

Monotone Strategyproofness*

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Abstract

We propose a way to compare the extent of preference misrepresentation between two strategies. We define a preference revelation mechanism to be *monotone strategyproof* if declaring a “more truthful” preference ordering dominates (with respect to the true preferences) declaring a “less truthful” preference ordering. Our main result states that a mechanism is strategyproof if, and only if, it is monotone strategyproof. This result holds for any deterministic social choice function on any domain; for probabilistic social choice functions it holds under a mild assumption on the domain.

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1. INTRODUCTION

Truthful revelation is a primary goal in mechanism design. Ideally, it is a dominant strategy to truthfully reveal one's preferences, and a mechanism that induces such a dominant strategy for all agents and all preference profiles is said to be **strategyproof**. Non-trivial strategyproof mechanisms do not always exist if other desired properties are also imposed (Gibbard (1978), Satterthwaite (1978)), but a number of environments have been identified for which non-trivial strategyproof mechanisms exist, e.g. voting, two-sided matching, house allocation, or auctions.¹

Strategyproof mechanisms induce a radical division between strategies, for they distinguish the truthful strategy from all other strategies. All non-truthful strategies are deemed undesired regardless of their other characteristics; a lie is a lie, whether big or small. This gave the prior literature little reason to scrutinize misrepresentations in strategyproof mechanisms, for instance by measuring how much they deviate from the truth. We argue that this is an important omission and we focus in this paper specifically on non-truthful strategies in strategyproof mechanisms.

We believe there is a need for a general tool to analyze misrepresentations. There is indeed now growing evidence that strategyproof mechanisms perform poorly in the laboratory (see Chen (2008) for a survey).² Actually, experimental data from games with a dominant strategy also exhibit seemingly irrational behavior.³ Overall, most experimental analysis of strategyproof mechanisms cannot go further than acknowledging the percentage of subjects not being truthful, and analyzing how this percentage varies when changing some environment parameters or the mechanism itself. However, the existing studies have not been able to rank non-truthful strategies on how close they are to the true preferences, save for some specific cases.⁴ This is a serious limitation because what makes strategyproof mechanisms appealing is, among other things, their ability to generate quality data about individuals' preferences.

¹See for instance Moulin (1980) for voting with single-peaked preferences, Dubins and Freedman (1981) and Roth (1982) for two-sided matching. See also Barberà (2011) for a recent survey.

²See for instance Cason *et al.* (2006) for the pivotal and the Groves-Clarke mechanisms, Chen and Sönmez (2006) or Calsamiglia *et al.* (2010) in a matching context.

³See Palacios-Huerta and Volij (2009) for the centipede game, Kagel and Levin (1986) for auction games or Andreoni (1995) for public good games.

⁴Chen and Sönmez (2006) and Calsamiglia *et al.* (2010) for instance analyze which type of alternative is likelier to be displaced in the preference orderings.

Such information is crucial if one wishes to run counterfactuals and test potential new policies. Policy makers (and econometricians) may prefer a mechanism with a large percentage of individuals not being truthful but “close” to the truth over a mechanism with a smaller percentage of misrepresentations but consisting of large deviations from the truth.

From a theoretical perspective we argue that studying misrepresentations can help understanding further the anatomy of strategyproof mechanisms. By its definition, strategyproofness imposes the existence of a dominant strategy in the mechanism. But does it also impose any structure on misrepresentations? To address this question we classify misrepresentations so as to be able to rank strategies on how much they misrepresent the true preferences. Our contention is that such a classification must be linked to the cost of misrepresenting preferences. Drawing on the intuition for strategyproofness, small misrepresentations should have a lower impact on agents’ welfare than large ones, or, put differently, small deviations should dominate large ones.⁵ We call a mechanism satisfying this property **monotone strategyproof**. One might conjecture that imposing monotonicity between payoffs and distance from the truth would be more restrictive than the usual incentive compatibility, i.e., that some strategyproof mechanisms may not be monotone strategyproof. Our main contribution here is to show that monotone strategyproofness is actually equivalent to strategyproofness. This seemingly counterintuitive result turns out to be straightforward to show and holds for a very general class of environments.

Our result is derived within a typical environment where each individual has a preference relation over a finite set of alternatives and participates in a strategyproof mechanism. We first devise a measure to compare the degree of preference misrepresentation. Given two preference orderings P_i and P'_i , we define the **Kemeny set** of P_i and P'_i as the pairs of alternatives that are not ordered in the same way under these two preferences.⁶ We compare the degree of misrepresentation by comparing Kemeny sets: Given a true preference ordering P_i , an ordering P'_i is defined to be more truthful than P''_i when the Kemeny set of P'_i and P_i is a subset of that of P''_i and P_i . That is, P'_i is more truthful than P''_i when P''_i has relatively more elements whose order disagrees with P_i . In this context, a mechanism is said to be monotone strategyproof if a more truthful strategy always dominates a less truthful one.⁷

⁵See Jackson (1992) for a similar argument in the case of in an exchange economy.

⁶The cardinality of this set is the well-known *Kemeny distance* (Kemeny, 1959).

⁷The equivalent definition for stochastic mechanisms simply replaces dominance with stochastic dominance.

It is straightforward to see that monotone strategyproofness implies strategyproofness. Our main result (Theorem 2) states that the reverse also holds under a mild assumption on the domain of the mechanism. For deterministic social choice functions this equivalence actually holds for *any* environment (Theorem 1).

We compare strategies by comparing their Kemeny sets. A natural question is whether a non-truthful strategy P'_i that dominates another non-truthful strategy P''_i is necessarily closer to the true preferences in the way we define it. In other words, is Kemeny set inclusion equivalent to the dominance relation? It turns out that this equivalence is true for deterministic mechanisms, but not for the general case. For non-deterministic mechanisms we show how one preference ordering may dominate another without Kemeny set inclusion. This observation illustrates the complication added by non-deterministic mechanisms.

Two closely related papers are Carroll (2012) and Sato (2013). Like us, they also compare “large” and “small” misrepresentations, but they address a different question than we do. Both Carroll and Sato characterize conditions under which “local” strategyproofness implies “global” strategyproofness, that is, conditions under which restricting misrepresentations that only switch the ranking of two consecutive alternatives in one’s preferences is enough to characterize strategyproofness. So their concern is more about the transitivity of strategyproofness.⁸ Another related paper is Cho (2014). While considering closely related issues to ours, the analysis in Cho (2014) is constrained by a more restrictive environment. Cho studies probabilistic assignment mechanisms (Carroll, Sato and us consider any social choice mechanism). Cho’s main contribution consists of proposing several ways to compare probabilistic assignments, and he shows that Sato’s result continue to hold with these new notions of assignment comparison. As a by-product, Cho finds equivalence between monotone strategyproofness (that he calls “lie-monotonicity”) and strategyproofness for stochastic mechanisms under specific domain conditions.⁹ This result is covered in our Corollary 1. We obtain results for a more general domain, and also consider deterministic social choice

⁸Another, somewhat less related paper, is Pathak and Sönmez (2012), who also focus on misrepresentation of preferences. However, Pathak and Sönmez are interested in comparing mechanisms—and therefore consider mechanisms that are not strategyproof—while we are interested in comparing misrepresentations under strategyproof mechanisms.

⁹Cho’s definition of lie-monotonicity is more restrictive than our notion of monotone strategyproofness, as he only compares preference orderings that differ only in the relative ranking of two consecutive alternatives. Under his domain restriction this turns out to be equivalent to monotone strategyproofness.

functions and mechanisms with cardinal types.

We outline the environment we consider in Section 2. Monotone strategyproofness is defined and shown to be equivalent to strategyproofness in Section 3. In Section 4 we discuss the relevance of using Kemeny sets to compare strategies and show how similar result can be obtained when agents have cardinal utility functions over outcomes. We conclude in Section 5.

2. PRELIMINARIES

Let N be a set of agents and X a finite set of alternatives. We shall focus in this paper on the incentives from an individual agent's perspective, henceforth called agent i .¹⁰ A preference P_i for agent i over X is a linear order on X . Given a preference relation P_i we denote by R_i the weak ordering associated with P_i , i.e., xR_ix' implies xP_ix' or $x = x'$.¹¹

A **preference profile** is a list P of preferences for each agent $j \in N$, $P = \times_{j \in N} P_j$. We follow the usual convention to denote by P_{-i} the profile $(P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_n)$. The set of all possible preferences, called the **universal domain**, is denoted \mathcal{P} . A **domain** is a non-empty subset of \mathcal{P} , which does not need to be a product of individual domains.

A domain \mathcal{D} is **connected** if, for any two preference orderings $P_i, P'_i \in \mathcal{D}$, there exists a sequence (P^1, \dots, P^ℓ) such that $P^1 = P_i$, $P^\ell = P'_i$ and for each $h < \ell$, P^h and P^{h+1} are adjacent. A sequence of preference orderings (P^1, \dots, P^ℓ) satisfies the **non-restoration property** if whenever for some $x, x' \in X$ and some $h < \ell$ we have xP^hx' and $x'P^{h+1}x$ then it implies that $x'P^{h'}x$ for each $h' > h + 1$.

A **lottery** is a vector of probabilities $\pi \in \mathbb{R}^{|X|}$ such that $\sum_{x \in X} \pi_x = 1$. We denote by $\Delta(X)$ the set of all lotteries over X . A **social choice function** (or a **mechanism**) on a domain \mathcal{D} is a mapping $\varphi : \times_{i \in N} \mathcal{D} \rightarrow \Delta(X)$. Given a profile P , we denote by $\varphi_x(P)$ the probability of alternative x under the lottery $\varphi(P)$. The social choice function is **deterministic** if for each $P \in \mathcal{D}^N$, $\varphi(P)$ is a degenerate lottery. In this case (abusing notation) we shall denote by $\varphi(P_i, P_{-i})$ the alternative x such that $\varphi_x(P_i, P_{-i}) = 1$.

¹⁰Thus, the set of individuals need not be finite nor countable.

¹¹Our model only describes the problem from a unique agent's perspective, so one can view an agent i 's preferences as a preference ordering over i 's *individual outcomes*. This way our model encompasses private good environments like two-sided matching models, where agents are defined as having preferences over potential partners while an outcome is a matching that involves *all* agents.

Given preference orderings P_i, P'_i, P''_i , we say that P'_i (**stochastically**) **dominates** P''_i **with respect to** P_i , denoted $P'_i \gg^{P_i} P''_i$, when

$$\text{for each } P_{-i}, \text{ for each } x \in X, \quad \sum_{x' : x' R_i x} \varphi_{x'}(P'_i, P_{-i}) \geq \sum_{x' : x' R_i x} \varphi_{x'}(P''_i, P_{-i}). \quad (1)$$

For a deterministic social choice function, condition (1) can be rewritten as

$$\text{for each } P_{-i}, \quad \varphi(P'_i, P_{-i}) R_i \varphi(P''_i, P_{-i}). \quad (1')$$

Definition 1 A social function φ is **strategyproof** on a domain \mathcal{D} if for each agent $i \in N$, and for each $P_i, P'_i \in \mathcal{D}$, P_i dominates P'_i with respect to P_i .

Observe that the sets of individuals, the (true) preference profile P , and a social choice function φ on a domain \mathcal{D} induce a strategic form game $\Gamma^\varphi = \langle N, \mathcal{D}, P \rangle$, where N is the set of players, \mathcal{D} is the set of (pure) strategy profiles, the outcome of a strategy profile P is given by $\varphi(P)$, and each player $i \in N$ evaluates the outcome $\varphi(P)$ using his true preferences P_i . In this context, a social choice function φ is strategyproof if in the game form Γ^φ the truthful strategy P_i is a (weakly) dominant strategy for each player i .

3. MONOTONE STRATEGYPROOFNESS

One natural way to compare two preference orderings is by counting the number of pairs of alternatives whose relative rank differ between the two orderings. This method is known as the *Kemeny distance* (Kemeny, 1959). We propose instead to compare preference orderings with what we call the **Kemeny sets** of the preference orderings.

Definition 2 Given two preference orderings P_i, P'_i , the **Kemeny set of P_i and P'_i** is the set of all pairs $(x, x') \in X \times X$ that are not ordered identically in P_i and P'_i ,

$$K(P_i, P'_i) = \{(x, x') \in X \times X : x' P_i x \text{ and } x P'_i x'\}. \quad (2)$$

We are now ready to introduce our main concept:

Definition 3 A social choice function is **monotone strategyproof** on a domain \mathcal{D} if for each $P_i \in \mathcal{D}$ and each pair $P'_i, P''_i \in \mathcal{D}$ such that $K(P'_i, P_i) \subset K(P''_i, P_i)$, P'_i dominates P''_i with respect to P_i .

Note that if a social choice function is monotone strategyproof it is obviously strategyproof. Indeed, $K(P_i, P_i) = \emptyset$ implies that P_i dominates any other preference ordering P'_i . The next theorem states that the converse also holds for deterministic social choice functions on any domain.

Theorem 1 *Let φ be a deterministic social choice function on a domain \mathcal{D} . Then φ is strategyproof if, and only if, it is monotone strategyproof.*

Proof Let P_i, P'_i and P''_i such that $K(P'_i, P_i) \subset K(P''_i, P_i)$ with $P_i, P'_i, P''_i \in \mathcal{D}$. Let P_{-i} be any profile, and let $x' = \varphi(P'_i, P_{-i})$ and $x'' = \varphi(P''_i, P_{-i})$ and assume that $x' \neq x''$.¹²

Observe that if $x'P''_ix''$, then φ cannot be strategyproof. This is because if P''_i were the true preferences, then individual i could benefit by reporting P'_i instead of P''_i . Similarly, it cannot be that $x''P'_ix'$. So it must be that $x'P'_ix''$ and $x''P''_ix'$. Since $K(P'_i, P_i) \subset K(P''_i, P_i)$, we have $x'P_ix''$. That is, $\varphi(P'_i, P_{-i})P_i\varphi(P''_i, P_{-i})$. ■

Observe that Theorem 1 holds for any domain, but only for deterministic social choice functions. If we want to consider non-deterministic social choice functions a result similar to Theorem 1 can be obtained under certain conditions on the domain. Before presenting those conditions some definitions are in order.

For any two preferences P_i and P'_i we first construct the set of connected components of the graph $G(P_i, P'_i) = (X, K(P_i, P'_i))$, where X is the set of vertices and $K(P_i, P'_i)$ is the set of edges. That is, in the graph $G(P_i, P'_i)$ there is an edge between x and x' if $(x, x') \in K(P_i, P'_i)$, i.e., if the relative order of x and x' differ between P_i and P'_i . Two alternatives x and x' are **connected** in $G(P_i, P'_i)$ if there exists a sequence (x_1, \dots, x_k) with $x = x_1, x' = x_k$ such that $(x_h, x_{h+1}) \in K(P_i, P'_i)$ for each $h < k$. A **connected component** is a set of alternatives $C \subseteq X$ such that any two alternatives in C are connected in $G(P_i, P'_i)$ and no alternative in C is connected with an alternative in $X \setminus C$.

For instance, if $K(P_i, P'_i) = \{(x_1, x_2), (x_2, x_4), (x_3, x_5)\}$, then the graph $G(P_i, P'_i)$ has three edges: between x_1 and x_2 , between x_2 and x_4 , and between x_3 and x_5 . The connected components in $G(P_i, P'_i)$ are $\{x_1, x_2, x_4\}$ and $\{x_3, x_5\}$.

For a subset of alternatives $C \subset X$, let P_i **restricted to C** , denoted $P_i|_C$, be a preference ordering defined on C such that for any $x, y \in C$, $xP_i|_C y$ if, and only if $xP_i y$. We say that a

¹²If $\varphi(P'_i, P_{-i}) = \varphi(P''_i, P_{-i})$ for any profile P_{-i} , then P'_i and P''_i are equivalent strategies and thus P'_i trivially dominates P''_i .

preference ordering P'_i is a **complete reversal** of P_i when for any x and y , xP_iy if, and only if yP'_ix .

A domain \mathcal{D} is **weakly connected** if for any two distinct preferences P_i and P'_i there exists a sequence (P^1, \dots, P^k) that satisfies the non-restoration property where $P^1 = P_i$, $P^k = P'_i$ and for each $h < k$, and the graph $G(P_i^h, P_i^{h+1})$ has exactly one connected component C , such that either $|C| \leq 3$, or $P^{h+1}|_C$ is a complete reversal of $P^h|_C$.¹³

Theorem 2 *Let φ be a social choice function on a weakly connected domain \mathcal{D} . Then φ is strategyproof if, and only if, it is monotone strategyproof.*

The proof of Theorem 2 will invoke the Lemmas 1, 2 and 3 that we present below.

Lemma 1 *Let P_i , P'_i and P''_i be preference orderings, and let $C \subset X$ be a unique connected component of the graph $G(P'_i, P''_i)$. If $P'_i|_C \gg^{P_i|_C} P''_i|_C$, then $P'_i \gg^{P_i} P''_i$.*

Proof Since C is the unique connected component, $P'_i|_{X \setminus C} = P''_i|_{X \setminus C}$. Let A be the set of alternatives that are above C in P'_i , $A = \{x : xP'_iy \text{ for all } y \in C\}$. The set C being the unique connected component implies that we also have $A = \{x : xP''_iy \text{ for all } y \in C\}$. Similarly, we can define the set B of alternatives below C , $B = X \setminus (A \cup C) = \{x : yP'_ix \text{ for all } y \in C\} = \{x : yP''_ix \text{ for all } y \in C\}$.

By strategyproofness, $P'_i \gg^{P_i} P''_i$ and $P''_i \gg^{P''_i} P'_i$ imply that, for any P_{-i} and any $x \in A$, $\sum_{yR'_ix} \varphi_y(P'_i, P_{-i}) \geq \sum_{yR'_ix} \varphi_y(P''_i, P_{-i})$ and $\sum_{yR''_ix} \varphi_y(P''_i, P_{-i}) \geq \sum_{yR''_ix} \varphi_y(P'_i, P_{-i})$, respectively. Since $P'_i|_A = P''_i|_A$, for each of the above inequalities both sides must be identical. It follows then that $\varphi_x(P'_i, P_{-i}) = \varphi_x(P''_i, P_{-i})$, for each $x \in A$.

Let x_1 be the highest alternative in $P'_i|_B$ (so x_1 is also the highest alternative ranked in $P''_i|_B$) and let x'_0 and x''_0 be the lowest alternatives ranked in $P'_i|_C$ and $P''_i|_C$, respectively. Clearly, $P'_i \gg^{P_i} P''_i$ implies $\sum_{yR'_ix'_0} \varphi_x(P'_i, P_{-i})$ and $P''_i \gg^{P''_i} P'_i$ implies $\sum_{yR''_ix''_0} \varphi_x(P''_i, P_{-i})$. Observe that $\{x : xR'_ix'_0\} = \{x : xP'_ix_1\} = A \cup C = \{x : xR''_ix''_0\} = \{x : xP''_ix_1\}$. So we have $\sum_{xP'_ix_1} \varphi_x(P'_i, P_{-i}) = \sum_{xP''_ix_1} \varphi_x(P''_i, P_{-i})$. Again using $P'_i \gg^{P_i} P''_i$ and $P''_i \gg^{P''_i} P'_i$ we obtain $\sum_{xR'_ix_1} \varphi_x(P'_i, P_{-i}) = \sum_{xR''_ix_1} \varphi_x(P''_i, P_{-i})$. Therefore, $\varphi_{x_1}(P'_i, P_{-i}) = \varphi_{x_1}(P''_i, P_{-i})$.

¹³We could allow in the definition that two consecutive preference ordering in the sequence, say, P_i^h and P_i^{h+1} are identical, in which case the graph $G(P_i^h, P_i^{h+1})$ would have no connected component. To avoid taking care of those trivial cases we require that along the sequence there is always at least one connected component.

Continuing with the alternatives ranked below x_1 (which are ordred identically in P'_i and P''_i) we then obtain, for each $x \in B$, $\varphi_x(P'_i, P_{-i}) = \varphi_x(P''_i, P_{-i})$.

We now show that $P'_i \gg^{P_i} P''_i$, i.e., for any $x \in X$,

$$\sum_{y:yR_ix} \varphi_y(P'_i, P_{-i}) \geq \sum_{y:yR_ix} \varphi_y(P''_i, P_{-i}). \quad (3)$$

Let x be any alternative in X . Since $\varphi_y(P'_i, P_{-i}) = \varphi_y(P''_i, P_{-i})$ for each $y \notin C$,

$$\sum_{y:yR_ix \& y \notin C} \varphi_y(P'_i, P_{-i}) = \sum_{y:yR_ix \& y \notin C} \varphi_y(P''_i, P_{-i}). \quad (4)$$

Moreover, $P'_i|_C \gg^{P_i|_C} P''_i|_C$ implies

$$\sum_{y:yR_ix \& y \in C} \varphi_y(P'_i, P_{-i}) \geq \sum_{y:yR_ix \& y \in C} \varphi_y(P''_i, P_{-i}) \quad (5)$$

Summing Eqs. (4) and (5) yields (3), the desired result. \blacksquare

Lemma 2 *Suppose P''_i is a complete reversal of P_i . Then for any P'_i , $K(P'_i, P_i) \subseteq K(P''_i, P_i)$, and for any stochastic strategyproof mechanism $P'_i \gg^{P_i} P''_i$.*

Proof Let $P_i = x_1, \dots, x_k$, then $P''_i = x_k, \dots, x_1$. So $K(P_i, P''_i) = \{(x_h, x_{h'}) \in X \times X : h \neq h'\}$, and thus we obviously have $K(P_i, P'_i) \subseteq K(P_i, P''_i)$ for any P'_i . Consider any profile P_{-i} and let π_h, π'_h and π''_h denote the probability of alternative x_h under the strategy P_i, P'_i and P''_i , respectively. We need to show that $P'_i \gg^{P_i} P''_i$, that is,

$$\pi'_1 \geq \pi''_1 \quad (a_1)$$

$$\pi'_1 + \pi'_2 \geq \pi''_1 + \pi''_2 \quad (a_2)$$

...

$$\pi'_1 + \dots + \pi'_{k-1} \geq \pi''_1 + \dots + \pi''_{k-1} \quad (a_{k-1})$$

$$\pi'_1 + \dots + \pi'_{k-1} + \pi'_k \geq \pi''_1 + \dots + \pi''_{k-1} + \pi''_k \quad (a_k)$$

By strategyproofness $P''_i \gg^{P''_i} P'_i$, i.e.,

$$\pi''_k \geq \pi'_k \quad (a'_k)$$

$$\pi''_k + \pi''_{k-1} \geq \pi'_k + \pi'_{k-1} \quad (a'_{k-1})$$

...

$$\pi''_k + \dots + \pi''_2 \geq \pi'_k + \dots + \pi'_2 \quad (a'_2)$$

$$\pi''_k + \dots + \pi''_2 + \pi''_1 \geq \pi'_k + \dots + \pi'_2 + \pi'_1 \quad (a'_1)$$

Since there are k alternatives

$$\sum_h \pi'_h = \sum_h \pi''_h = 1. \quad (6)$$

Note that (a_k) follows directly from (6). For $1 < l \leq k$, observe that Eq. (6) implies that (a'_l) is equivalent to $1 - \sum_{h < l} \pi''_h \geq 1 - \sum_{h < l} \pi'_h \Leftrightarrow \sum_{h < l} \pi'_h \geq \sum_{h < l} \pi''_h$, i.e., Eq. (a_{l-1}) . ■

Lemma 3 *Let $|X| = 3$, and let P_i, P'_i and P''_i such that $K(P'_i, P_i) \subset K(P''_i, P_i)$. For any strategyproof social choice function, $P'_i \gg^{P_i} P''_i$.*

Proof Let $|X| = 3$, and let P_i, P'_i and P''_i such that $K(P'_i, P_i) \subset K(P''_i, P_i)$. Note that $|K(P'_i, P''_i)| = 0$ implies $P'_i = P''_i$ and thus $P'_i \gg^{P_i} P''_i$ trivially holds.

If $|K(P'_i, P''_i)| = 1$, then the graph $G(P'_i, P''_i)$ has exactly one connected component, say, $C = \{x, y\}$, where xP'_iy and yP''_ix . Due to Kemeny set inclusion, xP_iy . Therefore, $P'_i|_C = P_i|_C$ and thus $P'_i|_C \gg^{P_i|_C} P''_i|_C$ is equivalent to $P'_i|_C \gg^{P_i|_C} P''_i|_C$. By Lemma 1, $P'_i \gg^{P_i} P''_i$.

If $|K(P'_i, P''_i)| = 3$ then P''_i is a complete reversal of P'_i . By Kemeny set inclusion, it must be that $P'_i = P_i$. Therefore, P''_i is a complete reversal of P_i , and by Lemma 2, $P'_i \gg^{P_i} P''_i$.

Consider now the case when $|K(P'_i, P''_i)| = 2$. By Kemeny set inclusion, either $|K(P'_i, P_i)| = 0$ or $|K(P'_i, P_i)| = 1$. In the former case, $P'_i = P_i$ and thus by strategyproofness, $P'_i \gg^{P_i} P''_i$. In the latter case, by Kemeny set inclusion $|K(P''_i, P_i)| = 3$, i.e., P''_i is a complete reversal of P_i . Then by Lemma 2, $P'_i \gg^{P_i} P''_i$. ■

Proof of Theorem 2 Let P_i, P'_i and P''_i such that $K(P'_i, P_i) \subseteq K(P''_i, P_i)$. Since the domain is weakly connected, there exist a sequence P_i^1, \dots, P_i^ℓ that satisfies the non-restoration property where $P'_i = P_i^1$, $P''_i = P_i^\ell$, and for each $h < \ell$, the graph $G(P_i^h, P_i^{h+1})$ has only one connected component C , such that either $|C| \leq 3$ or $P_i^{h+1}|_C$ is a complete reversal of $P_i^h|_C$. Notice that the non-restoration property implies that $K(P_i^h, P_i) \subseteq K(P_i^{h+1}, P_i)$. Since the stochastic dominance relation is transitive it is sufficient to show that for any $h < \ell$ we have $P_i^h \gg^{P_i} P_i^{h+1}$.

Let C be the (unique) connected component of $G(P_i^h, P_i^{h+1})$. If $|C| \neq 3$ then $P_i^{h+1}|_C$ is a complete reversal of $P_i^h|_C$ by weak connectedness. Therefore by Lemma 2 implies $P_i^h|_C \gg^{P_i|_C} P_i^{h+1}|_C$. When $|C| = 3$, then by Lemma 3 we have $P_i^h|_C \gg^{P_i|_C} P_i^{h+1}|_C$. Since C is a unique connected component of $G(P_i^h, P_i^{h+1})$ and $P_i^h|_C \gg^{P_i|_C} P_i^{h+1}|_C$, $P_i^h \gg^{P_i} P_i^{h+1}$ follows from Lemma 1. ■

A domain \mathcal{D} is **strongly connected** if for any two distinct preferences P_i and P'_i there exists a sequence (P^1, \dots, P^k) that satisfies the non-restoration property where $P^1 = P_i$, $P^k = P'_i$ and for each $h < k$, and the graph $G(P_i^h, P_i^{h+1})$ has exactly one connected component C such that $|C| = 2$.¹⁴ Sato (2013) showed that the single-peaked domain is strongly connected and Carroll (2012) showed that the (maximal) single-crossing domain is also strongly connected.¹⁵ Note that the universal domain is obviously strongly connected. Clearly, any strongly connected domain is also weakly connected. Therefore, following corollary holds.

Corollary 1 *Let φ be a social choice function on a strongly connected domain \mathcal{D} . Then φ is strategyproof if, and only if, it is monotone strategyproof.*

The next example shows that when the domain is not weakly connected then strategyproofness and monotone strategyproofness are no longer equivalent.

Example 1 Let $X = \{x_1, x_2, x_3, x_4\}$, and let \mathcal{D} be the domain composed of the three preference orderings depicted in Table 1. Note that $K(P'_i, P_i) = \{(x_3, x_4)\}$ and $K(P''_i, P_i) =$

P_i	P'_i	P''_i
x_1	x_1	x_2
x_2	x_2	x_4
x_3	x_4	x_3
x_4	x_3	x_1

Table 1: A domain not strongly connected

$\{(x_3, x_4), (x_1, x_2), (x_1, x_3), (x_1, x_4)\}$, i.e., $K(P'_i, P_i) \subset K(P''_i, P_i)$. However, the domain \mathcal{D} is not weakly connected.

Let φ be a mechanism such that, for any P_{-i} , the probability to obtain alternative $x \in X$ for each of the preferences in \mathcal{D} is given by the Table 2. It can be verified that φ is

¹⁴Sato (2013) calls a strongly connected domain a *connected domain that satisfies the non-restoration property*. The *strongly path-connected domain* defined by Chatterji *et al.* (2013) rests on a similar notion but the notion of connectedness is imposed on alternatives and not preferences.

¹⁵Both single-peakedness and single-crossingness assume the existence of an ordering of alternatives, and admissible preferences are obtained using this ordering. It is of course possible to have a (small) domain that satisfies the properties required by single-peakedness or single-crossingness but does not contain enough preferences such that any pair of preferences in the domain are connected.

	P_i	P'_i	P''_i
x_1	.51	.51	.1
x_2	.3	.3	.7
x_3	.18	.01	.1
x_4	.01	.18	.1

Table 2: Probabilities of each alternative under P_i , P'_i and P''_i

strategyproof, yet it is not monotone strategyproof as $P'_i \gg^{P_i} P''_i$ does not hold. Indeed, $.82 = \sum_{x : xR_ix_3} \varphi_x(P'_i, P_{-i}) < \sum_{x : xR_ix_3} \varphi_x(P''_i, P_{-i}) = .9$

4. DISCUSSION

4.1. Weak preferences

Until now we have only considered the case of strict preference domains. In this section we question whether our results extend to the case of weak preferences. A weak preference relation R_i for agent i over X is a complete, reflexive and transitive binary relation on X . Given a preference relation R_i we denote by P_i and I_i the corresponding strict and indifference preference relation, respectively. That is, xP_ix' if xR_ix' and *not* $x'R_ix$, and xI_ix' if both xR_ix' and $x'R_ix$ hold. We denote by \mathcal{R} the domain of all possible preference profiles over X .

The natural extension of the Kemeny set inclusion for weak preference relations —when comparing two preference orderings with respect to a third one— is the notion of *intermediate preferences* introduced by Grandmont (1978).

Definition 4 R'_i is between R_i and R''_i (noted as $R'_i \in (R_i, R''_i)$) if for all $x, x' \in X$,

- (a) xR_ix' and xR''_ix' imply xR'_ix' .
- (b) xP_ix' and xP''_ix' imply xP'_ix' .
- (c) $(xI_ix'$ and $xP''_ix')$ or $(xP_ix'$ and $xI''_ix')$ imply xR'_ix' .

One easily see that for a triple (P_i, P'_i, P''_i) of strict preferences $K(P'_i, P_i) \subseteq K(P''_i, P_i)$ implies that condition (b) of Definition 4 holds. In this case a natural definition of monotone strategyproofness would be that for any triple of preference relations (R_i, R'_i, R''_i) such that

$R'_i \in (R_i, R''_i)$, it holds that R'_i dominates R''_i with respect to R_i . One could then conjecture that monotone strategyproofness would be equivalent to strategyproofness in this setting. However, there exist situations where this property does not hold. To see this, let $R'_i \in (R_i, R''_i)$ such that for some preference profile R_{-i} we have $\varphi(R'_i, R_{-i}) = x'$ and $\varphi(R''_i, R_{-i}) = x''$, and assume that $x' \neq x''$. Suppose that we have $x'' P_i x'$, $x' I'_i x''$ and $x' I''_i x''$. That is, for the pair (x', x'') we have to consider the second part of condition (c) of Definition 4. It is then easy to see that the pair (x', x'') does not violate the fact that R'_i is between R_i and R''_i , yet we clearly have that R'_i cannot dominate R''_i with respect to R_i .

In other words, the equivalence between strategyproofness and monotone strategyproofness is not assured when considering indifferences. The reason behind it is that when an agent is indifferent between two alternatives, strategyproofness does not impose any particular selection among these two alternatives. That is, we may well have that under two preferences R'_i and R''_i individual i is indifferent between two alternatives, say, x' and x'' , yet x' is strictly preferred to x'' under R_i . Strategyproofness in this case does not impose that alternative x' should be chosen over x'' under either R'_i or R''_i for some preference profile P_{-i} .

4.2. Comparing preferences

Theorems 1 and 2 show that Kemeny set inclusion captures dominance relations between different strategies in a strategyproof mechanism. One natural question to address is whether the converse also holds, i.e., when a preference ordering P'_i dominates another ordering P''_i , is it necessarily the case that $K(P'_i, P_i)$ is a subset of $K(P''_i, P_i)$? In other words one may ask whether the partial order over preferences induced by the Kemeny set relation is the weakest possible order such that the equivalence between monotone strategyproofness and strategyproofness holds.

To investigate this question, first note that we may be limited in the set of alternatives we can compare. To see this, suppose that for some alternatives x and y , and preference orderings P'_i and P''_i , there is no P_{-i} such that $\varphi(P'_i, P_{-i}) = x$ and $\varphi(P''_i, P_{-i}) = y$. If this happens, it is impossible to know how P'_i and P''_i compare those two alternatives, and thus we cannot say anything about Kemeny set inclusion. So when comparing two preference orderings P'_i and P''_i , we can only consider pairs of alternatives x, y such that, for some P_{-i} , $\varphi(P'_i, P_{-i}) = x$ and $\varphi(P''_i, P_{-i}) = y$.

Given a mechanism φ , the **joint range** of two preference orderings P_i and P'_i is the set of

pairs of alternatives (v, v') for which there exists a profile P_{-i} such that $\varphi(P_i, P_{-i}) = v$ and $\varphi(P'_i, P_{-i}) = v'$. The joint range of P_i and P'_i for mechanism φ is denoted $J_\varphi(P_i, P'_i)$.

Definition 5 Given three preference orderings P_i, P'_i and P''_i , the **Kemeny set of P' with respect to P on joint range with P''** is the set of all pairs $(x, x') \in X \times X$ that are not ordered identically in P_i and P'_i and that belong to the joint range of P'_i and P''_i , i.e.,

$$\widehat{K}(P'_i, P_i, P''_i) \equiv K(P'_i, P_i) \cap J_\varphi(P'_i, P''_i). \quad (7)$$

Proposition 1 *Let φ be a deterministic strategyproof social choice function, and let (P_i, P'_i, P''_i) be any triple of preferences. Then P'_i dominates P''_i with respect to P_i if, and only if, $\widehat{K}(P'_i, P_i, P''_i) \subseteq \widehat{K}(P''_i, P_i, P'_i)$.*

Proof The *if* direction is a direct corollary of Theorem 1. Consider the *only if* direction, and let P_i, P'_i and P''_i such that $P'_i \gg^{P_i} P''_i$. We need to show that $\widehat{K}(P'_i, P_i, P''_i) \subseteq \widehat{K}(P''_i, P_i, P'_i)$. So we only need to check the Kemeny set inclusion for the pairs that are in the joint range of P'_i and P''_i . Accordingly, let (x, y) be any pairs of alternatives in $J_\varphi(P'_i, P''_i)$. We then have to show that

$$(x, y) \in K(P'_i, P_i) \quad \Rightarrow \quad (x, y) \in K(P''_i, P_i). \quad (8)$$

Observe that if $(x, y) \notin K(P'_i, P_i)$, then Eq. (8) is trivially satisfied. Suppose then that $(x, y) \in K(P'_i, P_i)$. Note that since $P'_i \gg^{P_i} P''_i$ and $x \neq y$ imply $xP_i y$.¹⁶ If $(x, x') \notin K(P''_i, P_i)$ then $xP''_i y$, which is equivalent to $\varphi(P'_i, P_{-i})P''_i \varphi(P''_i, P_{-i})$. This contradicts strategyproofness, so $yP''_i x$ and thus $(x, y) \in K(P''_i, P_i)$. ■

Kemeny set inclusion (or betweenness) is more restrictive than Kemeny set inclusion on the joint range. The next example illustrates this point with the median voter with single-peaked preferences, where P'_i dominating P''_i with respect to P_i does not imply that $K(P_i, P'_i) \subset K(P_i, P''_i)$.

Example 2 Let φ be the standard median voter social choice function, and let the domain be the single-peaked preference domain. It is well known that for the single-peaked domain the median voter rule is strategyproof (Moulin, 1980). Let L be the order of the alternatives

¹⁶Clearly, if $(x, y) \in J_\varphi(P'_i, P''_i)$ we must have by the definition of the joint range $x \neq y$.

under which the domain is single-peaked. For simplicity, assume here that there is an odd number of individuals.

Let i be an individual and P_i his true preferences where x is the most preferred alternative (the *peak*) according to P_i . Consider now two preference orderings (that are single-peaked under the order L), P'_i and P''_i , where x' and x'' are their respective peaks. Suppose that $xLx'Lx''$. Then P'_i dominates P''_i .¹⁷

Suppose that there exists a pair of alternatives, say x_1 and x_2 , such that $(x_1, x_2) \in K(P'_i, P_i)$ yet $(x_1, x_2) \notin K(P''_i, P_i)$. We claim that $(x_1, x_2) \notin J_\varphi(P'_i, P''_i)$. To see this, without loss of generality suppose that $\varphi(P'_i, P_{-i}) = x_1$ and $\varphi(P''_i, P_{-i}) = x_2$. Consider first the case when x_1Lx' . If x_1Lx' , then by choosing P''_i instead of P'_i individual i cannot change the outcome, so $x_1 = x_2$, a contradiction. So, $x'Lx_1$. Using symmetric argument we obtain x_2Lx'' . So we have $xLx'Lx_1Lx_2Lx''$, which contradicts $(x_1, x_2) \in K(P'_i, P_i)$. If x_2Lx_1 , a similar argument leads to a contradiction, too. It is important to note that $(x_1, x_2) \in K(P'_i, P_i)$ and $(x_1, x_2) \notin K(P''_i, P_i)$ does not contradict single-peakedness. So, even though P'_i dominates P''_i we can still have $K(P'_i, P_i) \not\subseteq K(P''_i, P_i)$. However, it cannot be that $\widehat{K}(P'_i, P_i, P''_i) \not\subseteq \widehat{K}(P''_i, P_i, P'_i)$. \square

The result in Proposition 1 does not hold for non-deterministic social choice functions, as the example below shows.

Example 3 Consider a non-deterministic social choice function φ and following preference profiles, where the numbers in parenthesis are the probabilities that the corresponding alternative obtains for any profile P_{-i} under φ (one can readily verify that in such case φ is strategyproof),

P_i	P'_i	P''_i
x_1 (.5)	x_2 (.5)	x_3 (.5)
x_2 (.3)	x_1 (.3)	x_1 (.2)
x_3 (.2)	x_3 (.2)	x_2 (.3)

It is easy to check that $P'_i \gg^{P_i} P''_i$. Yet we have $K(P'_i, P_i) \not\subseteq K(P''_i, P_i)$.

¹⁷Consider any profile P_{-i} . Observe that if $\varphi(P_i, P_{-i}) = \varphi(P''_i, P_{-i})$ then $\varphi(P'_i, P_{-i}) = \varphi(P_i, P_{-i})$, and thus $\varphi(P'_i, P_{-i})R_i\varphi(P''_i, P_{-i})$. So, assume $\varphi(P_i, P_{-i}) \neq \varphi(P''_i, P_{-i})$. If $\varphi(P'_i, P_{-i}) = \varphi(P_i, P_{-i})$ or $\varphi(P'_i, P_{-i}) = \varphi(P''_i, P_{-i})$ then again $\varphi(P'_i, P_{-i})R_i\varphi(P''_i, P_{-i})$. So, suppose that $\varphi(P_i, P_{-i}) \neq \varphi(P'_i, P_{-i}) \neq \varphi(P''_i, P_{-i})$. This implies that $\varphi(P_i, P_{-i}) = x$, $\varphi(P'_i, P_{-i}) = x'$ and $\varphi(P''_i, P_{-i}) = x''$. So, $\varphi(P'_i, P_{-i})P_i\varphi(P''_i, P_{-i})$.

4.3. Cardinal environments

The concept of monotone strategyproofness can be easily adapted to cardinal environments, i.e., when an individual is characterized by a utility vector over the set of alternatives and individuals have expected utility preferences over lotteries. A few more definitions are needed before going further.

A **type space** is a non-empty subset of $\times_{i \in N} \mathbb{R}^{|X|}$, and an **individual's type** is a vector in $\mathbb{R}^{|X|}$. Given a type space T , a **mechanism** is a mapping $\varphi : T \rightarrow \Delta(X)$. We denote by u_i a generic type of individual i , and $u = (u_i)_{i \in N}$ is a **type profile**. Given a (true) type profile u and a reported type profile u' , the expected utility of individual i is given by the inner product $u_i \cdot \varphi(u)$. A mechanism is **incentive compatible** on a type space T if, for each $i \in N$, for each $u \in T$ and each $u' \in T$ such that $u_{-i} = u'_{-i}$, we have $u_i \cdot (\varphi(u_i, u_{-i}) - \varphi(u'_i, u_{-i})) \geq 0$.

Definition 6 A mechanism is **monotone incentive compatible** on a type space T if, for each $i \in N$, for each $u \in T$ and each $u', u'' \in T$ such that $u_{-i} = u'_{-i} = u''_{-i}$ and $u'_i = \alpha \cdot u_i + (1 - \alpha) \cdot u''_i$ for some $\alpha \in [0, 1]$, we have $u_i \cdot (\varphi(u'_i, u_{-i}) - \varphi(u''_i, u_{-i})) \geq 0$.

We can now introduce the counterpart of Theorem 2 for cardinal mechanisms.¹⁸

Proposition 2 *A mechanism is incentive compatible if, and only if, it is monotone incentive compatible.*

Proof That a monotone incentive compatible mechanism is also incentive compatible is obvious.¹⁹ Consider then an incentive compatible mechanism φ on a type space T . Let u_i be any admissible type for individual i , and u'_i, u''_i such that $u'_i = (1 - \alpha) \cdot u_i + \alpha \cdot u''_i$ for some $\alpha \in [0, 1]$. Note that if $\alpha = 0$ or $\alpha = 1$ then $u'_i = u_i$ or u''_i and thus we trivially have $u_i \cdot (\varphi(u'_i, u_{-i}) - \varphi(u''_i, u_{-i})) \geq 0$. So assume $\alpha \in (0, 1)$. Since φ is incentive compatible,

$$\begin{aligned} u'_i \cdot (\varphi(u'_i, u_{-i}) - \varphi(u''_i, u_{-i})) &\geq 0 \\ u''_i \cdot (\varphi(u''_i, u_{-i}) - \varphi(u'_i, u_{-i})) &\geq 0 \end{aligned}$$

Multiplying the second constraint by α and adding up the two inequalities and rearranging yields

$$(u'_i - \alpha u''_i) \cdot (\varphi(u'_i, u_{-i}) - \varphi(u''_i, u_{-i})) \geq 0$$

¹⁸The proof of Proposition 2 is built on the proof of Proposition 1 in Carroll (2012).

¹⁹Take any u_i and u''_i and set $u'_i = 0 \cdot u_i + (1 - 0) \cdot u''_i$.

Note that $u'_i - \alpha u''_i = (1 - \alpha) \cdot u_i$. Since $\alpha \in (0, 1)$ we obtain $u_i \cdot (\varphi(u'_i, u_{-i}) - \varphi(u''_i, u_{-i})) \geq 0$.

■

A straightforward application of Proposition 2 is for incentive compatible auction mechanisms with private values. Consider the case when individuals' types are real numbers (their value of the good to be auctioned). Our result then simply says that if an individual's private value for the auctioned good is, say, x , then bidding $x' < x$ dominates bidding $x'' < x'$.

5. CONCLUSIONS

We showed that for strategyproof mechanisms one can meaningfully compare the extent of preference misrepresentation by comparing pairs of alternatives. We defined the concept of monotone strategyproofness, which captures the link between incentives and the extent of a misrepresentation: a larger extent of misrepresentation makes the individual (weakly) worse off. Remarkably, requiring monotone strategyproofness does not reduce the set of strategyproof social choice functions. This result shows that imposing strategyproofness (or incentive compatibility) does not only consist of imposing the existence of *one* dominating strategy (the one corresponding to the true type/preferences) but also imposes the existence of a large collection of dominance relations between strategies, thereby providing further evidence that strategyproofness is a very demanding property. The works of Nehring and Puppe (2007) or Barberá, Berga and Moreno (2010) share some similarities with ours in the sense that we all address the question of which additional property, or feature, is implied by strategyproofness. Barberá *et al.* characterize domains of preferences under which any strategyproof social choice function is also strategyproof, and Nehring and Puppe (2007) show that when the domain is a subdomain of generalized single-peaked preferences then any strategyproof social choice function takes the form of voting by issues.²⁰

Our results also shed light on the complications that arise when social choice functions are non-deterministic, or are defined on domains with indifferences. Strategyproofness in the non-deterministic case imposes that the truthful strategy stochastically dominates any other strategy. It is well known that the mere existence of a stochastically dominating strategy

²⁰The similarities between their works and ours stop here. The domains identified by Nehring and Puppe (2007) or Barberá *et al.* (2010) differ significantly from that of weakly connected preferences. Contrary to these two papers, our domain condition never consist of comparing or relating preferences *across* individuals.

can be very challenging in a general setting, so it is not a surprise that one needs to impose some constraints on the domain to obtain the equivalence between strategyproofness and monotone strategyproofness for stochastic mechanisms. As for the case of domains with weak preferences we encounter stronger hurdles. Our discussion in Section 4.1 indeed suggests that obtaining a similar result for the case of weak preferences seems beyond reach.

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