' expected payoffs, thereby depressing their incentives to acquire information; this result can be easily derived given the expressions for  $V_h$  in Proposition 1. As N increases,  $k^*$  decreases, and thus the probability that each buyer is informed,  $\lambda^* = G(k^*)$ , also falls. As N tends to infinity, for any  $\gamma$ ,  $V_h$  approaches zero and the buyers' incentive to acquire information evaporates. On the other hand, ceteris paribus, having more buyers implies more effective information aggregation. In particular, holding  $\lambda$  constant, increasing N reduces the risk that no buyer acquires information and, consequently, decreases the probability that socially valuable information is not generated.

Figure 5 plots the typical shape of  $I(\gamma)$  across different values of N, where we set  $\gamma = \gamma_I$  for each N. Unfortunately, a precise characterization of the optimal N depends on, e.g., the exact

<sup>&</sup>lt;sup>27</sup>The intuition is as follows: when  $\rho$  is moderately high, there is incentive for the policymaker to decrease  $\gamma$  in order to encourage information production along *both* the extensive margin (i.e., to increase  $k^*$ ) and the intensive margin (i.e., to reduce  $F_u(0)$ ). As  $\rho$  approaches one, the latter effect vanishes, and  $\gamma_I$  can actually increase.

shape of the distribution function G, and hence is not analytically tractable. However, as this example illustrates, the number of buyers is a second, important consideration for policymakers when designing a program of this type.



**Information Disclosure and Price Discovery.** The analysis above also assumes that the winning bid is observable, but that losing bids are not. This assumption was motivated by realism: whereas transaction prices can often be observed, losing bids are typically not.<sup>28</sup> From a theoretical point of view, however, it should be fairly obvious that publishing the full vector of bids would be more informative about the quality of the asset.

On the other hand, the fact that losing bids are not observed can be important for encouraging buyers to participate in the first place. More specifically, since investors in one auction could potentially compete in future auctions for similar assets, they may find it valuable to keep their bids (and thus some of the private information they acquired) private for use at a later time. It is worth noting, however, that the decision to publish the number of bidders in each auction is important; using the winning bid to back out the underlying signals of the bidders is known to be considerably more complicated when the number of bidders is unknown.<sup>29</sup>

<sup>&</sup>lt;sup>28</sup>This assumption is also consistent with the implementation of PPIP, as discussed earlier.

<sup>&</sup>lt;sup>29</sup>See, e.g., Athey and Haile [2007].

**Information Structure** We employed an extremely simple signal-generating process in our analysis: there were only two signals, and one signal was assumed to completely reveal the quality of the asset. Though this information structure greatly simplified our analysis, none of our central insights depends on it. In other words, the main lessons that arise from the analysis of our stylized environment carry over to a more general environment.

To see this, suppose an agent who chooses to incur the cost  $k_i$  can receive one of S signals. In this case, there are potentially S + 1 types of bidders (including uninformed bidders) at the bidding stage. Then, as long as these signals satisfy the monotone likelihood ratio property, the single crossing property that applies to buyers' interim beliefs and their subsequent bids continues to hold: buyers who receive "better" signals place higher bids than those who receive "worse" signals. Moreover, it is straightforward to show that the bids of these agents—perhaps with the exception of agents who received the lowest signal—converge to v as  $\gamma$  converges to zero. As a result, as in Proposition 3, the incentive for buyers to acquire information vanishes as  $\gamma$  converges to zero. Similarly, for  $\gamma$  close to one, informed bidders would place serious bids only if their signal was sufficiently high. This also reduces buyers' ex ante incentives to acquire information about the quality of the asset, which hinders price discovery.

Therefore, as in our simple environment, the value of  $\gamma$  that maximizes price discovery in an environment with a more general signal structure will be interior: it will provide some insurance for buyers to participate in the auction by making serious bids, but not so much insurance that they make those bids without first inspecting the asset. Notice that this reasoning is independent of the likelihood of the lowest signal, so that our insights do not depend on the assumption that a "bad" signal perfectly reveals the low quality of the asset.

**Budget-Constrained Buyers.** Our analysis focuses on the insurance role that nonrecourse lending plays in alleviating the problem of adverse selection. There is, however, an alternative theory for why markets crash, often called "cash-in-the-market pricing" (see, e.g., Allen and Gale [1994]). According to this theory, markets can experience a sudden decrease in prices and trading volume because the buyers in the market are budget constrained: though they would like to purchase assets at the current market prices, they cannot acquire the liquid assets required to do so. Interestingly, by allowing private investors to leverage and thus relaxing their budget constraints, the nonrecourse lending we study could also help address this second source of market freezes.

Suppose, for example, that each buyer i = 1, ..., N has liquid wealth  $w_i$ , which is a random draw from a distribution with support  $[0, \overline{w}]$ . Moreover, suppose that bids are constrained by the inequality

$$\gamma b_i \leq w_i,$$

so that each buyer is required to finance a fraction  $\gamma$  of the purchase price with his liquid wealth. Clearly, if  $\overline{w}$  is sufficiently small, then buyers will not be able to bid  $b_i \ge c$ , and thus no information will be produced. Therefore,  $\gamma < 1$  certainly has the ability to promote trade and price discovery. However, as in our benchmark model, decreasing  $\gamma$  too much can potentially be counterproductive.

The reason is that *budget constraints relax the winner's curse*. In particular, in our benchmark model, the expected quality of the asset conditional on winning the auction is revised down because there is a probability that other buyers received the signal  $\ell$ . However, in a model with budget constraints, this effect is diminished; when a buyer wins the auction, it could be because other buyers received a signal  $\ell$ , but it could also be because they received a signal h but their budget constraint was binding. Since decreasing  $\gamma$  relaxes budget constraints, it also attenuates the winner's curse and thus decreases  $V_h$ . As a result, as in our benchmark model,  $k^*(\gamma)$  can also be non-monotonic in an environment where the market is frozen due to cash-in-the-market pricing. Since the analysis of this issue is fairly complex, we explore it in greater detail in a separate paper.

## 6 Conclusion

One of the most important questions to emerge from the financial crisis of 2007-2008 was whether the government could (and should) intervene in a frozen market. As economists begin to grapple with this question, it is important to correctly identify the various costs and benefits of intervention. To date, most of the existing literature has identified the benefit of intervention as restoring gains from trade, while the costs of intervention have typically been associated with either a direct cost of taxpayer dollars or an indirect cost of encouraging risky behavior in the future (i.e., moral hazard).

This paper identifies and studies a margin that has been mostly ignored: price discovery. Infor-

mation produced in financial markets can have widespread effects on economic activity. We show that policymakers face an important trade-off between restoring gains from trade and maximizing price discovery; namely, while some amount of intervention may be required in order to incentivize buyers to participate in the market, too much intervention can erode the informational content of transaction prices. Hence, policymakers face a delicate balance, which must be calibrated based on, e.g., the severity of the lemons problem and the opacity of the assets for sale.

This paper is among the first to study the effects of government interventions in frozen markets on price discovery, and many important questions remain. For example, one might wonder exactly how the optimal policy depends on the details of the decision problem to which the information from asset prices is applied; in the working paper version, Camargo et al. [2013], we study one such decision problem and show that the basic insights generated here are preserved. It would also be interesting to study how dynamic considerations would change sellers' incentives to trade their assets and buyers' incentives to produce information at a given time. For example, if sellers have more than one asset, they may choose not to sell a portion of their assets to avoid marking other assets to market prices, as in Bond and Leitner [2014]. Similarly, if buyers expect information to be revealed and prices to rise, they may choose to delay bidding, as in Camargo and Lester [2011]. These considerations, and many others, are left for future work.

## Appendix

### **Proof of Lemma 1**

For each  $j \in \{L, H\}$ , let  $Q_i(b)$  be the ex-ante probability that a buyer bids b or less when the asset is of quality  $j \in \{L, H\}$ ; notice that  $Q_H(b) = (1 - \lambda)F_u(b) + \lambda F_h(b)$ , while  $Q_L(b) =$  $(1-\lambda)F_u(b) + \lambda[\rho + (1-\rho)F_h(b)]$ . Now, for each  $b \ge c$  and  $j \in \{L, H\}$ , let  $\xi_j(b)$  be such that

$$\xi_j(b) = \sum_{s=0}^{N-1} \frac{1}{s+1} \binom{N-1}{s} Q_j(b_-)^{N-1-s} [Q_j(b) - Q_j(b_-)]^s.$$

By construction,  $\xi_i(b)$  is the probability that a buyer who bids  $b \ge c$  wins the auction when the asset is of quality  $j \in \{L, H\}$ . It is easy to see that  $\xi_i(b)$  is nondecreasing in  $b^{30}$ . The following two facts are useful in the proof of (iii):

$$\xi_j(b) = \frac{1}{N} \sum_{s=0}^{N-1} Q_j(b)^s Q_j(b_-)^{N-1-s};$$
(11)

$$\frac{\xi_j(b)}{\xi_j(b_+)} = \frac{1}{N} \sum_{s=0}^{N-1} \left( \frac{Q_j(b_-)}{Q_j(b)} \right)^s.$$
(12)

*Proof.* Notice that

$$\sum_{s=0}^{N-1} \frac{1}{s+1} \binom{N-1}{s} a^s b^{N-1-s} = \frac{(a+b)^N - b^N}{Na}$$

for all  $a, b > 0.^{31}$  Hence,

$$\xi_j(b) = \frac{Q_j(b)^N - Q_j(b_-)^N}{N[Q_j(b) - Q_j(b_-)]}$$

and, since  $\xi_i(b_+) = Q_i(b)^{N-1}$ ,

$$\frac{\xi_j(b)}{\xi_j(b_+)} = \sum_{s=0}^{N-1} \frac{1}{s+1} \binom{N-1}{s} \left( \frac{Q_j(b_-)}{Q_j(b)} \right)^{N-1-s} \left[ 1 - \frac{Q_j(b_-)}{Q_j(b)} \right]^s = \frac{1 - (Q_j(b_-)/Q_j(b))^N}{N[1 - (Q_j(b_-)/Q_j(b))]}.$$

Equations (11) and (12) are now a consequence of the fact that  $c^N - d^N = (c - d) \sum_{s=0}^{N-1} c^s d^{N-1-s}$ for all c, d > 0.

(i) The result is obvious if  $\overline{b}_u = 0$ . Suppose then that  $\overline{b}_u \ge c$ , and let  $\pi_u = \pi$  and  $\pi_h = \tilde{\pi}$ . Since the expected payoff to a type  $s \in \{u, h\}$  buyer who bids b is

$$V_s(b) = \pi_s \xi_H(b)(v - b) - (1 - \pi_s)\xi_L(b)\gamma b,$$

<sup>&</sup>lt;sup>30</sup>Let b' > b. Since  $F_s(b'_-) \ge F_s(b)$  for each s, we then have that  $\xi_j(b') \ge Q_j(b'_-)^{N-1} \ge Q_j(b)^{N-1} \ge \xi_j(b)$ . <sup>31</sup>For a proof of this fact, let  $A(y) = \sum_{s=0}^{N-1} \frac{1}{1+s} {N-1 \choose s} (ya)^s b^{N-1-s}$  and B(y) = yA(y). The desired result holds since  $A(1) = \int_0^1 B'(y) dy$  and  $B'(y) = (b + ya)^N$  by the binomial formula.

we have that

$$V_h(b) = \frac{1}{\pi + (1 - \pi)(1 - \rho)} V_u(b) + \frac{(1 - \pi)\rho}{\pi + (1 - \pi)(1 - \rho)} \xi_L(b)\gamma b;$$
(13)

recall that  $\tilde{\pi} = \pi/(\pi + (1 - \rho)(1 - \pi))$ . The second term in the right-hand side of (13) is strictly increasing in b. In addition, by the optimality of  $\bar{b}_u$  for a type u buyer, we have that  $V_u(\bar{b}_u) \ge V_u(b)$  for all  $b \in [c, \bar{b}_u]$ . It then follows that  $V_h(\bar{b}_u) > V_h(b)$  for all  $b \in [c, \bar{b}_u]$ , which implies that  $\underline{b}_h \ge \bar{b}_u$ .

(*ii*) Suppose  $F_h(0) = 1$ . By (*i*), this implies that  $F_u(0) = 1$  as well. Hence, the payoff to a type h buyer who bids b = c is equal to  $\tilde{\pi}(v - c) - (1 - \tilde{\pi})\gamma c$ , which is greater than zero by Assumption 2. Thus, bidding b = 0 is suboptimal for a type h buyer, a contradiction.

(*iii*) We begin by establishing that there are no atoms on the relevant region of the support. First, notice that  $\pi_s(v-b) - (1-\pi_s)\gamma b > 0$  if b is a mass point of  $F_s$ . Indeed, since  $Q_H(b_-) < Q_L(b_-)$  when  $b \in [\underline{b}_u, \overline{b}_h]$  is a mass point of either  $F_u$  or  $F_h$ , equation (11) implies that  $\xi_H(b) < \xi_L(b)$  in this case. The desired result follows from the fact that

$$V_s(b) = [\xi_H(b) - \xi_L(b)]\pi_s(v - b) + \xi_L(b)[\pi_s(v - b) - (1 - \pi_s)\gamma b].$$

Now let  $\eta(b) = \xi_H(b)/\xi_L(b)$ . We claim that  $\eta(b_+) \ge \eta(b)$  for all  $b \in [\underline{b}_u, \overline{b}_h]$ . Indeed, by (12),

$$\eta(b_+) \ge \eta(b) \Leftrightarrow \frac{\xi_L(b)}{\xi_L(b_+)} \ge \frac{\xi_H(b)}{\xi_H(b_+)} \Leftrightarrow \frac{Q_L(b) - Q_L(b_-)}{Q_L(b)} \le \frac{Q_H(b) - Q_H(b_-)}{Q_H(b)}.$$

The desired result follows from the fact that  $Q_L(b) \ge Q_H(b)$  and

$$Q_L(b) - Q_L(b_-) = (1 - \lambda)(F_u(b) - F_u(b_-)) + \lambda(1 - \rho)(F_h(b) - F_h(b_-))$$
  
$$\leq (1 - \lambda)(F_u(b) - F_u(b_-)) + \lambda(F_h(b) - F_h(b_-)) = Q_H(b) - Q_H(b_-).$$

Suppose then that b is a mass point of  $F_s$ . This implies that

$$V_{s}(b_{+}) = \pi_{s}\xi_{h}(b_{+})(v-b)\left\{1 - \frac{(1-\pi_{s})\gamma b}{\pi_{s}(v-b)\eta(b_{+})}\right\}$$
  

$$\geq \pi_{s}\xi_{h}(b_{+})(v-b)\left\{1 - \frac{(1-\pi_{s})\gamma b}{\pi_{s}(v-b)\eta(b)}\right\} > \pi_{s}\xi_{h}(b)(v-b)\left\{1 - \frac{(1-\pi_{s})\gamma b}{\pi_{s}(v-b)\eta(b)}\right\},$$

where the strict inequality follows from the fact that  $\xi_H(b_+) > \xi_H(b)$  and  $\pi_s(v-b) > (1-\pi_s)\gamma b$ . Thus, bidding b is suboptimal for a type s buyer, a contradiction.

Now we establish that there are no gaps. Suppose  $F_u$  is constant in some interval  $[b_1, b_2] \subseteq (\max\{c, \underline{b}_u\}, \overline{b}_u]$ ; if  $b_1 = \max\{c, \underline{b}_u\}$ , then  $b_1$  is a mass point of  $F_u$ . In this case, a type u bidder strictly prefers bidding  $b_1$  to  $b_2$ , for both bids imply the same and positive probability of winning, while the first bid implies a smaller payment. Thus,  $F_u(b)$  is strictly increasing in b when  $b \in [\max\{c, \underline{b}_u\}, \overline{b}_u]$ . A similar argument applies to  $F_h$ .

(*iv*) Suppose  $\underline{b}_u > 0$  and consider a type u buyer who bids  $\underline{b}_u$ . By (*i*) and (*iii*), the buyer wins if, and only if, all other buyers are of type  $\ell$ , which is only possible if the asset is of low quality. So,

the expected payoff to the buyer is strictly negative, which cannot be the case.

#### **Proof of Proposition 1**

We know from Lemma 1 that if  $b \ge c$ , then

$$V_{s}(b) = \pi_{s} \left[ (1-\lambda)F_{u}(b) + \lambda F_{h}(b) \right]^{N-1} (v-b) - (1-\pi_{s}) \left[ (1-\lambda)F_{u}(b) + \lambda \left(\rho + (1-\rho)F_{s}(b)\right) \right]^{N-1} \gamma b,$$

where  $\pi_u = \pi$  and  $\pi_h = \tilde{\pi}$ . We also know from Lemma 1 that the following three mutually exclusive cases are also exhaustive:  $\bar{b}_u > c$ ,  $\bar{b}_u = 0$  and  $\underline{b}_h > c$ , and  $\underline{b}_h = 0$ .

Case 1:  $\overline{b}_u > c$ .

For each  $b \in [c, \bar{b}_u]$ ,  $F_u(b)$  is derived from the fact that  $V_u(b) = 0$ . In addition, combining  $F_u(\bar{b}_u) = 1$  with  $V_u(\bar{b}_u) = 0$ , we obtain

$$\bar{b}_u = \frac{\pi (1-\lambda)^{N-1} v}{\pi (1-\lambda)^{N-1} + (1-\pi)(1-\lambda+\lambda\rho)^{N-1} \gamma}.$$

We see immediately that  $\underline{b}_h = \overline{b}_u$  when  $\underline{b}_u > c$ . Hence,  $V_h$  is determined by considering a type h buyer who bids  $\overline{b}_u$ . From (13) in the proof of Lemma 1 and  $F_h(\overline{b}_u) = 0$ , we find that

$$V_h = \frac{(1-\pi)\rho}{\pi + (1-\pi)(1-\rho)} (1-\lambda + \lambda\rho)^{N-1} \gamma \bar{b}_u.$$

Substituting  $\overline{b}_u$  in the above expression for  $V_h$  and arranging the terms,  $V_h$  is obtained as in (6). For each  $b \in [\underline{b}_h, \overline{b}_h]$ ,  $F_h(b)$  is derived from the fact that  $V_h(b) = V_h$ .

A necessary and sufficient condition for the equilibrium described in the above paragraph to exist is that  $F_u(c) \in (0, 1)$ . From  $V_u(c) = 0$ , we obtain

$$\left[1 + \frac{\lambda \rho}{(1-\lambda)F_u(c)}\right]^{N-1} = \frac{\pi(v-c)}{(1-\pi)\gamma c}.$$

Hence,  $F_u(c) > 0$  if, and only if,  $\gamma < \hat{\gamma} = \pi (v - c)/(1 - \pi)c$ , and  $F_u(c) < 1$  if, and only if,  $\lambda < \underline{\lambda}(\gamma)$ ; note that  $\underline{\lambda}(\gamma) > 0$  if, and only if,  $\gamma < \hat{\gamma}$ .

Suppose now that  $\lambda < \underline{\lambda}(\gamma)$ , so that  $\gamma < \hat{\gamma}$  a fortiori. Then  $\overline{b}_u = 0$  implies that the payoff to a type u buyer from bidding b = c is at least

$$\pi (1-\lambda)^{N-1} (v-c) - (1-\pi)(1-\lambda+\lambda\rho)^{N-1} \gamma c,$$

which is positive given that  $\lambda < \underline{\lambda}(\gamma)$ . Thus,  $\overline{b}_u > c$ . Case 3:  $\underline{b}_h = 0$ .

Note that if type h buyers are indifferent between bidding b = 0 and bidding  $b \in [c, \overline{b}_h]$ , then

 $F_h(b)$  must be such that

$$V_h(b) = \tilde{\pi} \left[ 1 - \lambda + \lambda F_h(b) \right]^{N-1} (v - b) - \tilde{\pi} \left[ 1 - \lambda + \lambda \left( \rho + (1 - \rho) F_h(b) \right) \right]^{N-1} \gamma b = 0.$$

A necessary and sufficient condition for this equilibrium to exist is that  $F_h(c) > 0$ . Straightforward algebra shows that  $F_h(c) > 0$  is equivalent to  $\lambda > \overline{\lambda}(\gamma)$ .

Suppose now that  $\lambda > \overline{\lambda}(\gamma)$ . Then  $\underline{b}_h > 0$  implies that

$$V_h(c) \le \tilde{\pi}(1-\lambda)^{N-1}(v-c) - (1-\tilde{\pi})(1-\lambda+\lambda\rho)^{N-1}\gamma b < 0.$$

a contradiction. Thus,  $\underline{b}_h = 0$ .

Case 2:  $\overline{b}_u = 0$  and  $\underline{b}_h > c$ .

We know from above that  $\overline{b}_u = 0$  and  $\underline{b}_h > c$  if, and only if,  $\lambda \in [\underline{\lambda}(\gamma), \overline{\lambda}(\gamma)]$ . Moreover, the analysis in the main text shows that there exists a unique equilibrium when  $\lambda \in [\underline{\lambda}(\gamma), \overline{\lambda}(\gamma)]$  and that  $V_h$  is given by (7) in this equilibrium.

#### **Proof of Proposition 3**

Noticee that  $V_I(G(k), \gamma) \leq V_I(G(0), \gamma)$  for all  $k \geq 0$  by Lemma 2. Moreover,  $V_I(G(0), \gamma)$  converges to zero as  $\gamma$  decreases to zero by Proposition 1 and the fact that  $G(0) < \underline{\lambda}(\gamma)$  if  $\gamma$  is small enough. Hence,  $k^*(\gamma)$  converges to zero as  $\gamma$  decreases to zero. A straightforward argument shows that  $k^*(\gamma)$  is also continuous in  $\gamma$ .

Now note that  $\underline{k}(\gamma)$  is continuous and nonincreasing in  $\gamma$ , with  $\underline{k}(0) = 1$  and  $\underline{k}(\gamma) = 0$  if  $\gamma \ge \pi(v-c)/(1-\pi)c$ . Since  $k^*(\gamma) > 0$  for  $\gamma > 0$  and  $k^*(\gamma) < 1$  if  $\gamma$  is small enough, there exists  $\gamma \in (0,1)$  with  $k^*(\gamma) = \underline{k}(\gamma)$ . Let  $\tilde{\gamma} \in (0,1)$  be the greatest value of  $\gamma$  for which  $k^*(\gamma) = \underline{k}(\gamma)$ . Then  $G(k^*(\tilde{\gamma})) = \underline{\lambda}(\tilde{\gamma})$ , as  $\underline{k}(\tilde{\gamma}) = k^*(\tilde{\gamma}) > 0$ . So, by the reasoning in the main text,  $k^*(\gamma) < k^*(\tilde{\gamma})$  if  $\gamma < \tilde{\gamma}$ . Given that  $\underline{k}(\gamma) \ge \underline{k}(\gamma^*)$ , there exists no other  $\gamma$  such that  $k^*(\gamma) = \underline{k}(\gamma)$ .

#### **Proposition 7 and Proof**

**Proposition 7.** The equilibrium cutoff cost for inspecting the asset is  $k^* = 0$  when  $\gamma = 0$ .

We prove that if  $\gamma = 0$ , then the expected payoff to a type h buyer is zero regardless of the probability  $\lambda$  that the other buyers become informed. Since  $V_h = 0$  implies that  $V_I(\lambda, 0) = 0$ , it immediately follows that  $k^* = 0$  when  $\gamma = 0$ .

Suppose, by contradiction, that  $V_h > 0$ , so that  $\underline{b}_h \ge c$  and  $\overline{b}_h < v$ . Given that

$$V_s(b) = \pi_s \xi_H(b)(v-b)$$

for all  $b \ge c$ , we then have  $V_u \ge V_u(\underline{b}_h) > 0$ , so that  $\underline{b}_u \ge c$  and  $\overline{b}_u < v$  as well. Therefore,  $\underline{b} = \min\{\underline{b}_u, \underline{b}_h\} \in [c, v)$ . It is now easy to see that  $\xi_H$  has no mass point. This, however, implies that  $V_u(\underline{b}) = V_h(\underline{b}) = 0$ , so that either  $V_u = 0$  or  $V_h = 0$ , a contradiction.

### **Proof of Lemma 3**

Consider first the case where  $\lambda \in (0, \underline{\lambda}(\gamma))$ , so that  $\overline{b}_u > c$ . Suppose p = 0. In this case, all buyers must be either of type  $\ell$  or of type u. Therefore,

$$\phi(0;\lambda,\gamma) = \frac{\pi}{\pi + (1-\pi) \left[1 + \frac{\lambda\rho}{(1-\lambda)F_u(0)}\right]^N}$$

Now suppose  $p \in [c, \overline{b}_u]$ . In this case, the winner must be uninformed, while all other buyers are either of type  $\ell$  or of type u bidding below p, so that

$$\phi(p;\lambda,\gamma) = \frac{\pi}{\pi + (1-\pi) \left[1 + \frac{\lambda\rho}{(1-\lambda)F_u(p)}\right]^{N-1}}.$$

Since  $F_u(c) = F_u(0) > 0$  and  $F_u(p)$  is strictly increasing in p when  $p \in [c, \overline{b}_u]$ , it is easy to see that  $\phi(0; \lambda, \gamma) < \phi(c; \lambda, \gamma)$  and that  $\phi(p; \lambda, \gamma)$  is strictly increasing in p when  $p \in [c, \overline{b}_u]$ .

Finally, if  $p \in [\underline{b}_h, \overline{b}_h]$ , then the winner must be of type h, while any other buyer can be of type  $\ell$ , of type u, or of type h bidding less than p. Therefore,

$$\phi(p;\lambda,\gamma) = \frac{\pi}{\pi + (1-\pi)(1-\rho)\left\{1 + \frac{\lambda\rho[1-F_h(p)]}{1-\lambda+\lambda F_h(p)}\right\}^{N-1}}$$

It is easy to see that  $\phi(\underline{b}_h; \lambda, \gamma) > \phi(\overline{b}_u; \lambda, \gamma)$ . Moreover, since  $F_h(p)$  is strictly increasing in p when  $p \in [\underline{b}_h, \overline{b}_h]$ , we have that  $\phi(p; \lambda, \gamma)$  is strictly increasing in p when  $p \in [\underline{b}_h, \overline{b}_h]$ . To finish, note that  $F_h(\overline{b}_h) = 1$  implies that  $\phi(\overline{b}_h; \lambda, \gamma) = \tilde{\pi}$ .

Consider now the case where  $\lambda \in [\underline{\lambda}(\gamma), \overline{\lambda}(\gamma)]$ . Then, since now  $F_u(0) = 1$ , we have that

$$\phi(0;\lambda,\gamma) = \frac{\pi}{\pi + (1-\pi) \left[1 + \frac{\lambda\rho}{1-\lambda}\right]^N}$$

and

$$\phi(p;\lambda,\gamma) = \frac{\pi}{\pi + (1-\pi)(1-\rho) \left\{ 1 + \frac{\lambda\rho[1-F_h(p)]}{1-\lambda+\lambda F_h(p)} \right\}^{N-1}}$$

for all  $p \in [c, \overline{b}_h]$ . We see immediately from the analysis above that  $\phi(0; \lambda, \gamma) < \phi(c; \lambda, \gamma)$ ,  $\phi(p; \lambda, \gamma)$  is strictly increasing in p when  $p \in [c, \overline{b}_h]$ , and  $\phi(\overline{b}_h; \lambda, \gamma) = \tilde{\pi}$ .

### Derivation of $\Omega^*_L(\pi^+,\gamma)$

The only change from  $\Omega_H^*(\cdot;\gamma)$  is that now each informed buyer is of type h with probability  $\lambda(1-\rho)$  and of type l with probability  $\lambda\rho$ . Therefore, if  $\gamma < \tilde{\gamma}$ , then

$$\Omega_{L}^{*}(\pi^{+};\gamma) = \begin{cases}
0 & \text{if } \pi^{+} \in [0,\phi(0)) \\
[(1-\lambda^{*}(\gamma))F_{u}^{*}(0) + \lambda^{*}(\gamma)\rho]^{N} & \text{if } \pi^{+} \in [\phi(0),\phi(c)) \\
[(1-\lambda^{*}(\gamma))F_{u}^{*}(\phi^{-1}(\pi^{+})) + \lambda^{*}(\gamma)\rho]^{N} & \text{if } \pi^{+} \in [\phi(c),\phi(\overline{b}_{u})) \\
(1-\lambda^{*}(\gamma) + \lambda^{*}(\gamma)\rho)^{N} & \text{if } \pi^{+} \in [\phi(\overline{b}_{u}),\phi(\underline{b}_{h})) \\
\{1-\lambda^{*}(\gamma) + \lambda^{*}(\gamma)[\rho + (1-\rho)F_{h}^{*}(\phi^{-1}(\pi^{+}))]\}^{N} & \text{if } \pi^{+} \in [\phi(\underline{b}_{h}),\phi(\overline{b}_{h})]
\end{cases}$$
(14)

and if  $\gamma \geq \tilde{\gamma}$ , then

$$\Omega_{L}^{*}(\pi^{+};\gamma) = \begin{cases}
0 & \text{if } \pi^{+} \in [0,\phi(0)) \\
(1-\lambda^{*}(\gamma)+\lambda^{*}(\gamma)\rho)^{N} & \text{if } \pi^{+} \in [\phi(0),\phi(c)) \\
\{1-\lambda^{*}(\gamma)+\lambda^{*}(\gamma)[\rho+(1-\rho)F_{h}^{*}(\phi^{-1}(\pi^{+}))]\}^{N} & \text{if } \pi^{+} \in [\phi(c),\phi(\overline{b}_{h})]
\end{cases}$$
(15)

#### **Proof of Proposition 5**

The proof consists of three steps. We first show that  $I(\gamma)$  is strictly decreasing in  $\gamma$  if  $\gamma > \tilde{\gamma}$ . We then show that  $I(\tilde{\gamma} - \varepsilon) > I(\tilde{\gamma})$  for  $\varepsilon$  positive but sufficiently small. We finally show that  $I(\tilde{\gamma}) > \lim_{\gamma \to 0} I(\gamma) = 0$ .

Step 1.  $I'(\gamma) < 0$  if  $\gamma > \tilde{\gamma}$ .

Suppose  $\gamma > \tilde{\gamma}$ . We begin by establishing some properties of  $\Omega^*(\cdot; \gamma)$  that are useful in the argument that follows. A straightforward consequence of Lemma 3 is that if  $\pi^+ \in [\phi(c), \tilde{\pi}]$ , then  $\phi(p) = \phi(p; \lambda^*(\gamma), \gamma) \leq \pi^+$  if, and only if,

$$a = \left[\frac{(1-\pi^{+})\pi}{\pi^{+}(1-\pi)(1-\rho)}\right]^{\frac{1}{N-1}} \le 1 + \frac{\lambda^{*}(\gamma)\rho\left[1-F_{h}^{*}(p)\right]}{1-\lambda^{*}(\gamma)+\lambda^{*}(\gamma)F_{h}^{*}(p)};$$

note that  $a \ge 1$  since  $\pi^+ \le \tilde{\pi}$ . Hence,

$$F_h^*(\phi^{-1}(\pi^+)) = 1 - \frac{1}{\lambda^*(\gamma)} \frac{a-1}{a-1+\rho},$$

and so  $\lambda^*(\gamma)[1 - F_h^*(\phi^{-1}(\pi^+))]$  is independent of  $\gamma$  for all  $\pi^+ \in [\phi(c), \tilde{\pi}]$ . Therefore, (10) and (15) imply that  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\phi(c), \tilde{\pi}]$ . Another consequence of (10) and (15) is that

$$\Omega^*(\pi^+;\gamma) = \pi (1 - \lambda^*(\gamma))^N + (1 - \pi)(1 - \lambda^*(\gamma) + \lambda^*(\gamma)\rho)^N$$

for all  $\pi^+ \in [\phi(0), \phi(c))$ . Thus, from Proposition 3, we have that  $\Omega^*(\pi^+; \gamma)$  is strictly increasing in  $\gamma$  when  $\pi^+ \in [\phi(0), \phi(c))$ .

We now compute the derivative of  $\mathbb{E}[H(\pi^+)]$  with respect to  $\gamma$ . First note that

$$\begin{split} \mathbb{E}[H(\pi^{+})] &= \Omega^{*}(\phi(0);\gamma)H(\phi(0)) + \int_{\phi(c)}^{\tilde{\pi}} H(\pi^{+})d\Omega^{*}(\pi^{+};\gamma) \\ &= H(\tilde{\pi}) + \Omega^{*}(\phi(0);\gamma)\left[H(\phi(0)) - H(\phi(c))\right] - \int_{\phi(c)}^{\tilde{\pi}} H'(\pi^{+})\Omega^{*}(\pi^{+};\gamma)d\pi^{+}, \end{split}$$

where the second equality follows from integration by parts and the fact that  $\Omega^*(\phi(c);\gamma) = \Omega^*(\phi(0);\gamma)$ . Given that

$$\frac{d}{d\gamma} \int_{\phi(c)}^{\tilde{\pi}} H'(\pi^+) \Omega^*(\pi^+;\gamma) d\pi^+ = -\Omega^*(\phi(c);\gamma) H'(\phi(c)) \frac{d\phi(c)}{d\gamma}$$

by the Fundamental Theorem of Calculus and the fact that  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\phi(c), \tilde{\pi}]$ , we then have that

$$\frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} = \frac{d\Omega^*(\phi(0);\gamma)}{d\gamma} \left[H(\phi(0)) - H(\phi(c))\right] + \Omega^*(\phi(0);\gamma)H'(\phi(0))\frac{d\phi(0)}{d\gamma}$$

Since  $H(\phi(c)) < H(\phi(0)) + H'(\phi(0))(\phi(c) - \phi(0))$  by the strictly concavity of  $H(\phi)$  and  $d\Omega^*(\phi(0), \gamma)/d\gamma > 0$ , the equation for  $d\mathbb{E}[H(\pi^+)]/d\gamma$  derived above implies that

$$\frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} > H'(\phi(0)) \left\{ -\frac{d\Omega^*(\phi(0);\gamma)}{d\gamma}\phi(c) + \frac{d}{d\gamma} \left[\Omega^*(\phi(0);\gamma)\phi(0)\right] \right\}.$$

We claim that the right-hand side of the above equation is zero. Indeed,

$$\frac{d\Omega^*(\phi(0);\gamma)}{d\gamma} = -N \left[ \pi (1 - \lambda^*(\gamma))^{N-1} + (1 - \pi)(1 - \rho)(1 - \lambda^*(\gamma) + \lambda^*(\gamma)\rho)^{N-1} \right] \frac{d\lambda^*(\gamma)}{d\gamma}$$

and so Lemma 3 implies that

$$-\frac{d\Omega^*(\phi(0);\gamma)}{d\gamma}\phi(c) = N\pi(1-\lambda^*(\gamma))^{N-1}\frac{d\lambda^*(\gamma)}{d\gamma}$$

The desired result follows from the fact that Lemma 3 also implies that  $\Omega^*(\phi(0); \gamma)\phi(0) = \pi(1 - \lambda^*(\gamma))^N$ . We can then conclude that  $\mathbb{E}[H(\pi^+)]$  is strictly increasing in  $\gamma$  when  $\gamma \in (\tilde{\gamma}, 1]$ . This implies that  $I'(\gamma) < 0$  if  $\gamma > \tilde{\gamma}$ .

Step 2.  $I(\tilde{\gamma} - \varepsilon) > I(\tilde{\gamma})$  for  $\varepsilon$  positive but sufficiently small.

Suppose that  $\gamma < \tilde{\gamma}$ . By the same argument as in Step 1,  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\underline{b}_h, \tilde{\pi}]$ . In addition, as in Step 1, if  $\pi^+ \in [\phi(c), \phi(\overline{b}_h)]$ , then

$$F_u^*(\phi^{-1}(\pi^+)) = \frac{\lambda^*(\gamma)}{(1 - \lambda^*(\gamma))(\hat{a} - 1)}$$

where  $\hat{a} > 1$  is the only variable that depends on  $\pi^+$ . Hence, when  $\pi^+ \in [\phi(c), \phi(\overline{b}_u)]$ , (9) and (14) imply that  $\Omega^*(\pi^+; \gamma) = \Psi(\pi^+, \lambda^*(\gamma))$ , where  $\Psi(\pi^+, \lambda)$  is strictly increasing in  $\lambda$ . In particular,  $\Omega^*(\pi^+; \gamma)$  is strictly increasing in  $\gamma$  when  $\pi^+ \in [\phi(c), \phi(\overline{b}_u)]$  according to Proposition 3. Now observe from the proof of Proposition 1 that

$$(1 - \lambda^*(\gamma))F_u^*(0) = \frac{\lambda^*(\gamma)\rho}{(\hat{\gamma}/\gamma)^{1/(N-1)} - 1},$$

where  $\hat{\gamma} = \pi (v - c)/(1 - \pi)c$ . Hence, (9) and (14) together with Proposition 3 also imply that  $\Omega^*(\phi(0), \gamma)$  is strictly increasing in  $\gamma$ .

We now compute  $d\mathbb{E}[H(\pi^+)]/d\gamma$ . Integration by parts implies that

$$\mathbb{E}[H(\pi^{+})] = H(\tilde{\pi}) + \Omega^{*}(\phi(0);\gamma) \left[H(\phi(0)) - H(\phi(c))\right] + \Omega^{*}(\phi(\bar{b}_{u});\gamma) \left[H(\phi(\bar{b}_{u})) - H(\phi(\underline{b}_{h}))\right] \\ - \int_{\phi(c)}^{\phi(\bar{b}_{u})} H'(\pi^{+})\Omega^{*}(\pi^{+};\gamma)d\pi^{+} - \int_{\phi(\underline{b}_{h})}^{\tilde{\pi}} H'(\pi^{+})\Omega^{*}(\pi^{+};\gamma)d\pi^{+},$$

where we used the fact that  $\Omega^*(\phi(c); \gamma) = \Omega^*(\phi(0); \gamma)$  and  $\Omega^*(\phi(\underline{b}_h); \gamma) = \Omega^*(\phi(\overline{b}_u); \gamma)$ . By the Fundamental Theorem of Calculus and the fact that  $\Omega^*(\pi^+; \gamma)$  is independent of  $\gamma$  when  $\pi^+ \in [\phi(c), \tilde{\pi}]$ , we then have

$$\frac{d\mathbb{E}[H(\pi^+)]}{d\gamma} = \frac{d\Omega^*(\phi(0);\gamma)}{d\gamma} \left[H(\phi(0)) - H(\phi(c))\right] + \Omega^*(\phi(0);\gamma)H'(\phi(0))\frac{d\phi(0)}{d\gamma} + \frac{d\Omega^*(\phi(\overline{b}_u);\gamma)}{d\gamma} \left[H(\phi(\overline{b}_u)) - H(\phi(\underline{b}_h))\right] - \int_{\phi(c)}^{\phi(\overline{b}_u)} H'(\pi^+)\frac{d\Omega^*(\pi^+;\gamma)}{d\gamma}d\pi^+.$$

Since  $\Omega^*(\phi(0); \gamma)$  is strictly increasing in  $\gamma$  and  $H(\phi)$  is strictly concave in  $\phi$ ,

$$\frac{d\mathbb{E}[H(\pi^{+})]}{d\gamma} > H'(\phi(0)) \left\{ -\frac{d\Omega^{*}(\phi(0);\gamma)}{d\gamma}\phi(c) + \frac{d}{d\gamma} \left[\Omega^{*}(\phi(0);\gamma)\phi(0)\right] \right\} + \frac{d\lambda^{*}(\gamma)}{d\gamma} \left\{ \frac{\partial\Psi(\pi^{+};\lambda^{*}(\gamma))}{\partial\lambda} \left[H(\phi(\bar{b}_{u})) - H(\phi(\underline{b}_{h}))\right] - \int_{\phi(c)}^{\phi(\bar{b}_{u})} H'(\pi^{+}) \frac{\partial\Psi(\pi^{+};\lambda^{*}(\gamma))}{\partial\lambda} d\gamma d\pi^{+} \right\}.$$

Now observe that  $\Omega^*(\phi(0); \gamma)\phi(0) = \pi[(1 - \lambda^*(\gamma))F_u^*(0)]^N$ . Moreover, straightforward algebra shows that

$$\frac{d\Omega^*(\phi(0);\gamma)}{d\gamma}\phi(c) = \frac{d}{d\gamma}[\Omega^*(\phi(0);\gamma)\phi(0)] + \phi(c)(1-\pi)N[(1-\lambda^*(\gamma))F_u^*(0) + \lambda^*(\gamma)\rho]^{N-1}\rho\frac{d\lambda^*(\gamma)}{d\gamma}.$$

Given that  $d\lambda^*(\tilde{\gamma})/d\gamma = 0$ , we can then conclude that

$$\frac{d\mathbb{E}[H(\pi^+)]}{d\gamma}\bigg|_{\gamma=\tilde{\gamma}} > 0.$$

Hence, there exists  $\varepsilon > 0$  such that  $I(\gamma)$  is strictly decreasing in  $\gamma$  when  $\gamma \in (\tilde{\gamma} - \varepsilon, \tilde{\gamma})$ .

Step 3.  $I(\tilde{\gamma}) > \lim_{\gamma \to 0} I(\gamma) = 0.$ 

Since  $I(\tilde{\gamma}) > 0$ , it suffices to show that  $I(\gamma)$  converges to zero as  $\gamma$  tends to zero. The result is a straightforward consequence of Proposition 3:  $\lim_{\gamma \to 0} k^*(\gamma) = 0$ , and so  $\Omega^*(\cdot; \gamma)$  converges to the degenerate distribution that assigns probability one to  $\pi$  as  $\gamma$  tends to zero.

#### **Proof of Proposition 6**

Let  $\tilde{k} = k^*(\tilde{\gamma})$ . Notice that:

$$\pi (1 - G(\tilde{k}))^{N-1} (v - c) - (1 - \pi)(1 - G(\tilde{k}) + \rho G(\tilde{k}))^{N-1} \tilde{\gamma} c = 0$$
(16)

and that

$$\pi (1 - G(\tilde{k}))^{N-1} (v - c) - (1 - \pi)(1 - \rho)(1 - G(\tilde{k}) + \rho G(\tilde{k}))^{N-1} \tilde{\gamma} c = \tilde{k}.$$
 (17)

Equation (16) follows from the fact that type u buyers are indifferent between bidding zero and bidding b = c when  $\gamma = \tilde{\gamma}$ , while (17) follows from the fact that  $\tilde{k} = [\pi + (1 - \pi)(1 - \rho)]V_h(G(\tilde{k}), \tilde{\gamma})$ . Combining (16) and (17), we obtain that

$$\tilde{k} = \pi \rho (1 - G(\tilde{k}))^{N-1} (v - c)$$
(18)

and that

$$\tilde{k} = (1 - \pi)\rho(1 - G(\tilde{k}) + \rho G(\tilde{k}))^{N-1}\tilde{\gamma}c.$$
 (19)

It is clear from (18) that  $\tilde{k}$  is strictly increasing in both  $\pi$  and  $\rho$ . It is also clear from (19) that  $\tilde{\gamma}$  is strictly increasing in  $\pi$ . Now observe that (18) and (19) together imply that

$$\tilde{\gamma} = \frac{\pi}{1-\pi} \frac{v-c}{c} \left( 1 + \frac{\rho G(\tilde{k})}{1-G(\tilde{k})} \right)^{\frac{1}{N-1}}.$$

Since  $\tilde{k}$  is strictly increasing in  $\rho$ , this last equation implies that  $\tilde{\gamma}$  is strictly decreasing in  $\rho$ .

## References

- Franklin Allen and Douglas Gale. Limited market participation and volatility of asset prices. *The American Economic Review*, pages 933–955, 1994.
- Kenneth J. Arrow. The value of and demand for information. *Decision and Organization*, 2: 131–139, 1972.
- Susan Athey and Philip A. Haile. Nonparametric approaches to auctions. *Handbook of Econometrics*, 6:3847–3965, 2007.
- Susan Athey and Jonathan Levin. The value of information in monotone decision problems. *Working paper, MIT*, 1998.
- Tor-Erik Bakke and Toni M Whited. Which firms follow the market? an analysis of corporate investment decisions. *Review of Financial Studies*, 23(5):1941–1980, 2010.
- Lawrence M Benveniste, Alexander Ljungqvist, William J Wilhelm, and Xiaoyun Yu. Evidence of information spillovers in the production of investment banking services. *The Journal of Finance*, 58(2):577–608, 2003.
- Dirk Bergemann and Juuso Valimaki. Information acquisition and efficient mechanism design. *Econometrica*, 70(3):1007–1033, 2002.
- Dirk Bergemann, Xianwen Shi, and Juuso Välimäki. Information acquisition in interdependent value auctions. *Journal of the European Economic Association*, 7(1):61–89, 2009.
- Philip Bond and Itay Goldstein. Government intervention and information aggregation by prices. *Working Paper, University of Pennsylvania and University of Minnesota*, 2012.
- Philip Bond and Yaron Leitner. Market run-ups, market freezes, and leverage. *fothcoming, Journal of Financial Economics*, 2014.
- Philip Bond, Alex Edmans, and Itay Goldstein. The real effects of financial markets. *Annu. Rev. Financ. Econ.*, 4(1):339–360, 2012.
- Antonio Cabrales, Olivier Gossner, and Roberto Serrano. Entropy and the value of information for investors. *American Economic Review*, 103(1):360–77, 2013.
- Braz Camargo and Benjamin Lester. Trading dynamics in decentralized markets with adverse selection. *Federal Reserve Bank of Philadelphia, Working Paper No. 11-36*, 2011.
- Braz Camargo, Kyungmin Kim, and Benjamin Lester. Subsidizing price discovery. *Federal Reserve Bank of Philadelphia, Working Paper No. 13-20*, 2013.
- Melanie Cao and Shouyong Shi. Screening, bidding, and the loan market tightness. *European Finance Review*, 5(1-2):21–61, 2001.
- Giovanni Cespa and Thierry Foucault. Illiquidity contagion and liquidity crashes. *forthcoming, Review of Financial Studies*, 2013.

- V.V. Chari, Ali Shourideh, and Ariel Zetlin-Jones. Adverse selection, reputation and sudden collapses in secondary loan markets. *NBER Working Paper*, 2010.
- Qi Chen, Itay Goldstein, and Wei Jiang. Price informativeness and investment sensitivity to stock price. *Review of Financial Studies*, 20(3):619–650, 2007.
- Jonathan Chiu and Thorsten V. Koeppl. Market making of last resort: Optimal intervention when financial markets are frozen. *Working Paper, Bank of Canada and Queen's University*, 2011.
- Tri Vi Dang, Gary Gorton, and Bengt Holmström. Ignorance, debt and financial crises. Unpublished, Yale SOM, 2012.
- Douglas W Diamond and Raghuram G Rajan. Illiquid banks, financial stability, and interest rate policy. *Journal of Political Economy*, 120(3):552–591, 2012.
- James Dow and Gary Gorton. Stock market efficiency and economic efficiency: Is there a connection? *The Journal of Finance*, 52(3), 1997.
- Emmanuel Farhi and Jean Tirole. Collective moral hazard, maturity mismatch and systemic bailouts. *The American Economic Review*, 102:60–93, 2012.
- Mark Flannery. Stabilizing large financial institutions with contingent capital certificates. *CARE-FIN Research Paper*, (04), 2010.
- Thierry Foucault and Laurent Frésard. Cross-listing, investment sensitivity to stock price, and the learning hypothesis. *Review of Financial Studies*, 25(11):3305–3350, 2012.
- Thierry Foucault and Thomas Gehrig. Stock price informativeness, cross-listings, and investment decisions. *Journal of Financial Economics*, 88(1):146–168, 2008.
- Itay Goldstein and Ady Pauzner. Demand-deposit contracts and the probability of bank runs. *Journal of Finance*, 60(3):1293–1327, 2005.
- Gary Gorton. Information, liquidity, and the (ongoing) panic of 2007. *American Economic Review*, 99(2):567–72, 2009.
- Sanford Grossman. On the efficiency of competitive stock markets where trades have diverse information. *The Journal of Finance*, 31(2):573–585, 1976.
- Veronica Guerrieri and Robert Shimer. Competitive equilibrium in asset markets with adverse selection. *University of Chicago, Working Paper*, 2011.
- Oliver Hart and Luigi Zingales. A new capital regulation for large financial institutions. *American Law and Economics Review*, 13(2):453–490, 2011.
- FA Hayek. The use of knowledge in society. American Economic Review, 35(4), 1945.
- Martin F Hellwig. On the aggregation of information in competitive markets. *Journal of Economic Theory*, 22(3):477–498, 1980.

- Christopher House and Yusufcan Masatlioglu. Managing markets for toxic assets. *NBER Working Paper*, 2010.
- Matthew O. Jackson. Efficiency and information aggregation in auctions with costly information. *Review of Economic Design*, 8(2):121, 2003.
- Ilan Kremer. Information aggregation in common value auctions. *Econometrica*, 70(4):1675–1682, 2002.
- Stephan Lauermann and Asher Wolinsky. Search and adverse selection. *mimeo*, 2013.
- Jacob Marschak. *Remarks on the Economics of Information*. Defense Technical Information Center, 1959.
- Steven Matthews. Information acquisition in discriminatory auctions. Bayesian models in economic theory, ed. by M. Boyer, and R. Kihlstrom, 49:1477–1500, 1984.
- Robert L McDonald. Contingent capital with a dual price trigger. *Journal of Financial Stability*, 9 (2):230–241, 2013.
- Paul R Milgrom. A convergence theorem for competitive bidding with differential information. *Econometrica: Journal of the Econometric Society*, pages 679–688, 1979.
- Paul R Milgrom. Rational expectations, information acquisition, and competitive bidding. *Econo*metrica: Journal of the Econometric Society, pages 921–943, 1981.
- Nicola Persico. Information acquisition in auctions. *Econometrica*, 68(1):135–148, 2000.
- Wolfgang Pesendorfer and Jeroen M Swinkels. The loser's curse and information aggregation in common value auctions. *Econometrica: Journal of the Econometric Society*, pages 1247–1281, 1997.
- Wolfgang Pesendorfer and Jeroen M Swinkels. Efficiency and information aggregation in auctions. *American Economic Review*, pages 499–525, 2000.
- Thomas Philippon and Philipp Schnabl. Efficient recapitalization. Journal of Finance, forthcoming., 2011.
- Thomas Philippon and Vasiliki Skreta. Optimal interventions in markets with adverse selection. *The American Economic Review*, 102(1):1–28, 2012.
- Claude E. Shannon. A mathematical theory of communication. *Bell Systems Technical Journal*, 27:379–423, 1948.
- Christopher A. Sims. Implications of rational inattention. *Journal of Monetary Economics*, 50(3): 665–690, 2003.
- Phillip Swagel. The financial crisis: an inside view. *Brookings Papers on Economic Activity*, 2009 (1):1–63, 2009.

- Jean Tirole. Overcoming adverse selection: How public intervention can restore market functioning. *The American Economic Review*, 102(1):29–59, 2012.
- Laura L. Veldkamp. *Information Choice in Macroeconomics and Finance*. Princeton University Press, 2011.
- Robert Wilson. A bidding model of perfect competition. *The Review of Economic Studies*, pages 511–518, 1977.